CHAPTER 4

FLIGHT PERFORMANCE

This chapter deals with the performance of rocket-propelled vehicles such as missiles, spacecraft, space launch vehicles, or projectiles. It is intended to give the reader an introduction to the subject from a rocket propulsion point of view. Rocket propulsion systems provide forces to a flight vehicle and cause it to accelerate (or decelerate), overcome drag forces, or change flight direction. They are usually applied to several different flight regimes: (1) flight within the atmosphere (air-to-surface missiles or sounding rockets); (2) near-space environment (earth satellites); (3) lunar and planetary flights; and (4) sun escape; each is discussed further. References 4–1 to 4–4 give background on some of these regimes. The appendices give conversion factors, atmosphere properties, and a summary of key equations. The chapters begins with analysis of simplified idealized flight trajectories, then treats more complex flight path conditions, and discusses various flying vehicles.

4.1. GRAVITY-FREE, DRAG-FREE SPACE FLIGHT

This simple rocket flight analysis applies to an outer space environment, where there is no air (thus no drag) and essentially no significant gravitational attraction. The flight direction is the same as the thrust direction (along the axis of the nozzle), namely, a one-dimensional, straight-line acceleration path; the propellant mass flow $\dot{m}$, and thus the thrust F, remain constant for the propellant burning duration t_p. For a constant propellant flow the flow rate is m_p/t_p, where m_p is the total usable propellant mass. From Newton's second law and for an instantaneous vehicle mass m and a vehicle velocity u.

$$F = m\,du/dt \tag{4–1}$$

For a rocket where the propellant flow rate is constant the instantaneous mass of the vehicle m can be expressed as a function of the initial mass of the full vehicle m_0, m_p, t_p, and the instantaneous time t.

$$m = m_0 - \frac{m_p}{t_p}t = m_0\left(1 - \frac{m_p}{m_0}\frac{t}{t_p}\right) \tag{4–2}$$

$$= m_0\left(1 - \zeta\frac{t}{t_p}\right) = m_0\left[1 - (1 - \mathbf{MR})\frac{t}{t_p}\right] \tag{4–3}$$

Equation 4–3 expresses the vehicle mass in a form useful for trajectory calculations. The vehicle mass ratio **MR** and the propellant mass fraction ζ have been defined by Eqs. 2–7 and 2–8. They are related by

$$\zeta = 1 - \mathbf{MR} \tag{4–4}$$

A definition of the various masses is shown in Fig. 4–1. The initial mass at takeoff m_0 equals the sum of the useful propellant mass m_p plus the empty or final vehicle mass m_f; m_f in turn equals the sum of the inert masses of the engine system (such as nozzles, tanks, cases, or unused, residual propellant), plus the guidance, control, electronics, and related equipment, and the payload.

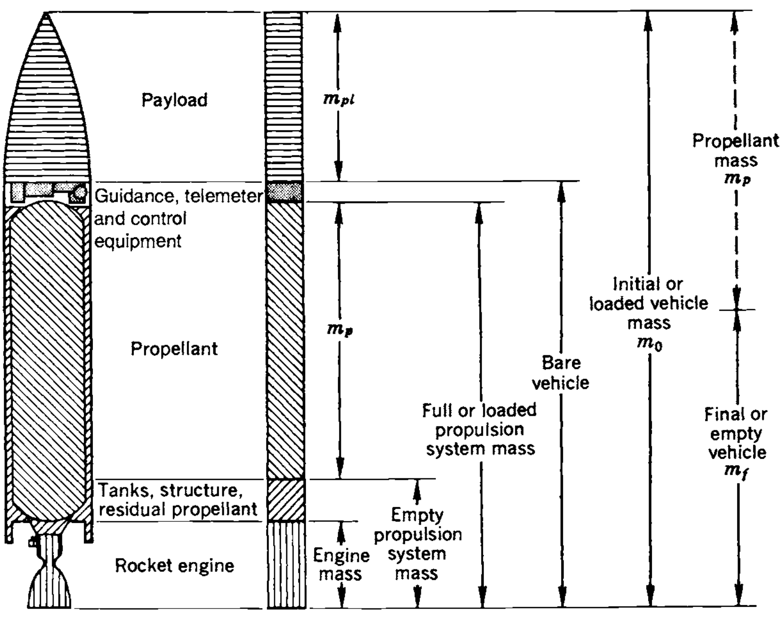


FIGURE 4–1. Definitions of various vehicle masses.

For constant propellant flow $\dot{m}$ and a finite propellant burning time the total propellant mass m_p is $\dot{m}t_p$ and the instantaneous vehicle mass $m = m_0 - \dot{m}t$. Equation 4–1 can be written as

$$\begin{aligned} du &= (F/m)dt = (c\dot{m}/m)\,dt \\ &= \frac{(c\dot{m})\,dt}{m_0 - m_p t/t_p} = \frac{c(m_p/t_p)\,dt}{m_0(1 - m_p t/m_0 t_p)} = \frac{c\zeta/t_p}{1 - \zeta t/t_p}\,dt \end{aligned}$$

Integration leads to the maximum vehicle velocity at propellant burnout u_p that can be attained in a gravity-free vacuum. When $u_0 \neq 0$ it is often called the velocity increment Δu.

$$\Delta u = -c\ \ln(1 - \zeta) + u_0 = c\ln(m_0/m_f) + u_0 \tag{4–5}$$

If the initial velocity u_0 is assumed to be zero, then

$$\begin{aligned} u_p &= \Delta u = -c\ln(1 - \zeta) = -c\ln[m_0/(m_0 - m_p)] \\ &= -c\ln \mathbf{MR} = c\ln(1/\mathbf{MR}) \\ &= c\ln(m_0/m_f) \end{aligned} \tag{4–6}$$

This is the maximum velocity increment Δu that can be obtained in a gravity-free vacuum with constant propellant flow, starting from rest with $u_0 = 0$. The effect of variations in c, I_s, and ζ on the flight velocity increment are shown in Fig. 4–2. An alternate way to write Eq. 4–6 uses e, the base of the natural logarithm.

$$e^{\Delta u/c} = 1/\mathbf{MR} = m_0/m_f \tag{4–7}$$

The concept of the maximum attainable flight velocity increment Δu in a gravity-free vacuum is useful in understanding the influence of the basic parameters. It is used in comparing one propulsion system or vehicle with another, one flight mission with another, or one proposed upgrade with another possible design improvement.

From Eq. 4–6 it can be seen that *propellant mass fraction* has a logarithmic effect on the vehicle velocity. By increasing this ratio from 0.80 to 0.90, the interplanetary maximum vehicle velocity in gravitationless vacuum is increased by 43%. A mass fraction of 0.80 would indicate that only 20% of the total vehicle mass is available for structure, skin, payload, propulsion hardware, radios, guidance system, aerodynamic lifting surfaces, and so on; the remaining 80% is useful propellant. It requires careful design to exceed 0.85; mass fraction ratios approaching 0.95 appear to be the probable practical limit for single-stage vehicles and currently known materials. When the mass fraction is 0.90, then $\mathbf{MR} = 0.1$ and $1/\mathbf{MR} = 10.0$. This marked influence of mass fraction or mass ratio on the velocity at power cutoff, and therefore also the range,

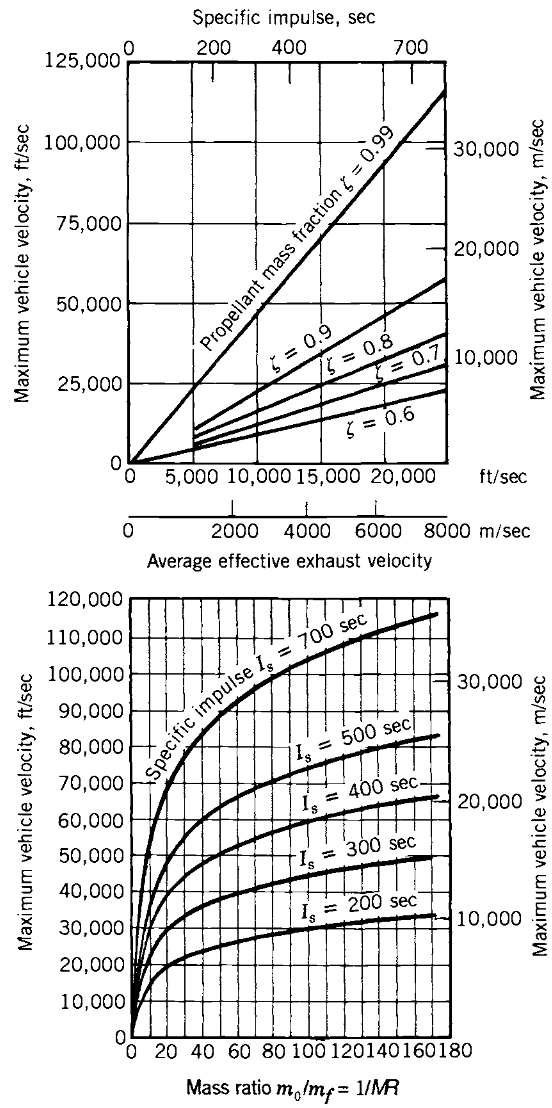


FIGURE 4–2. Maximum vehicle velocity in a gravitationless, drag-free space for different mass ratios and specific impulses (plot of Eq. 4–6). Single-state vehicles can have values of 1/**MR** up to about 20 and multistage vehicles can exceed 200.

not only is true of interplanetary spaceships in a vacuum but applies to almost all types of rocket-powered vehicles. For this reason, importance is placed on saving inert mass on every vehicle component, including the propulsion system.

Equation 4–6 can be modified and solved for the effective propellant mass m_p required to achieve a desired velocity increment for a given initial takeoff mass or a final burnout mass of the vehicle. The final mass consists of the payload, the structural mass of the vehicle, the empty propulsion system mass (which includes residual propellant), plus a small additional mass for guidance, communications, and control devices. Here $m_p = m_0 - m_f$.

$$m_p = m_f(e^{\Delta u/c} - 1) = m_0(1 - e^{(-\Delta u/c)}) \quad (4\text{–}8)$$

The flight velocity increment u_p is proportional to the effective exhaust velocity c and, therefore, to the specific impulse. Thus any improvement in I_s (such as better propellants, more favorable nozzle area ratio, or higher chamber pressure) reflects itself in improved vehicle performance, provided that such an improvement does not also cause an excessive increase in rocket propulsion system inert mass, which causes a decrease in the effective propellant fraction.

4.2. FORCES ACTING ON A VEHICLE IN THE ATMOSPHERE

The external forces commonly acting on vehicles flying in the earth's atmosphere are thrust, aerodynamic forces, and gravitational attractions. Other forces, such as wind or solar radiation pressure, are small and generally can be neglected for many simple calculations.

The *thrust* is the force produced by the power plant, such as a propeller or a rocket. It usually acts in the direction of the axis of the power plant, that is, along the propeller shaft axis or the rocket nozzle axis. The thrust force of a rocket with constant mass flow has been expressed by Eq. 2–6 as a function of the effective exhaust velocity c and the propellant flow rate $\dot{m}$. In many rockets the mass rate of propellant consumption $\dot{m}$ is essentially constant, and the starting and stopping transients are usually very short and can be neglected. Therefore, the thrust is

$$F = c\dot{m} = cm_p/t_p \quad (4\text{–}9)$$

As explained in Chapter 3, for a given propellant the value of the effective exhaust velocity c or specific impulse I_s depends on the nozzle area ratio and the altitude. The value of c can increase by a relatively small factor of between 1.2 and 1.6 as altitude is increased.

The *drag D* is the *aerodynamic force* in a direction opposite to the flight path due to the resistance of the body to motion in a fluid. The *lift L* is the aerodynamic force acting in a direction normal to the flight path. They are expressed as functions of the flight speed u, the mass density of the fluid in which the vehicle moves ρ, and a typical surface area A.

$$L = C_L \tfrac{1}{2} \rho A u^2 \quad (4\text{–}10)$$

$$D = C_D \tfrac{1}{2} \rho A u^2 \quad (4\text{–}11)$$

C_L and C_D are lift and drag coefficients, respectively. For airplanes and winged missiles the area A is understood to mean the wing area. For wingless missiles or space launch vehicles it is the maximum cross-sectional area normal to the missile axis. The lift and drag coefficients are primarily functions of the vehicle configuration, flight Mach number, and angle of attack, which is the angle between the vehicle axis (or the wing plane) and the flight direction. For low flight speeds the effect of Mach number may be neglected, and the drag and lift coefficients are functions of the angle of attack. The variation of the drag and lift coefficients for a typical supersonic missile is shown in Fig. 4–3. The values of these coefficients reach a maximum near a Mach number of unity. For wingless vehicles the angle of attack α is usually very small ($0 < \alpha < 1°$). The density and other properties of the atmosphere are listed in Appendix 2. The density of the earth's atmosphere can vary by a factor up to two (for altitudes of 300 to 1200 km) depending on solar activity and night-to-day temperature variations. This introduces a major unknown in the drag. The aerodynamic forces are affected by the flow and pressure distribution of the rocket exhaust gases, as explained in Chapter 18.

For space launch vehicles and ballistic missiles the drag loss, when expressed in terms of Δu, is typically 5 to 10% of the final vehicle velocity increment. This relatively low value is due to the fact that the air density is low at high altitudes, when the velocity is high, and at low altitudes the air density is high but the flight velocity and thus the dynamic pressure are low.

Gravitational attraction is exerted upon a flying space vehicle by all planets, stars, the moon, and the sun. Gravity forces pull the vehicle in the direction of the center of mass of the attracting body. Within the immediate vicinity of the earth, the attraction of other planets and bodies is negligibly small compared to the earth's gravitational force. This force is the *weight*.

If the variation of gravity with the geographical features and the oblate shape of the earth are neglected, the acceleration of gravity varies inversely as the square of the distance from the earth's center. If R_0 is the radius of the earth's surface and g_0 the acceleration on the earth's surface at the earth's effective radius R_0, the gravitational attraction g is

$$\begin{aligned} g &= g_0(R_0/R)^2 \\ &= g_0[R_0/(R_0 + h)]^2 \end{aligned} \tag{4–12}$$

where h is the altitude. At the equator the earth's radius is 6378.388 km and the standard value of g_0 is 9.80665 m/sec^2. At a distance as far away as the moon, the earth's gravity acceleration is only about $3.3 \times 10^{-4}\ g_0$.

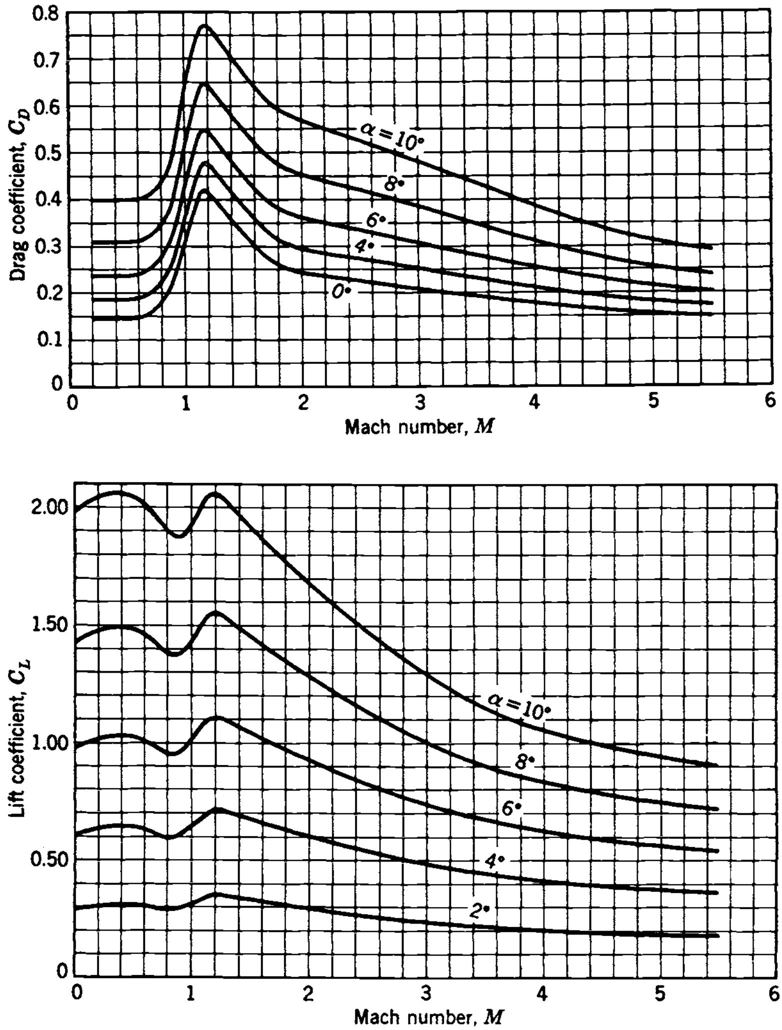


FIGURE 4–3. Variation of lift and drag coefficient with Mach number of the German V-2 missile based on body cross-sectional area with jet off and without exhaust plume effects at several angles of attack α.

4.3. BASIC RELATIONS OF MOTION

For a vehicle that flies within the proximity of the earth, the gravitational attraction of all other heavenly bodies may usually be neglected. Let it be assumed that the vehicle is moving in rectilinear equilibrium flight and that all control forces, lateral forces, and moments that tend to turn the vehicle are zero. The trajectory is two-dimensional and is contained in a fixed plane. The vehicle has wings that are inclined to the flight path at an angle of attack α and that give a lift in a direction normal to the flight path. The direction of flight does not coincide with the direction of thrust. Figure 4–4 shows these conditions schematically.

Let θ be the angle of the flight path with the horizontal and ψ the angle of the direction of thrust with the horizontal. In the direction of the flight path the product of the mass and the acceleration has to equal the sum of all forces, namely the propulsive, aerodynamic, and gravitational forces:

$$m(du/dt) = F\cos(\psi - \theta) - D - mg\sin\theta \tag{4–13}$$

The acceleration perpendicular to the flight path is $u(d\theta/dt)$; for a constant value of u and the instantaneous radius R of the flight path it is u^2/R. The equation of motion in a direction normal to the flight velocity is

$$mu(d\theta/dt) = F\ \sin(\psi - \theta) + L - mg\cos\theta \tag{4–14}$$

By substituting from Equations 4–10 and 4–11, these two basic equations can be solved for the accelerations as

$$\frac{du}{dt} = \frac{F}{m}\cos(\psi - \theta) - \frac{C_D}{2m}\rho u^2 A - g\sin\theta \tag{4–15}$$

$$u\frac{d\theta}{dt} = \frac{F}{m}\sin(\psi - \theta) + \frac{C_L}{2m}\rho u^2 A - g\cos\theta \tag{4–16}$$

No general solution can be given to these equations, since t_p, u, C_D, C_L, p, θ, or ψ can vary independently with time, mission profile, or altitude. Also, C_D and C_L are functions of velocity or Mach number. In a more sophisticated analysis other factors may be considered, such as the propellant used for nonpropulsive purposes (e.g., attitude control or flight stability). See Refs. 4–1 to 4–5 for a

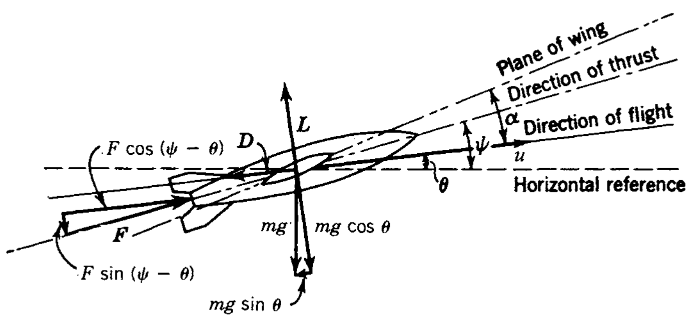


FIGURE 4–4. Two-dimensional free-body force diagram for flying vehicle with wings and fins.

background of flight performance in some of the flight regimes. Different flight performance parameters are maximized or optimized for different rocket flight missions or flight regimes, such as Δu, range, time-to-target, or altitude. Rocket propulsion systems are usually tailored to fit specific flight missions.

Equations 4–15 and 4–16 are general and can be further simplified for various special applications, as shown in subsequent sections. Results of such iterative calculations of velocity, altitude, or range using the above two basic equations often are adequate for rough design estimates. For actual trajectory analyses, navigation computation, space-flight path determination, or missile-firing tables, this two-dimensional simplified theory does not permit sufficiently accurate results. The perturbation effects, such as those listed in Section 4.6 of this chapter, must then be considered in addition to drag and gravity, and digital computers are necessary to handle the complex relations. An arbitrary division of the trajectory into small elements and a step-by-step or numerical integration to define a trajectory are usually indicated. The more generalized three-body theory includes the gravitational attraction among three masses (for example, the earth, the moon, and the space vehicle) and is considered necessary for many space-flight problems (see Refs. 4–2 and 4–3). When the propellant flow and the thrust are not constant, the form and the solution to the equations above become more complex.

A form of Eqs. 4–15 and 4–16 can also be used to determine the actual thrust or actual specific impulse during actual vehicle flights from accurately observed trajectory data, such as from optical or radar tracking data. The vehicle acceleration (du/dt) is essentially proportional to the net thrust and, by making an assumption or measurement on the propellant flow (which usually varies in a predetermined manner) and an analysis of aerodynamic forces, it is possible to determine the rocket propulsion system's actual thrust under flight conditions.

When integrating Eqs. 4–15 and 4–16 one can obtain actual histories of velocities and distances traveled and thus complete trajectories. The more general case requires six equations; three for translation along each of three perpendicular axes and three for rotation about these axes. The choice of coordinate systems and the reference points can simplify the mathematical solutions (see Refs. 4–2 and 4–4).

For a wingless rocket projectile, a space launch vehicle, or a missile with constant thrust and propellant flow, these equations can be simplified. In Fig. 4–5 the flight direction θ is the same as the thrust direction and lift forces for a symmetrical, wingless, stably flying vehicle can be assumed to be zero of zero angle of attack. For a two-dimensional trajectory in a single plane (no wind forces) and a stationary earth, the acceleration in the direction of flight is as follows:

$$\frac{du}{dt} = \frac{c\zeta/t_p}{1 - \zeta t/t_p} - g\sin\theta - \frac{C_D \frac{1}{2}\rho u^2 A/m_0}{1 - \zeta t/t_p} \tag{4–17}$$

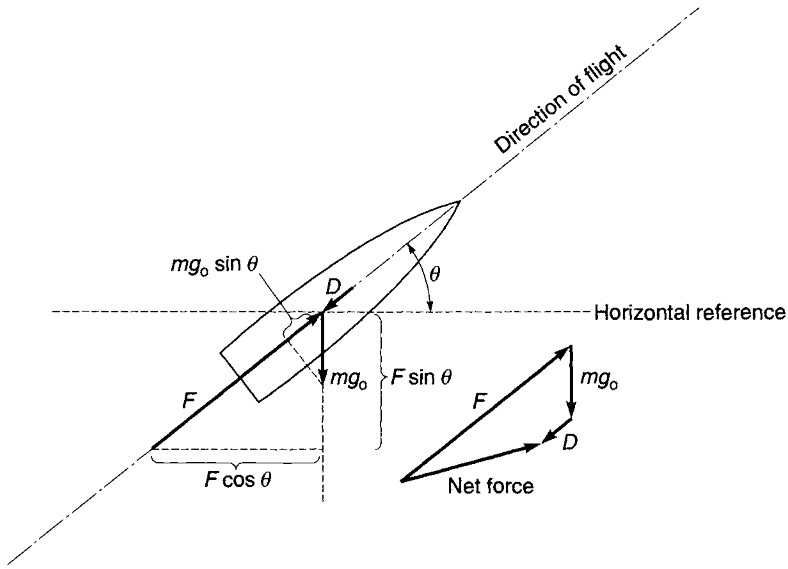


FIGURE 4–5. Simplified free-body force diagram for vehicle without wings or fins. The force vector diagram shows the net force on the vehicle.

A force vector diagram in Fig. 4–5 shows the net force (by adding thrust, drag and gravity vectors) to be at an angle to the flight path, which will be curved. These types of diagram form the basis for iterative trajectory numerical solutions.

The relationships in this Section 4.3 are for a two-dimensional flight path, one that lies in a single plane. If maneuvers out of that plane are also made (e.g., due to solar attraction, thrust misalignment, or wind) then the flight paths become three-dimensional and another set of equations will be needed to describe these flights. Reference 4–1 describes equations for the motion of rocket projectiles in the atmosphere in three dimensions. It requires energy and forces to push a vehicle out of its flight plane. Trajectories have to be calculated accurately in order to reach the intended flight objective and today almost all are done with the aid of a computer. A good number of *computer programs* for analyzing flight trajectories exit and are maintained by aerospace companies or Government agencies. Some are two-dimensional, relatively simple, and are used for making preliminary estimates or comparisons of alternative flight paths, alternative vehicle designs, or alternative propulsion schemes. Several use a stationary flat earth, while others use a rotating curved earth. Three-dimensional programs also exit, are used for more accurate flight path analyses, include some or all perturbations, orbit plane changes, or flying at angles of attack. As explained in Ref. 4–3, they are more complex.

If the flight trajectory is vertical (as for a sounding rocket), Eq. 4–17 is the same, except that $\sin\theta = 1.0$, namely

$$\frac{du}{dt} = \frac{c\zeta/t_p}{1 - \zeta t/t_p} - g - \frac{C_D \frac{1}{2}\rho u^2 A/m_0}{1 - \zeta t/t_p} \tag{4–18}$$

The velocity at the end of burning can be found by integrating between the limits of $t = 0$ and $t = t_p$ when $u = u_0$ and $u = u_p$. The first two terms can readily be integrated. The last term is of significance only if the vehicle spends a considerable portion of its time within the atmosphere. It can be integrated graphically or by numerical methods, and its value can be designated as BC_DA/m_0 such that

$$B = \int_0^{t_p} \frac{\frac{1}{2}\rho u^2}{1 - \zeta t/t_p} dt$$

The cutoff velocity or velocity at the end of propellant burning u_p is then

$$u_p = -\bar{c}\ln(1 - \zeta) - \bar{g}t_p - \frac{BC_DA}{m_0} + u_0 \tag{4–19}$$

where u_0 is the initial velocity, such as may be given by a booster, $\bar{g}$ is an average gravitational attraction evaluated with respect to time and altitude from Eq. 4–12, and $\bar{c}$ is a time average of the effective exhaust velocity, which is a function of altitude.

There are always a number of trade-offs in selecting the best trajectory for a rocket projectile. For example, there is a trade-off between burning time, drag, payload, maximum velocity, and maximum altitude (or range). Reference 4–6 describes the trade-offs between payload, maximum altitude, and flight stability for a sounding rocket.

If aerodynamic forces outside the earth's atmosphere are neglected (operate in a vacuum) and no booster or means for attaining an initial velocity ($u_0 = 0$) is assumed, the velocity at the end of the burning reached in a vertically ascending trajectory will be

$$\begin{aligned} u_p &= -\bar{c}\ln(1 - \zeta) - \bar{g}t_p \\ &= -\bar{c}\ln \mathbf{MR} - \bar{g}t_p \\ &= \bar{c}\ln(1/\mathbf{MR}) - \bar{g}t_p \end{aligned} \tag{4–20}$$

The first term is usually the largest and is identical to Eq. 4–6. It is directly proportional to the effective rocket exhaust velocity and is very sensitive to changes in the mass ratio. The second term is always negative during ascent, but its magnitude is small if the burning time t_p is short or if the flight takes place in high orbits or in space where $\bar{g}$ is comparatively small.

For a flight that is not following a vertical path, the gravity loss is a function of the angle between the flight direction and the local horizontal; more specifically, the gravity loss is the integral of $g \sin\theta\, dt$, as shown by Eq. 4–15.

For the simplified two-dimensional case the net acceleration a for vertical takeoff at sea level is

$$a = (F_0 g_0 / w_0) - g_0 \tag{4–21}$$

$$a/g_0 = (F_0/w_0) - 1 \tag{4–22}$$

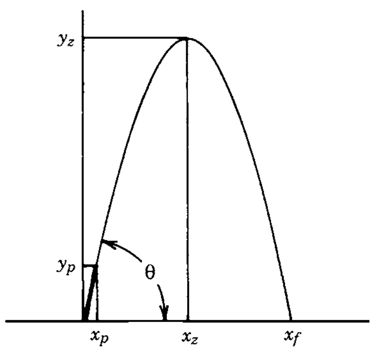
where a/g_0 is the *initial takeoff acceleration* in multiples of the sea level gravitational acceleration g_0, and F_0/w_0 is the thrust-to-weight ratio at takeoff. For large surface-launched vehicles, this initial-thrust-to-initial-weight ratio has values between 1.2 and 2.2; for small missiles (air-to-air, air-to-surface, and surface-to-air types) this ratio is usually larger, sometimes even as high as 50 or 100. *The final* or *terminal acceleration* a_f of a vehicle in vertical ascent usually occurs just before the rocket engine is shut off and before the propellant is completely consumed.

$$a_f/g_0 = (F_f/w_f) - 1 \tag{4–23}$$

In a gravity-free environment this equation becomes $a_f/g_0 = F_f/w_f$. In rockets with constant propellant flow the final acceleration is usually also the maximum acceleration, because the vehicle mass to be accelerated has its minimum value just before propellant exhaustion, and for ascending rockets the thrust usually increases with altitude. If this terminal acceleration is too large (and causes overstressing of the structure, thus necessitating an increase in structure mass), then the thrust can be designed to a lower value for the last portion of the burning period.

Example 4–1. A simple single-stage rocket for a rescue flare has the following characteristics and its flight path nomenclature is shown in the sketch.

Launch weight	4.0 lbf
Useful propellant mass	0.4 lbm
Effective specific impulse	120 sec
Launch angle (relative to horizontal)	80°
Burn time (with constant thrust)	1.0 sec



Drag is to be neglected, since the flight velocities are low. Assume no wind. Assume the local acceleration of gravity to be equal to the sea level g_0 and invariant throughout the flight.

Solve for the initial and final acceleration of powered flight, the maximum trajectory height, the time to reach maximum height, the range or distance to impact, and the angle at propulsion cutoff and at impact.

SOLUTION. Divide the flight path into three portions: the powered flight for 1 sec, the unpowered ascent after cutoff, and the free-fall descent. The thrust is obtained from Eq. 2–5:

$$F = I_s w/t_p = 120 \times 0.4/1 = 48 \text{ lbf}$$

The initial accelerations along the x and y directions are, from Eq. 4.22,

$$(a_0)_y = g_0[(F \sin\theta/w) - 1] = 32.2[(48/4)\sin 80^\circ - 1] = 348 \text{ ft/sec}^2$$

$$(a_0)_x = g_0(F/w)\cos\theta = 32.2(48/4)\cos 80^\circ = 67.1 \text{ ft/sec}^2$$

The initial acceleration in the flight direction is

$$a_0 = \sqrt{(a_0)_x^2 + (a_0)_y^2} = 354.4 \text{ ft/sec}^2$$

The direction of thrust and the flight path are the same. The vertical and horizontal components of the velocity u_p at the end of powered flight is obtained from Eq. 4–20. The vehicle mass has been diminished by the propellant that has been consumed.

$$(u_p)_y = c \ln(w_0/w_f)\sin\theta - g_0 t_p = 32.2 \times 120 \ln(4/3.6)0.984 - 32.2 = 375 \text{ ft/sec}$$

$$(u_p)_x = c \ln(w_0/w_f)\cos\theta = 32.2 \times 120 \ln(4/3.6)0.1736 = 70.7 \text{ ft/sec}$$

The trajectory angle with the horizontal at rocket cutoff for a dragless flight is

$$\tan^{-1}(375/70.7) = 79.3^\circ$$

Final acceleration is $a_f = Fg_0/w = 48 \times 32.2/3.6 = 429$ ft/sec^2. For the short duration of the powered flight the coordinates at propulsion burnout y_p and x_p can be calculated approximately by using an average velocity (50% of maximum) for the powered flight.

$$y_p = \tfrac{1}{2}(u_p)_y t_p = \tfrac{1}{2} \times 375 \times 1.0 = 187.5 \text{ ft}$$

$$x_p = \tfrac{1}{2}(u_p)_x t_p = \tfrac{1}{2} \times 70.7 \times 1.0 = 35.3 \text{ ft}$$

The unpowered part of the trajectory has a zero vertical velocity at its zenith. The initial velocities, the x and y values for this parabolic trajectory segment, are those of propulsion termination ($F = 0, u = u_p, x = x_p, y = y_p$); at the zenith $(u_y)_z = 0$.

$$(u_y)_z = 0 = -g_0(t_z - t_p) + (u_p)_y \sin\theta$$

At this zenith $\sin\theta = 1.0$. Solving for t_z yields

$$t_z = t_p + (u_p)_y/g_0 = 1 + 375/32.2 = 12.6 \text{ sec}$$

The trajectory maximum height or zenith can be determined:

$$\begin{aligned} y_z &= y_p + (u_p)_y(t_z - t_p) - \tfrac{1}{2}g_0(t_z - t_p)^2 \\ &= 187.5 + 375(11.6) - \tfrac{1}{2}32.2(11.6)^2 = 2370 \text{ ft} \end{aligned}$$

The range during ascent to the zenith point is

$$\begin{aligned} x_z &= (u_p)_x(t_z - t_p) + x_p \\ &= 70.7 \times 11.6 + 35.3 = 855 \text{ ft} \end{aligned}$$

The time of flight for the descent is, using $y_z = \frac{1}{2}g_0 t^2$,

$$t = \sqrt{2y_z/g_0} = \sqrt{2 \times 2370/32.2} = 12.1 \text{ sec}$$

The final range or x distance to the impact point is found by knowing that the initial horizontal velocity at the zenith $(u_z)_x$ is the same as the horizontal velocity at propulsion termination $(u_p)_x$:

$$x_f = (u_p)_x(t_{\text{descent}}) = 70.7 \times 12.1 = 855 \text{ ft}$$

The total range for ascent and descent is 855 + 855 = 1710. The time to impact is 12.6 + 12.1 = −24.7 sec. The vertical component of the impact or final velocity u_f is

$$u_f = g_0(t_f - t_z) = 32.2 \times 12.1 = 389.6 \text{ ft/sec}$$

The impact angle θ_f can be found:

$$\theta_f = \tan^{-1}(389.6/70.7) = 79.7^\circ$$

If drag had been included, it would have required an iterative solution for finite elements of the flight path and all velocities and distances would be somewhat lower in value. A set of flight trajectories for a sounding rocket is given in Ref. 4–5.

4.4. EFFECT OF PROPULSION SYSTEM ON VEHICLE PERFORMANCE

This section gives several methods for improving flight vehicle performance. Most of these enhancements, listed below, are directly influenced by the selection or design of the propulsion system. A few of the flight vehicle performance improvements do not depend on the propulsion system. Most of those listed below apply to all missions, but some are peculiar to some missions only.

1. The *effective exhaust velocity* c or the *specific impulse* I_s usually have a direct effect on the vehicle's flight performance. For example the vehicle final velocity increment Δu can be inceased by a higher I_s. This can be done by using a more energetic propellant (see Chapter 7 and 12), by a higher chamber pressure and, for upper stages operating at high altitudes, also by a larger nozzle area ratio.
2. The mass ratio m_0/m_f has a logarithmic effect. It can be increased in several ways. One way is by reducing the final mass m_f, which consists of the inert hardware plus the nonusable, residual propellant mass. Reducing the inert mass implies lighter structures, smaller payloads, lighter guidance/control devices, or less unavailable residual propellant; this means going to stronger structural materials at higher stresses, more efficient power supplies, or smaller electronic packages. During design there is always great emphasis to reduce all hardware masses and the residual propellants to their practical minima. Another way is to increase the initial mass, namely by increasing the thrust and adding more propellant, but with a minimum increase in the structure or propulsion system masses. It is possible to improve the effective mass ratio greatly by using two or more stages, as will be explained in Section 4.7.
3. Reducing the burning time (i.e., increasing the thrust level) will reduce the gravitational loss. However, the higher acceleration usually requires more structural and propulsion system mass, which in turn causes the mass ratio to be less favorable.
4. The *drag*, which can be considered as a negative thrust, can be reduced in at least four ways. The drag has several components: (a) The form drag depends on the aerodynamic shape. A slender pointed nose or sharp, thin leading edges of fins or wings have less drag than a stubby, blunt shape. (b) A vehicle with a small cross-sectional area has less drag. A propulsion design that can be packaged in a long, thin shape will be preferred. (c) The drag is proportional to the cross-sectional or frontal vehicle area. A higher propellant density will decrease the propellant volume and therefore will allow a smaller cross section. (d) The skin drag is caused by the friction of the air flowing over all the vehicle's outer surfaces. A smooth contour and a polished surface are usually better. The skin drag is also influenced by the propellant density, because it gives a smaller volume and thus a lower surface area. (e) The base drag is the fourth component; it is a function of the local ambient air pressure acting over the surface of the vehicle's base or bottom plate. It is influenced by the nozzle exit design (exit pressure) and the geometry of the vehicle base design. It is discussed further in Chapter 18.
5. The length of the propulsion nozzle often is a significant part of the overall vehicle or stage length. As was described in Chapter 3, there is an optimum nozzle contour and length, which can be determined by

trade-off analysis. A shorter nozzle length allows a somewhat shorter vehicle; on many designs this implies a somewhat lighter vehicle structure and a slightly better vehicle mass ratio.

6. The final vehicle velocity at propulsion termination can be increased by increasing the initial velocity u_0. By launching a satellite in an eastward direction the rotational speed of the earth is added to the final satellite orbital velocity. This tangential velocity of the earth is about 464 m/sec or 1523 ft/sec at the equator and about 408 m/sec or 1340 ft/sec for an easterly launch at Kennedy Space Center (latitude of 28.5° north). Conversely, a westerly satellite launch has a negative initial velocity and thus requires a higher-velocity increment. Another way to increase u is to launch a spacecraft from a satellite or an aircraft, which increases the initial vehicle velocity and allows launching in the desired direction, or to launch an air-to-surface missile from an airplane.
7. For vehicles that fly in the atmosphere it is possible to increase the range when aerodynamic lift is used to counteract gravity and reduce gravity losses. Using a set of wings or flying at an angle of attack increases the lift, but is also increases the drag. This lift can also be used to increase the maneuverability and trajectory flexibility.
8. When the flight velocity u is close to the rocket's effective exhaust velocity c, the propulsive efficiency is the highest (Eq. 2–23) and more of the rocket exhaust gas energy is transformed into the vehicle's flight energy. Trajectories where u is close in value to c for a major portion of the flight therefore need less propellant.

Several of these influencing parameters can be optimized. Therefore, for every mission of flight application there is an optimum propulsion system design and the propulsion parameters that define the optimum condition are dependent on vehicle or flight parameters.

4.5. SPACE FLIGHT

Newton's law of gravitation defines the attraction of gravitational force F_g between two bodies in space as follows:

$$F_g = Gm_1 m_2 / R^2 = \mu m_2 / R^2 \qquad (4\text{–}24)$$

Here G is the universal gravity constant ($G = 6.670 \times 10^{-11}$ m^3/ kg-sec^2), m_1 and m_2 are the masses of the two attracting bodies (such as the earth and the moon, the earth and a spacecraft, or the sun and a planet) and R is the distance between their centers of mass. The earth's gravitational constant μ is the product of Newton's universal constant G and the mass of the earth m_1 (5.974×10^{24} kg). It is $\mu = 3.98600 \times 10^{14}$ m^3/sec^2.

The rocket offers a means for escaping the earth for lunar and interplanetary travel, for escaping our solar system, and for creating a stationary or moving station in space. The flight velocity required to escape from the earth can be found by equating the kinetic energy of a moving body to the work necessary to overcome gravity, neglecting the rotation of the earth and the attraction of other celestial bodies.

$$\tfrac{1}{2}mu^2 = m\int g\, dR$$

By substituting for g from Eq. 4–12 and by neglecting air friction the following relation for the escape velocity is obtained:

$$u_e = R_0\sqrt{\frac{2g_0}{R_0 + h}} = \sqrt{\frac{2\mu}{R}} \qquad (4\text{–}25)$$

Here R_0 is the effective earth radius (6374.2 km), h is the orbit altitude above sea level, and g is the acceleration of gravity at the earth surface (9.806 m/sec). The spacecraft radius R measured from the earth's center is $R = R_0 + h$. The velocity of escape at the earth's surface is 11,179 m/sec or 36,676 ft/sec and does not vary appreciably within the earth's atmosphere, as shown by Fig. 4–6. Escape velocities for surface launch are given in Table 4–1 for the sun, the

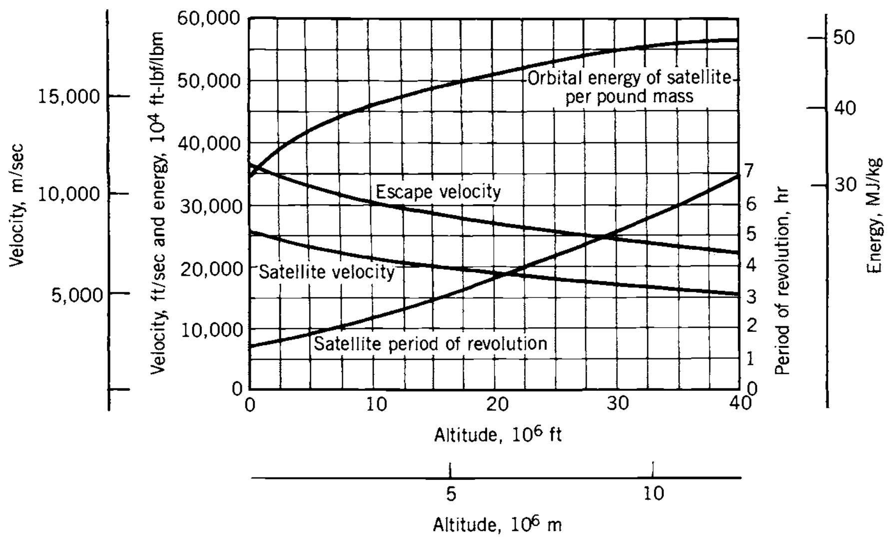


FIGURE 4–6. Orbital energy, orbital velocity, period of revolution, and earth escape velocity of a space vehicle as a function of altitude for circular satellite orbits. It is based on a spherical earth and neglects the earth's rotation and atmospheric drag.

TABLE 4–1. Characteristic Data for Several Heavenly Bodies

Name	Mean Radius of Orbit (million km)	Period of Revolution	Mean Diameter (km)	Relative Mass (Earth = 1.0)	Specific Gravity	Acceleration of Gravity at Surface (m/sec^2)	Escape Velocity at Surface (m/sec)
Sun	—	—	1,393,000	332,950	1.41	273.4	616,000
Moon	0.383	27.3 days	3475	0.012	3.34	1.58	2380
Mercury	57.87	87.97 days	4670	0.06	5.5	3.67	4200
Venus	108.1	224.70 days	12,400	0.86	5.3	8.67	10,300
Earth	149.6	365.256 days	12,742	1.00[a]	5.52	9.806	11,179
Mars	227.7	686.98 days	6760	0.15	3.95	3.749	6400
Jupiter	777.8	11.86 yr	143,000	318.4	1.33	26.0	59,700
Saturn	1486	29.46 yr	121,000	95.2	0.69	11.4	35,400
Uranus	2869	84.0 yr	47,100	15.0	1.7	10.9	22,400
Neptune	4475	164.8 yr	50,700	17.2	1.8	11.9	31,000
Pluto	5899	284.8 yr	5950	0.90	4	7.62	10,000

Source: in part from Refs 4–2 and 4–3.
[a]Earth mass is 5.976×10^{24} kg.

planets, and the moon. Launching from the earth's surface at escape velocity is not practical. As a vehicle ascends through the earth's atmosphere, it is subject to severe aerodynamic heating and dynamic pressures. A practical launch vehicle has to traverse the atmosphere at relatively low velocity and accelerate to the high velocities beyond the dense atmosphere. For example, during a portion of the Space Shuttle's ascent, its main engines are actually throttled to a lower thrust to avoid excessive pressure and heating. Alternatively, an escape vehicle can be launched from an orbiting space station or from an orbiting Space Shuttle.

A rocket spaceship can become a *satellite* of the earth and revolve around the earth in a fashion similar to that of the moon. Satellite orbits are usually elliptical and some are circular. Low earth orbits, typically below 500 km altitude, are designated by the letters LEO. Satellites are useful as communications relay stations for television or radio, weather observation, or reconnaissance observation. The altitude of the orbit is usually above the earth's atmosphere, because this minimizes the expending of energy to overcome the drag which pulls the vehicle closer to the earth. The effects of the radiation in the Van Allen belt on human beings and sensitive equipment sometimes necessitate the selection of an earth orbit at low altitude.

For a circular trajectory the velocity of a satellite must be sufficiently high so that its centrifugal force balances the earth's gravitational attraction.

$$mu_s^2/R = mg$$

For a circular orbit, the satellite velocity u_s is found by using Eq. 4–12,

$$u_s = R_0\sqrt{g_0/(R_0+h)} = \sqrt{\mu/R} \tag{4–26}$$

which is smaller than the escape velocity by a factor of $\sqrt{2}$. The period τ in seconds of one revolution for a circular orbit relative to a stationary earth is

$$\tau = 2\pi(R_0+h)/u_s = 2\pi(R_0+h)^{3/2}/(R_0\sqrt{g_0}) \tag{4–27}$$

The energy E necessary to bring a unit of mass into a circular satellite orbit neglecting drag, consists of kinetic and potential energy, namely,

$$\begin{aligned} E &= \tfrac{1}{2}u_s^2 + \int_{R_0}^{R} g\, dR \\ &= \tfrac{1}{2}R_0^2\frac{g_0}{R_0+h} + \int_{R_0}^{R} g_0\frac{R_0^2}{R^2}\, dR = \tfrac{1}{2}R_0 g_0\frac{R_0+2h}{R_0+h} \end{aligned} \tag{4–28}$$

The escape velocity, satellite velocity, satellite period, and satellite orbital energy are shown as functions of altitude in Fig. 4–6.

A satellite circulating around the earth at an altitude of 300 miles or 482.8 km has a velocity of about 7375 m/sec or 24,200 ft/sec, circles a stationary earth in 1.63 hr, and ideally requires an energy of 3.35×10^7 J to place 1 kg of spaceship mass into its orbit. An equatorial satellite in a circular orbit at an altitude of 6.611 earth radii (about 26,200 miles, 42,200 km, or 22,700 nautical miles) has a period of revolution of 24 hr. It will appear stationary to an observer on earth. This is known as a *synchronous* satellite in *geo-synchronous earth orbit*, usually abbreviated as GEO. It is used extensively for communications satellite applications. In Section 4.7 on launch vehicles we will describe how the payload of a given space vehicle diminishes as the orbit circular altitude is increased and as the inclination (angle between orbit plane and earth equatorial plane) is changed.

Elliptical Orbits

The circular orbit described above is a special case of the more general elliptic orbit shown in Fig. 4–7; here the earth (or any other heavenly body around which another body is moving) is located at one of the focal points of this ellipse. The equations of motion may be derived from Kepler's laws, and the elliptical orbit can be described as follows, when expressed in polar coordinates:

$$u = \left[\mu\left(\frac{2}{R} - \frac{1}{a}\right)\right]^{1/2} \tag{4–29}$$

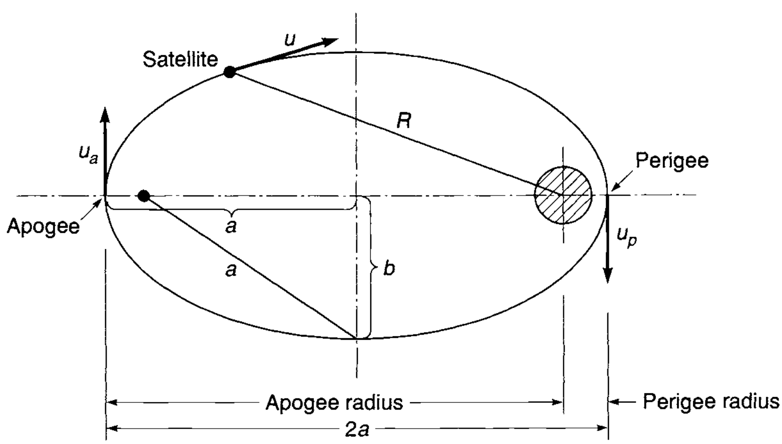


FIGURE 4–7. Elliptical orbit; the attracting body is at one of the focal points of the ellipse.

where u is the velocity of the body in the elliptical orbit, R is the instantaneous radius from the center of the attracting body (a vector quantity, which changes direction as well as magnitude), a is the major axis of the ellipse, and μ is the earth's gravitational constant with a value of 3.986×10^{14} m^3/sec^2. The symbols are defined in Fig. 4–7. From this equation it can be seen that the velocity u_p is a maximum when the moving body comes closest to its focal point at the orbit's *perigee* and that its velocity u_a is a minimum at its *apogee*. By substituting for R in Eq. 4–29, and by defining the ellipse's shape factor e as the *eccentricity of the ellipse*, $e = \sqrt{a^2 - b^2}/a$, then the apogee and perigee velocities can be expressed as

$$u_a = \sqrt{\frac{\mu(1-e)}{a(1+e)}} \tag{4–30}$$

$$u_b = \sqrt{\frac{\mu(1+e)}{a(1-e)}} \tag{4–31}$$

Another property of an elliptical orbit is that the product of velocity and instantaneous radius remains constant for any location a or b on the ellipse, namely, $u_a R_a = u_b R_b = uR$. The exact path that a satellite takes depends on the velocity (magnitude and vector orientation) with which it is started or injected into its orbit.

For interplanetary transfers the ideal mission can be achieved with minimum energy in a simple transfer ellipse, as suggested originally by Hohmann (see Ref. 4–6). Assuming the planetary orbits about the sun to be circular and coplanar, it can be demonstrated that the path of minimum energy is an ellipse tangent to the planetary orbits as shown in Fig. 4–8. This operation requires a velocity increment (relatively high thrust) at the initiation and another at ter-

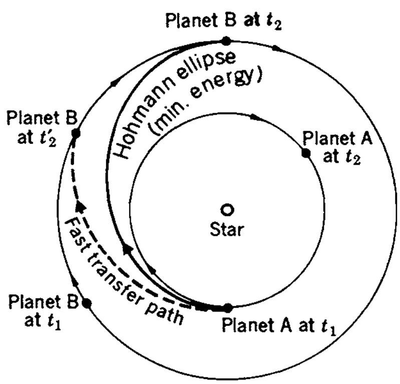


FIGURE 4–8. Schematic diagram of interplanetary transfer paths. These same transfer maneuvers apply when going from a low-altitude earth satellite orbit to a higher orbit.

mination; both increments are the velocity differences between the respective circular planetary velocities and the perigee and apogee velocity which define the transfer ellipse. The thrust levels at the beginning and end maneuvers of the Hohmann ellipse must be high enough to give a short operating time and the acceleration of at least 0.01 g_0, but preferably more. With electrical propulsion these accelerations would be about 10^{-5} g_0, the operating time would be weeks or months, and the best transfer trajectories would be very different from a Hohmann ellipse; they are described in Chapter 19.

The departure date or the *relative positions of the launch planet and the target planet* for a planetary transfer mission is critical, because the spacecraft has to meet with the target planet when it arrives at the target orbit. The Hohmann transfer time $(t_2 - t_1)$ starting on earth is about 116 hours to go to the moon and about 259 days to Mars. If a faster orbit (shorter transfer time) is desired (see dashed lines in Fig. 4–8), it requires more energy than a Hohmann transfer ellipse. This means a larger vehicle with a larger propulsion system that has more total impulse. There also is a *time window* for a launch of a spacecraft that will make a successful rendezvous. For a Mars mission an earth-launched spacecraft may have a launch time window of more than two months. A Hohmann transfer ellipse or a faster transfer path apply not only to planetary flight but also to earth satellites, when an earth satellite goes from one circular orbit to another (but within the same plane). Also, if one spacecraft goes to a rendezvous with another spacecraft in a different orbit, the two spacecraft have to be in the proper predetermined positions prior to the launch for simultaneously reaching their rendezvous. When the launch orbit (or launch planet) is not in the same plane as the target orbit, then additional energy will be needed by applying thrust in a direction normal to the launch orbit plane.

Example 4–2. A satellite is launched from a circular equatorial parking orbit at an altitude of 160 km into a coplanar circular synchronous orbit by using a Hohmann transfer ellipse. Assume a homogeneous spherical earth with a radius of 6374 km. Determine the velocity increments for entering the transfer ellipse and for achieving the synchronous orbit at 42,200 km altitude. See Fig. 4–8 for the terminology of the orbits.

SOLUTION. The orbits are $R_A = 6.531 \times 10^6$ m; $R_B = 48.571 \times 10^6$ m. The major axis a of the transfer ellipse

$$a_{te} = \tfrac{1}{2}(R_A + R_B) = 27.551 \times 10^6 \text{ m/sec}$$

The orbit velocities of the two satellites are

$$u_A = \sqrt{\mu/R_A} = [3.986005 \times 10^{14}/6.571 \times 10^6]^{\frac{1}{2}} = 7788 \text{ m/sec}$$
$$u_B = \sqrt{\mu/R_B} = 2864.7 \text{ m/sec}$$

The velocities needed to enter and exit the transfer ellipse are

$$(u_{te})_A = \sqrt{\mu}[(2/R_A) - (1/a)]^{\frac{1}{2}} = 10,337 \text{ m/sec}$$
$$(u_{te})_B = \sqrt{\mu}[(2/R_B) - (1/a)]^{1/2} = 1394 \text{ m/sec}$$

The changes in velocity going from parking orbit to ellipse and from ellipse to final orbit are:

$$\Delta u_A = |(u_{te})_A - u_A| = 2549 \text{ m/sec}$$
$$\Delta u_B = |u_B - (u_{te})_B = 1471 \text{ m/sec}$$

The total velocity change for the transfer maneuvers is:

$$\Delta u_{\text{total}} = \Delta u_A + \Delta u_B = 4020 \text{ m/sec}$$

Figure 4–9 shows the elliptical transfer trajectory of a ballistic missile or a satellite ascent vehicle. During the initial powered flight the trajectory angle is adjusted by the guidance system to an angle that will allows the vehicle to reach the apogee of its elliptical path exactly at the desired orbit altitude. For the ideal satellite *orbit injection* the simplified theory assumes an essentially instantaneous application of the total impulse as the ballistic trajectory reaches its apogee or zenith. In reality the rocket propulsion system operates over a finite time, during which gravity losses and changes in altitude occur.

Deep Space

Lunar and *interplanetary* missions include circumnavigation, landing, and return flights to the moon, Venus, Mars, and other planets. The energy necessary to escape from earth can be calculated as $\frac{1}{2}mv_e^2$ from Eq. 4–25. It is 6.26×10^7J/kg, which is more than that required for a satellite. The gravitational attraction of various heavenly bodies and their respective escape velocities depends on their masses and diameters; approximate values are listed in Table 4–1. An idealized diagram of an interplanetary landing mission is shown in Fig. 4–10.

The *escape from the solar system* requires approximately 5.03×10^8 J/kg. This is eight times as much energy as is required for escape from the earth. There is technology to send small, unmanned probes away from the sun to outer space; as yet there needs to be an invention and demonstrated proof of a long duration, novel, rocket propulsion system before a mission to the nearest star can be achieved. The trajectory for a spacecraft to escape from the sun is either a parabola (minimum energy) or a hyperbola.

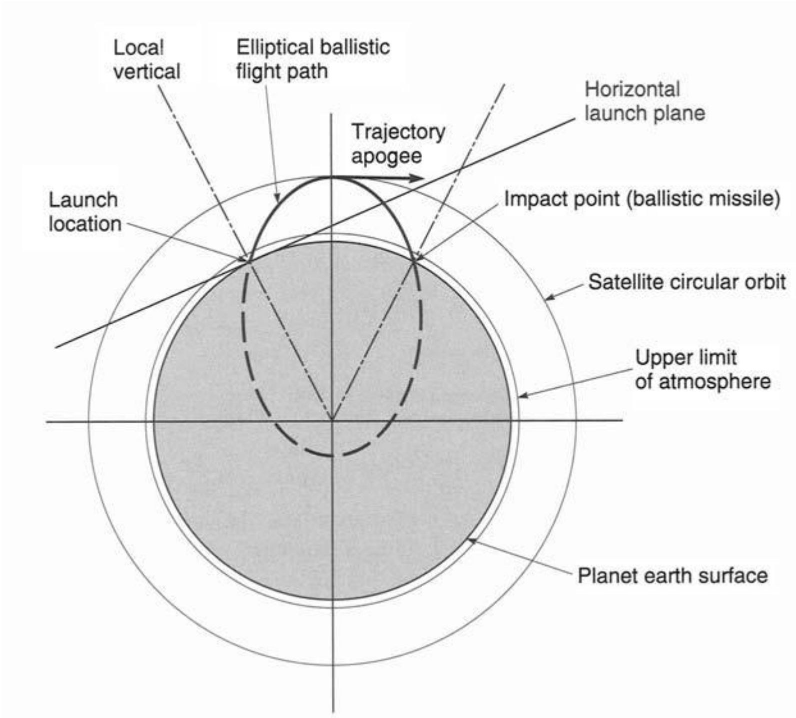


FIGURE 4–9. Long-range ballistic missiles follow an elliptical free-flight trajectory (in a drag-free flight) with the earth's center as one of the focal points. The surface launch is usually vertically up (not shown here), but the trajectory is quickly tilted during early powered flight to enter into the ellipse trajectory. The ballistic range is the arc distance on the earth's surface. For satellites, another powered flight period occurs (called orbit injection) just as the vehicle is at its elliptical apogee (as indicated by the velocity arrow), causing the vehicle to enter an orbit.

Perturbations

This section gives a brief discussion of the disturbing torques and forces which cause perturbations or deviations from any space flight path or satellite's flight trajectory. For a more detailed treatment of flight paths and their perturbations, see Refs. 4–2 and 4–3. A system is needed to measure the satellite's position and deviation from the intended flight path, to determine the needed periodic correction maneuver and then to counteract, control, and correct them. Typically, the corrections are performed by a set of small reaction control thrusters which provide predetermined total impulses into the desired directions. These corrections are needed throughout the life of the spacecraft (for 1 to 20 years) to overcome the effects of the disturbances and maintain the intended flight regime.

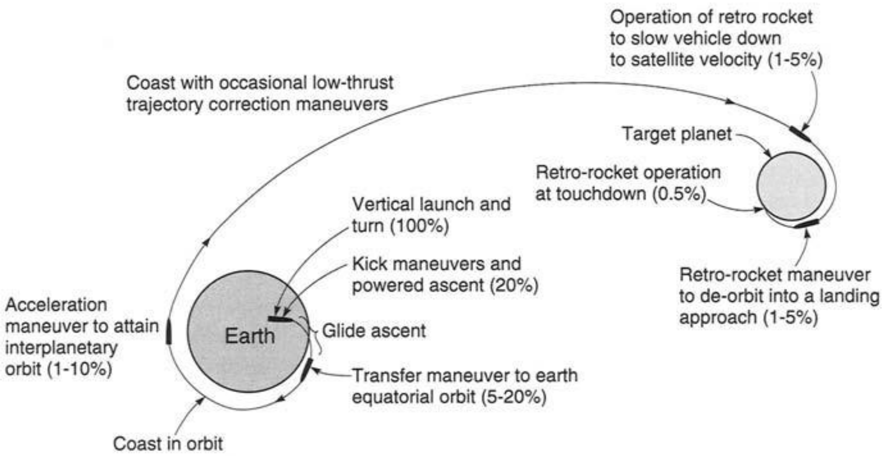


FIGURE 4–10. Schematic diagram of typical powered flight maneuvers during a hypothetical interplanetary mission with a landing. The numbers indicate typical thrust magnitudes of the maneuvers in percent of launch takeoff thrust. This is not drawn to scale. Heavy lines show powered flight segments.

Perturbations can be cateogirzed as short-term and long-term. The daily or orbital period oscillating forces are called *diurnal* and those with long periods are called *secular*.

High-altitude each satellites (36,000 km and higher) experience perturbing forces primarily as gravitational pull from the sun and the moon, with the forces acting in different directions as the satellite flies around the earth. This third-body effect can increase or decrease the velocity magnitude and change its direction. In extreme cases the satellite can come very close to the third body, such as the moon, and undergo what is called a hyperbolic maneuver that will radically change the trajectory. This encounter can be used to increase or decrease the energy of the satellite and intentionally change the velocity and the shape of the orbit.

Medium- and low-altitude satellites (500 to 35,000 km) experience perturbations because of the earth's oblateness. The earth bulges in the vicinity of the equator and a cross section through the poles is not entirely circular. Depending on the inclination of the orbital plane to the earth equator and the altitude of the satellite orbit, two perturbations result: (1) the regression of the nodes, and (2) shifting of the apsides line (major axis). Regression of the nodes is shown in Fig. 4–11 as a rotation of the plane of the orbit in space, and it can be as high as 9° per day at relatively low altitudes. Theoretically, regression does not occur in equatorial orbits.

Figure 4–12 shows an exaggerated shift of the apsidal line, with the center of the earth remaining as a focus point. This perturbation may be visualized as the movement of the prescribed elliptical orbit in a fixed plane. Obviously, both the apogee and perigee points change in position, the rate of change being a func-

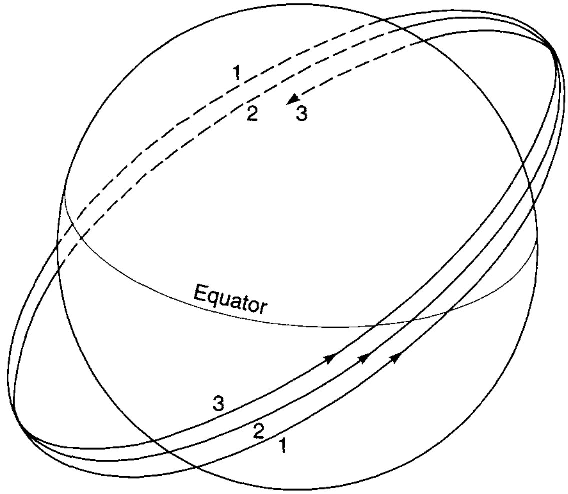


FIGURE 4–11. The regression of nodes is shown as a rotation of the plane of the orbit. The direction of the movement will be opposite to the east–west components of the earth's satellite motion.

tion of the satellite altitude and plane inclination angle. At an apogee altitude of 1000 nautical miles (n.m.) and a perigee of 100 n.m. in an equatorial orbit, the apsidal drift is approximately 10° per day.

Satellites of modern design, with irregular shapes due to protruding antennas, solar arrays, or other asymmetrical appendages, experience torques and forces that tend to perturb the satellite's position and orbit throughout its orbital life. The principal torques and forces result from the following factors:

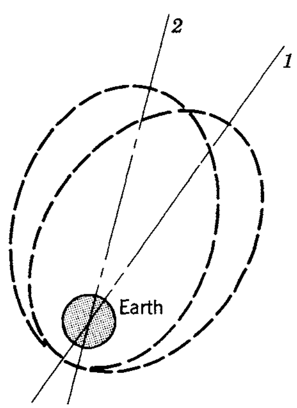


FIGURE 4–12. Shifting of the apsidal line of an elliptic orbit from position 1 to 2 because of the oblateness of the earth.

1. *Aerodynamic drag*. This factor is significant at orbital altitudes below 500 km and is usually assumed to cease at 800 km above the earth. Reference 4–7 gives a detailed discussion of aerodynamic drag which, in addition to affecting the attitude of unsymmetrical vehicles, causes a change in elliptical orbits known as apsidal drift, a decrease in the major axis, and a decrease in eccentricity of orbits about the earth.
2. *Solar radiation*. This factor dominates at high altitudes (above 800 km) and is due to impingement of solar photons upon satellite surfaces. The solar radiation pressure p (N/m^2) on a given surface of the satellite in the vicinity of the earth exposed to the sun can be determined as

$$p = 4.5 \times 10^{-6} \cos\theta[(1 - k_s)\cos\theta + 0.67k_d] \qquad (4\text{–}32)$$

   where θ is the angle (degrees) between the incident radiation vector and the normal to the surface, and k_s and k_d are the specular and diffuse coefficients of reflectivity. Typical values are 0.9 and 0.5, respectively, for k_s and k_d on the body and antenna, and 0.25 and 0.01 respectively, for k_s and k_d with solar array surfaces. The radiation intensity varies as the square of the distance from the sun (see Ref. 4–8). The torque T on the vehicle is given by $T = pAl$, where A is the projected area and l is the offset distance between the spacecraft's center of gravity and the center of solar pressure.
3. *Gravity gradients*. Gravitational torque in spacecraft results from a variation in the gravitational force on the distributed mass of a spacecraft. Determination of this torque requires knowledge of the gravitational field and the distribution of spacecraft mass. This torque decreases as a function of the orbit radius and increases with the offset distances of masses within the spacecraft (including booms and appendages), it is most significant in large spacecraft or space stations operating in relatively low orbits (see Ref. 4–9).
4. *Magnetic field*. The earth's magnetic field and any magnetic moment within the satellite interact to produce torque. The earth's magnetic field precesses about the earth's axis but is very weak (0.63 and 0.31 gauss at poles and equator, respectively). This field is continually fluctuating in direction and intensity because of magnetic storms and other influences. Since the field strength decreases with $1/R^3$ with the orbital altitude, magnetic field forces are often neglected in the preliminary design of satellites (see Ref. 4–10).
5. *Internal accelerations*. Deployment of solar array panels, the shifting of propellant, movement of astronauts or other mass within the satellite, or the "unloading" of reaction wheels produce torques and forces.

We can categorize satellite propulsion needs according to function as listed in Table 4–2, which shows the total impulse "budget" applicable to a typical

TABLE 4.2. Propulsion Functions and Total Impulse Needs of a 2000-lbm Geosynchronous Satellite with a 7-Year Life

Function	Total Impulse (N-sec)
Acquisition of orbit	20,000
Attitude control (rotation)	4,000
Station keeping, E–W	13,000
Station keeping, N–S	270,000
Repositioning (Δu, 200 ft/sec)	53,000
Control apsidal drift (third body attraction)	445,000
Deorbit	12,700
Total	817,700

high altitude, elliptic orbit satellite. The control system designer often distinguishes two different kinds of stationary-keeping orbit corrections needed to keep the satellite in a synchronous position. The east–west correction refers to a correction that moves the point at which a satellite orbit intersects the earth's equatorial plane in an east or west direction; it usually corrects forces caused largely by the oblateness of the earth. The north–south correction counteracts forces usually connected with the third-body effects of the sun and the moon.

In many satellite missions the gradual changes in orbit caused by perturbation forces are not of concern. However, in certain missions it is necessary to compensate for these perturbing forces and maintain the satellite in a specific orbit and in a particular position in that orbit. For example, a synchronous communications satellite in a GEO needs to maintain its position and its orbit, so it will be able to (1) keep covering a specific area of the earth or communicate with the same stations on earth within its line of sight, and (2) not become a hazard to other satellites in this densely occupied synchronous equatorial orbit. Another example is a LEO communications satellite system with several coordinated satellites; here at least one satellite has to be in a position to receive and transmit RF signals to specific points on earth. Their orbits, and the positions of these several satellites with respect to each other, need to be controlled and maintained (see Refs. 4–11 to 4–13).

Orbit maintenance means applying small correcting forces and torques periodically; for GEO it is typically every few months. Typical velocity increments for the orbit maintenance of synchronous satellites require a Δu between 10 and 50 m/sec per year. For a satellite mass of about 2000 kg a 50 m/sec correction for a 10-year orbit life would need a total impulse of about 100,000 N-sec, which corresponds to a propellant mass of 400 to 500 kg (about a quarter of the satellite mass) if done by a small monopropellant or bipropellant thrust. It would require much less propellant if electrical propulsion were used, but in some spacecraft the inert mass of the power supply would increase.

Mission Velocity

A convenient way to describe the magnitude of the energy requirement of a space mission is to use the concept of the *mission velocity*. It is the sum of all the flight velocity increments needed to attain the mission objective. In the simplified sketch of a planetary landing mission of Fig. 4–10, it is the sum of all the Δu velocity increments shown by the heavy lines (rocket-powered flight segments) of the trajectories. Even though some of the velocity increments were achieved by retro-action (a negative propulsion force to decelerate the flight velocity), these maneuvers required energy and their absolute magnitude is counted in the mission velocity. The initial velocity from the earth's rotation (464 m/sec at the equator and 408 m/sec at a launch station at 28.5° latitude) does not have to be provided by the vehicle's propulsion systems. For example, the required mission velocity for launching at Cape Kennedy, bringing the space vehicle into an orbit at 110 km, staying in orbit for a while, and then entering a de-orbit maneuver has the Δu components shown in Table 4–3.

The required mission velocity is the sum of the absolute values of all translation velocity increments that have forces going through the center of gravity of the vehicle (including turning maneuvers) during the flight of the mission. It is the theoretical hypothetical velocity that can be attained by the vehicle in a gravity-free vacuum, if all the propulsive energy of the momentum-adding thrust chambers in all stages were to be applied in the same direction. It is useful for comparing one flight vehicle design with another and as an indicator of the mission energy.

The required mission velocity has to be equal to the "supplied" mission velocity, that is, the sum of all the velocity increments provided by the propulsion systems of each of the various vehicle stages. The total velocity increment to be "supplied" by the shuttle's propulsion systems for the shuttle mission described below (solid rocket motor strap-on boosters, main engines and, for orbit injection, also the increment from the orbital maneuvering system—all shown in Fig. 1–13) has to equal or exceed 9621 m/sec. With chemical propulsion systems and a single stage, we can achieve a space mission velocity of 4000

TABLE 4–3. Space Shuttle Incremental Flight Velocity Breakdown

Ideal satellite velocity	7790 m/sec
Δu to overcome gravity losses	1220 m/sec
Δu to turn the flight path from the vertical	360 m/sec
Δu to counteract aerodynamic drag	118 m/sec
Orbit injection	145 m/sec
Deorbit maneuver to re-enter atmosphere and aerodynamic braking	60 m/sec
Correction maneuvers and velocity adjustments	62 m/sec
Initial velocity provided by the earth's rotation at 28.5° latitude	−408 m/sec
Total required mission velocity	9347 m/sec

to 13,000 m/sec, depending on the payload, vehicle design, and propellant. With two stages it can be between perhaps 12,000 and 22,000 m/sec.

Rotational maneuvers, described later, do not change the flight velocity and are not usually added to the mission velocity requirements. Also, maintaining a satellite in orbit against long-term perturbing forces (see prior section) is often not counted as part of the mission velocity. However, the designers need to provide additional propulsion capability and propellants for these purposes. These are often separate propulsion systems, called reaction control systems.

Typical vehicle velocities required for various interplanetary missions have been estimated as shown in Table 4–4. By starting interplanetary journeys from a space satellite station, a considerable saving in this vehicle velocity can be achieved, namely, the velocity necessary to achieve the earth-circling satellite orbit. As the space-flight objective becomes more ambitious, the mission velocity is increased. For a given single or multistage vehicle it is possible to increase the vehicle's terminal velocity, but usually only at the expense of payload. Table 4–5 shows some typical ranges of payload values for a given multistage vehicle as a percentage of a payload for a relatively simple earth orbit. Thus a vehicle capable of putting a substantial payload into a near-earth orbit can only land a very small fraction of this payload on the moon, since it has to have additional upper stages, which displace payload mass. Therefore, much larger vehicles are required for space flights with high mission velocities if compared to a vehicle of less mission velocity but identical payload. The values listed in Tables 4–4 and 4–5 are only approximate because they depend on specific vehicle design features, the propellants used, exact knowledge of the

TABLE 4–4. Vehicle Mission Velocities for Typical Interplanetary Missions

Mission	Ideal Velocity (km/sec)	Approximate Actual Velocity (1000 m/sec)
Satellite orbit around earth (no return)	7.9–10	9.1–12.5
Escape from earth (no return)	11.2	12.9
Escape from moon	2.3	2.6
Earth to moon (soft landing on moon, no return)	13.1	15.2
Earth to Mars (soft landing)	17.5	20
Earth to Venus (soft landing)	22	25
Earth to moon (landing on moon and return to earth[a])	15.9	17.7
Earth to Mars (landing on Mars, and return to earth [a])	22.9	27

[a]Assumes air braking within atmospheres.

TABLE 4–5. Relative Payload–Mission Comparison Chart for High-Energy Chemical Multistage Rocket Vehicles

Mission	Relative Payload[a] (%)
Earth satellite	100
Earth escape	35–45
Earth 24-hr orbit	10–25
Moon landing (hard)	35–45
Moon landing (soft)	10–20
Moon circumnavigation (single fly-by)	30–42
Moon satellite	20–30
Moon landing and return	1–4
Moon satellte and return	8–15
Mars flyby	20–30
Mars satellite	10–18
Mars landing	0.5–3

[a]300 nautical miles (555.6 km) earth orbit is 100% reference.

trajectory–time relation, and other factors that are beyond the scope of this short treatment. Further information on space flight can be found in Refs. 4–2 to 4–4 and 4–11 to 4–13.

For example, for a co-planar earth–moon and return journey it is necessary to undertake the following steps in sequence and provide an appropriate velocity increment for each. This is similar in concept to the diagram for interplanetary flight of Fig. 4–10. For the ascent from the earth and the entry into an earth satellite orbit, the vehicle has to be accelerated ideally to approximately 7300 m/sec; to change to the transfer orbit requires roughly another 2900 m/sec; to slow down and put the spacecraft into an approach to the moon (retro-action) and enter into an orbit about the moon is about 1000 m/sec; and to land on the moon is about another 1600 m/sec. The ascent from the moon and the entry into an earth return orbit is about 2400 m/sec. Aerodynamic drag is used to slow down the earth reentry vehicle and this maneuver does not require the expenditure of propellant. Adding these together and allowing 300 m/sec for various orbit adjustments comes to a total of about 14,500 m/sec, which is the approximate cumulative total velocity needed for the mission. Tables 4–3 and 4–4 compare very rough values of mission velocities and payloads for several space missions.

4.6. FLIGHT MANEUVERS

In this section we describe different flight maneuvers and relate them to specific propulsion system types. The three categories of *maneuvers* are:

1. In *translation maneuvers* the rocket propulsion thrust vector goes through the center of gravity of the vehicle. The vehicle momentum is changed in the direction of the flight velocity. An example of several powered (translational maneuvers) and unpowered (coasting) segments of a complex space flight trajectory is shown in schematic, simplified form in Fig. 4–10. To date, most maneuvers have used chemical propulsion systems.
2. In *truly rotational maneuvers* there is no net thrust acting on the vehicle. These are true couples that apply only torque. It requires four thrusters to be able to rotate the vehicle in either direction about any one axis (two thrusters apart, firing simultaneously, but in opposite directions). These types of maneuver are usually provided by reaction control systems. Most have used multiple liquid propellant thrusters, but in recent years many space missions have used electrical propulsion.
3. A combination of categories 1 and 2, such as a large misaligned thrust vector that does not go exactly through the center of gravity of the vehicle. The misalignment can be corrected by changing the vector direction of the main propulsion system (thrust vector control) during powered flight or by applying a simultaneous compensating torque from a separate reaction control system.

The following types of *space flight maneuvers and vehicle accelerations* use rocket propulsion. All propulsion operations are controlled (started, monitored, and stopped) by the vehicle's guidance and control system.

a. *First stage* and its *upper stage propulsion systems* add momentum during launch and ascent. They require rocket propulsion of high or medium thrusts and limited durations (typically 0.7 to 8 minutes). To date all have used chemical propulsion systems. They constitute the major mass of the space vehicle and are discussed further in the next section.

b. *Orbit injection or transferring from one orbit to another* requires accurately predetermined total impulses. It can be performed by the main propulsion system of the top stage of the launch vehicle. More often it is done by a separate propulsion system at lower thrust levels than the upper stages in item (a) above. Orbit injection can be a single thrust operation after ascent from an earth launch station. If the flight path is a Hohmann transfer ellipse (minimum energy) or a faster transfer orbit, then two thrust application periods are necessary, one at the beginning and one at the end of the transfer path. For injection into earth orbit, the thrust levels are typically between 200 and 45,000 N or 50 and 11,000 lbf, depending on the payload size transfer time, and the specific orbit. If the new orbit is higher, then the thrusts are applied in the flight direction. If the new orbit is at a lower altitude, then the thrusts must be applied in a direction opposite to the flight velocity vector. The transfer orbits can also be achieved with a very low thrust level (0.001 to 1 N) using an electric propulsion system, but the flight paths will be very different (multi-loop spiral) and the transfer duration will be much longer. This is explained in Chapter 19. Similar maneuvers are also performed with lunar or interplanetary flight missions, as the planetary landing mission shown schematically in Fig. 4–10.

c. *Velocity vector adjustment and minor in-flight correction maneuvers* are usually performed with low thrust, short duration and intermittent (pulsing) operations, using a reaction control system with multiple small liquid propellant thrusters, both for translation and rotation. The vernier rockets on a ballistic missile are used to accurately calibrate the terminal velocity vector for improved target accuracy. The reaction control rocket systems in a space launch vehicle will allow accurate orbit injection adjustment maneuvers after it is placed into orbit by another, less accurate propulsion system. Mid-course guidance-directed *correction maneuvers* for the trajectories of deep space vehicles fall also into this category. Propulsion systems for *orbit maintenance maneuvers*, also called *station keeping maneuvers* (to overcome perturbing forces), keeping a spacecraft in its intended orbit and orbital position and are also considered to be part of this category.

d. *Reentry and landing maneuvers* can take several forms. If the landing occurs on a planet that has an atmosphere, then the drag of the atmosphere will slow down the reentering vehicle. For an elliptical orbit the drag will progressively reduce the perigee altitude and the perigee velocity on every orbit. Landing at a precise, preplanned location requires a particular velocity vector at a predetermined altitude and distance from the landing site. The vehicle has to be rotated into the right position and orientation, so as to use its heat shield correctly. The precise velocity magnitude and direction prior to entering the denser atmosphere are critical for minimizing the heat transfer (usually to the vehicle's heat shield) and to achieve touchdown at the intended landing site or, in the case of ballistic missiles, the intended target. This usually requires a relatively minor maneuver (low total impulse). If there is very little or no atmosphere (for instance, landing on the moon or Mercury), then a reverse thrust has to be applied during descent and touchdown. The rocket propulsion system usually has variable thrust to assure a soft landing and to compensate for the decrease in vehicle mass as propellant is consumed during descent. The lunar landing rocket engine, for example, had a 10 to 1 thrust variation.

e. *Rendezvous and docking* involve both rotational and translational maneuvers of small reaction control thrusters. Rendezvous and its time windows were discussed on page 123. Docking (sometimes called lock-on) is the linking up of two spacecraft and requires a gradual gentle approach (low thrust, pulsing node thrusters) so as not to damage the spacecraft.

f. *A change of plane of the flight trajectory* requires the application of a thrust force (through the vehicle center of gravity) in a direction normal to the original plane of the flight path. This is usually performed by a propulsion system that has been rotated (by the reaction control system) into the proper orientation. This maneuver is done to change the plane of a satellite orbit or when going to a planet, such as Mars, whose orbit is inclined to the plane of the earth's orbit.

g. *Simple rotational maneuvers* rotate the vehicle on command into a specific angular position so as to orient or point a telescope, instrument, solar panel, or antenna for purposes of observation, navigation, communication, or solar power reception. Such a maneuver is also used to keep the orientation of a satellite in a specific direction; for example, if an antenna needs to be continuously pointed at the center of the earth, then the satellite needs to be rotated around its own axis once every satellite revolution. Rotation is also used to point a nozzle of the primary propulsion system into its intended direction just prior to its start. It can also provide for achieving flight stability, or for correcting angular oscillations, that would otherwise increase drag or cause tumbling of the vehicle. Spinning or rolling a vehicle will improve flight stability, but will also average out the misalignment in a thrust vector. If the rotation needs to be performed quickly, then a chemical multi-thruster reaction control system is used. If the rotational changes can be done over a long period of time, then an electrical propulsion system with multiple thrusters is often preferred.

h. *De-orbiting and disposal of used or spent spacecraft* is required today to remove space debris. The spent spacecraft should not become a hazard to other spacecraft. A relatively small thrust will cause the vehicle to go to a low enough elliptical orbit so that atmospheric drag will cause further slowing. In the dense regions of the atmosphere the reentering, expended vehicle will typically break up or overheat (burn up).

i. *Emergency or alternative mission.* If there is a malfunction in a spacecraft and it is decided to abort the mission, such as a premature quick return to the earth without pursuing the originally intended mission, then some of the rocket engines can be used for an alternate mission. For example, the main rocket engine in the Apollo lunar mission service module is normally used for retroaction to attain a lunar orbit and for return from lunar orbit to the earth; it can be used for emergency separation of the payload from the launch vehicle and for unusual midcourse corrections during translunar coast, enabling an emergency earth return.

Table 4–6 lists the maneuvers that have just been described, together with some others, and shows the various types of rocket propulsion system (as mentioned in Chapter 1) that have been used for each of these maneuvers. The table omits several propulsion systems, such as solar thermal or nuclear

TABLE 4–6. Types of Rocket Propulsion System Commonly Used for Different Flight Maneuvers

Propulsion System → / Flight Maneuvers and Applications ↓	High thrust, liquid propellant rocket engine, with turbopump	Medium to low thrust, liquid propellant rocket engine	Pulsing liquid propellant, multiple small thrusters	Large solid propellant rocket motor, often segmented	Medium to small solid propellant motors	Arc jet, resisto jet	Ion propulsion, Electromagnetic propulsion	Pulsed plasma jet
Launch vehicle booster	××			××				
Strap-on motor/engine	××			××				
Upper stages of launch vehicle	××	××		×	××			
Satellite orbit injection and transfer orbits		××			××	×		
Flight velocity adjustments, Flight path corrections, Orbit raising		×	××			×	×	
Orbit/position maintenance, rotation of spacecraft			××			××	×	×
Docking of two spacecraft			××					
Reentry and landing, Emergency maneuvers		×	×					
Deorbit		×	×		×	×		
Deep space, Sun escape		×	×				×	
Tactical missiles					××			
Strategic missiles	×	×	×	××				
Missile defense			×	××		××		

Legend: × = in use: ×× = preferred for use.

rocket propulsion, because these have not yet flown in a real space mission. The electrical propulsion systems have very high specific impulse (see Table 2–1), which makes them very attractive for deep space missions, but they can be applied only to missions with sufficiently long thrust action time for reaching the desired vehicle velocity with very small acceleration. The items with a double mark "××" have been the preferred methods in recent years.

Reaction Control System

The functions of a reaction control system have been described in the previous section on flight maneuvers. They are used for the maneuvers identified by paragraphs c, e, and g. In some vehcle designs they are also used for tasks described in b, part of d, and f, if the thrust levels are low.

A *reaction control system* (RCS), often called an *auxiliary rocket propulsion system*, is needed to provide for trajectory corrections (small Δu additions), as well as correcting the rotational or attitude position of almost all spacecraft and all major launch vehicles. If only rotational maneuvers are made, it has been called an *attitude control system*. The nomenclature has not been consistent throughout the industry or the literature.

An RCS can be incorporated into the payload stage and each of the stages of a multiple stage vehicle. In some missions and designs the RCS is built into only the uppermost stage; it operates throughout the flight and provides the control torques and forces for all the stages. Liquid propellant rocket engines with multiple thrusters have been used for almost all launch vehicles and the majority of all spacecraft. Cold gas systems were used with early spacecraft design. In the last decade an increasing number of electrical propulsion systems have been used, primarily on spacecraft, as described in Chapter 19. The life of an RCS may be short (when used on an individual vehicle stage), or it may see use throughout the mission duration (perhaps 10 years) when part of an orbiting spacecraft.

The vehicle attitude has to be controlled about three mutually perpendicular axes, each with two degrees of freedom (clockwise and counterclockwise rotation), giving a total of six degrees of rotational freedom. *Pitch* control raises or lowers the nose of the vehicle, *yaw* torques induce a motion to the right or the left side, and *roll* torques will rotate the vehicle about its axis, either clockwise or counterclockwise. In order to apply a true torque it is necessary to use two thrust chambers of exactly equal thrust and equal start and stop times, placed an equal distance from the center of mass. Figure 4–13 shows a simple spherical spacecraft attitude control system; thrusters $x - x$ or $x' - x'$ apply torques that rotate about the X-axis. There is a minimum of 12 thrusters in this system, but some spacecraft with geometrical or other limitations on the placement of these nozzles or with provisions for redundancy may actually have more than 12. The same system can, by operating a different set of nozzles, also provide translation forces; for example, if one each of the thrust units x and x' were operated simultaneously, the resulting forces would propel the vehicle in the direction of the Y-axis. With clever design it is possible to use fewer thrusters.

An RCS usually contains the following major subsystems: (1) sensing devices for determining the attitude, velocity, and position of the vehicle with respect to a reference direction at any one time, such as provided by gyroscopes, star-trackers, or radio beacons; (2) a control-command system that compares the actual space and rotary position with the desired or programmed position and issues command signals to change the vehicle position within a desired time period; and (3) devices for changing the angular position, such as a set of high-speed gyroscopic wheels and a set of attitude control thrust-providing devices. See Refs. 4–12 and 4–14.

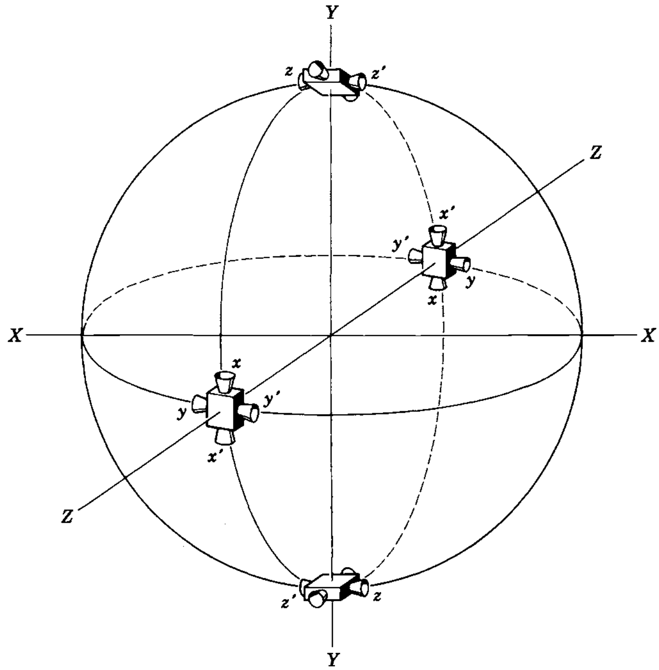


FIGURE 4–13. Simplified attitude control system diagram for spacecraft. It requires 12 thrusters (identified as x, y, z) to allow the application of pure torques about three perpendicular axes. The four unlabeled thrusters are needed for translation maneuvers along the z axis. They are shown here in four clusters.

A precise attitude angular correction can also be achieved by the use of an inertial or high-speed rotating reaction wheel, which applies torque when its rotational speed is increased or decreased. While these wheels are quite simple and effective, the total angular momentum change they can supply is generally small. By using a pair of supplementary attitude control thrust rocket units it is possible to unload or respin each wheel so it can continue to supply small angular position corrections as needed.

The torque T of a pair of thrust chambers of thrust F and a separation distance l is applied to give the vehicle with an angular or rotational moment of inertia M_a an angular acceleration of magnitude α:

$$T = Fl = M_a \alpha \tag{4–33}$$

For a cylinder of equally distributed mass $M_a = \frac{1}{2}mr^2$ and for a homogeneous sphere it is $M_a = \frac{2}{5}mr^2$. The largest possible practical value of moment arm l will minimize the thrust and propellant requirements. If the angular acceleration is constant over a time period t, the vehicle will move at an angular speed ω and through a displacement angle θ, namely

$$\omega = \alpha t \qquad \text{and} \qquad \theta = \tfrac{1}{2}\alpha t^2 \tag{4–34}$$

Commonly a control system senses a small angular disturbance and then commands an appropriate correction. For this detection of an angular position change by an accurate sensor it is actually necessary for the vehicle to undergo a slight angular displacement. Care must be taken to avoid overcorrection and hunting of the vehicle position or the control system. For this reason many spacecraft require extremely short multiple pulses (0.010 to 0.030 sec) and low thrust (0.01 to 100 N) (see Refs. 4–13 and 4–14).

Reaction control systems can be characterized by the magnitude of the total impulse, the number, thrust level, and direction of the thrusters, and by their duty cycles. The *duty cycle* refers to the number of thrust pulses, their operating times, the times between thrust applications, and the timing of these short operations during the mission operating period. For a particular thruster, a 30% duty cycle means an average active cumulative thrust period of 30% during the propulsion system's flight duration. These propulsion parameters can be determined from the mission, the guidance and control approach, the desired accuracy, flight stability, the likely thrust misalignments of the main propulsion systems, the three-dimensional flight path variations, the perturbations to the trajectory, and several other factors. Some of these parameters are often difficult to determine.

4.7. FLIGHT VEHICLES

As mentioned, the vast majority of rocket propelled vehicles are simple, single stage, and use solid propellant rocket motors. Most are used in military applications, as described in the next section. This section discusses more sophisticated multistage space launch vehicles and mentions others, such as large ballistic missiles (often called strategic missiles) and some sounding rockets. All have some intelligence in their guidance and navigation system. The total number of multistage rocket vehicles produced world wide in the last few years has been between 140 and 220 per year.

A single stage to orbit (LEO) is limited in the payload it can carry. Figure 4–2 shows that a high-performance single-stage vehicle with a propellant fraction of 0.95 and an average I_s of 400 sec can achieve an ideal terminal velocity of about 12,000 m/sec without payload. If the analysis includes drag and gravity forces, a somewhat higher value of I_s, maneuvers in the trajectory, and an attitude control system, it is likely that the payload would be between 0.2 and 1.4 percent of the gross take-off mass, depending on the design. For a larger percentage of payload, and for ambitious missions, we use vehicles with two or more stages as described here.

Multistage Vehicles

Multistep or *multistage rocket vehicles* permit higher vehicle velocities, more payload for space vehicles, and improved performance for long-range ballistic missiles. After the useful propellant is fully consumed in a particular stage, the remaining empty mass of that expended stage is dropped from the vehicle and the operation of the propulsion system of the next step or stage is started. The last or top stage, which is usually the smallest, carries the payload. The empty mass of the expended stage or step is separated from the remainder of the vehicle, because it avoids the expenditure of additional energy for further accelerating a useless mass. As the number of steps is increased, the initial takeoff mass can be decreased; but the gain in a smaller initial mass becomes less apparent when the total number of steps is large. Actually, the number of steps chosen should not be too large, because the physical mechanisms become more numerous, complex, and heavy. The most economical number of steps is usually between two and six, depending on the mission. Several different multi-stage launch vehicle configurations have been used successfully and four are shown in Fig. 4–14. Most are launched vertically, but a few have been launched from an airplane, such as the three-stage Pegasus space vehicle.

The payload of a multistage rocket is essentially proportional to the takeoff mass, even though the payload is only a very small portion of the initial mass. If a payload of 50 kg requires a 6000-kg multistage rocket, a 500-kg payload would require a 60,000-kg rocket unit with an identical number of stages, and a similar configuration with the same payload fraction. When the operation of the upper stage is started, immediately after thrust termination of the lower stage, then the total ideal velocity of a multistage vehicle of tandem or series-stage arrangement is simply the sum of the individual stage velocity increments. For n stages, the final velocity increment Δu_f is

$$\Delta u_f = \sum_{1}^{n} \Delta u = \Delta u_1 + \Delta u_2 + \Delta u_3 + \cdots \qquad (4\text{–}35)$$

The individual velocity increments are given by Eq. 4–6. For the simplified case of a vacuum flight in a gravity-free field this can be expressed as

$$\Delta u_f = c_1 \ln(1/\mathbf{MR}_1) + c_2 \ln(1/\mathbf{MR}_2) + c_3 \ln(1/\mathbf{MR}_3) + \cdots \qquad (4\text{–}36)$$

This equation defines the maximum velocity an ideal multistage vehicle can attain in a gravity-free vacuum environment. For more accurate actual trajectories the individual velocity increments can be determined by integrating Eqs. 4–15 and 4–16, which consider drag and gravity losses. Other losses or trajectory perturbations can also be included, as mentioned earlier in this chapter. Such an approach requires numerical solutions.

For two- or three-stage vehicles the overall vehicle mass ratio (initial mass at takeoff to final mass of last stage) can reach values of over 100 (corresponding to an equivalent single-stage propellant mass fraction ζ of 0.99). Figure 4–2 can be thus divided into regions for single- and multistage vehicles.

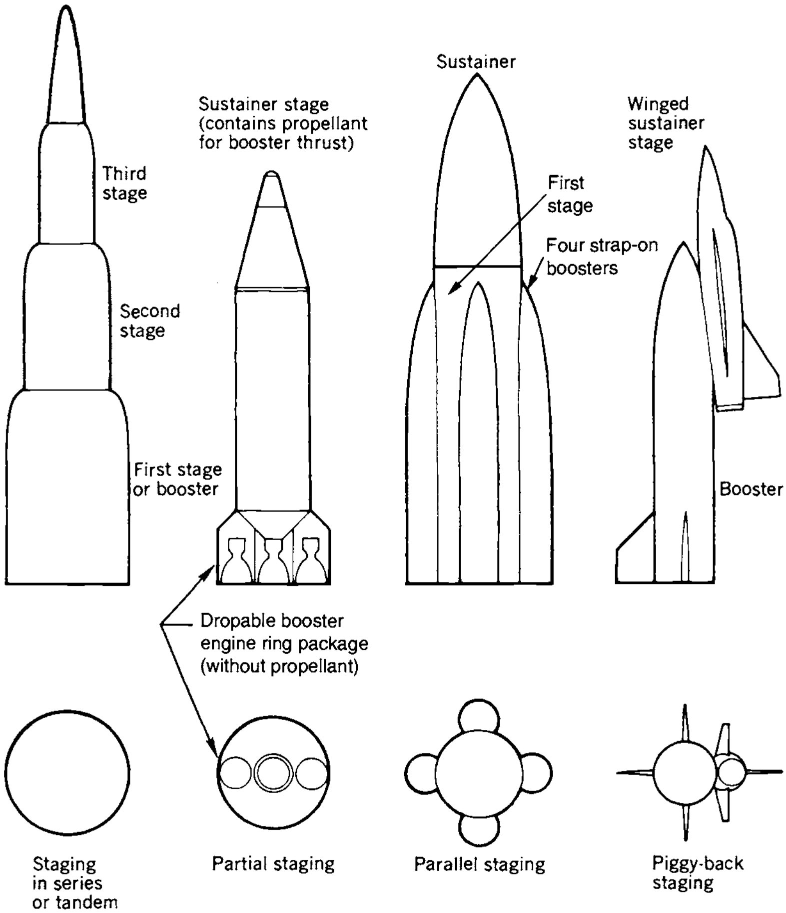


FIGURE 4–14. Simplified schematic sketches of four geometric configurations for assembling individual stages into a launch vehicle. The first is very common and the stages are stacked vertically on top of each other, as in the Minuteman long-range missile or the Delta launch vehicle. Partial staging was used on early versions of the Atlas; it allows all engines to be started at launching, thus avoiding a start during flight, and it permits the shut-off of engines on the launch stand if a failure is sensed prior to lift-off. The two booster engines, arranged in a doughnut-shaped assembly, are dropped off in flight. In the third sketch there are two or more separate "strap-on" booster stages attached to the bottom stage of a vertical configuration and this allows an increase in vehicle performance. The piggy-back configuration concept on the right is used in the Space Shuttle.

For multistage vehicles the stage mass ratios, thrust levels, propulsion durations, and the location or travel of the center of gravity of the stages are usually optimized, often using a complex trajectory computer program. The high specific impulse rocket engine (e.g., using hydrogen–oxygen propellants) is normally employed in upper stages of space launch vehicles, because a small increase in specific impulse is more effective there than in lower stages.

Example 4–3. A two-stage planetary exploration vehicle is launched from a high-orbit satellite into a gravity-free vacuum trajectory. The following notations are used and explained in the diagram.

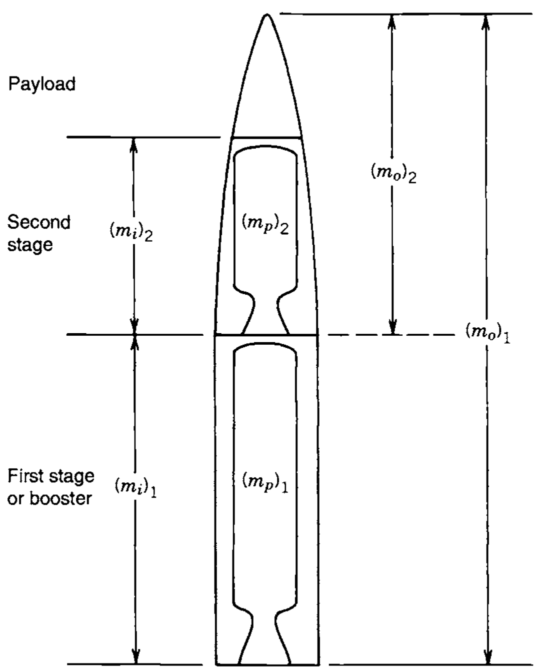
m_0 = initial mass of vehicle (or stage) at launch

m_p = useful propellant mass of stage

m_i = initial mass of stage(s)

m_f = final mass of stage (after rocket operation); it includes the empty propulsion system with its residual propellant, the structures of the vehicle and the propulsion system, the control, guidance, and payload masses.

m_{pl} = payload mass; it includes the guidance, control and communications equipment, antennas, scientific instruments, research apparatus, power supply, solar panels, sensors, etc.



Subscripts 1 and 2 refer to first and second stages. The following are given:

Flight and velocity increment in gravity-free vacuum	6200 m/sec
Specific impulse, I_s	310 sec
Effective exhaust velocity, c (all stages)	3038 m/sec
Initial launch vehicle mass	4500 kg
Propellant mass fraction, ζ (each stage)	0.88
Structural mass fraction, $(1-\zeta)$ (each stage)	0.12

Determine the payload for two cases: (1) when the two stage masses are equal, and (2) when the mass ratios of the two stages are equal.

SOLUTION. For launch the takeoff mass (m_0) equals the loaded first-stage mass $(m_i)_1$ plus the loaded second-stage mass $(m_i)_2$ plus the payload (m_{pl}). The propellant mass fraction ζ is 0.88. For case (1) the first and second stages are identical. Thus

$$m_i = (m_i)_1 = (m_i)_2$$
$$m_p = (m_p)_1 = (m_p)_2 = 0.88 m_i$$
$$(m_p)_1 = 0.88(m_i)_1$$
$$(m_0)_1 = 4500 \text{ kg} = 2m_i + m_{pl}$$
$$e^{\Delta u/c} = e^{6200/3038} = 7.6968 = \frac{(m_0)_1}{(m_0)_1 - (m_p)_1} \cdot \frac{(m_0)_2}{(m_0)_2 - (m_p)_2}$$

From these relationships it is possible to solve for the payload mass m_{pl}, which is 275 kg.

$$m_i = (4500 - 275)/2 = 2113 \text{ kg each stage}$$
$$m_p = 0.88 m_i = 1855 \text{ kg each stage}$$

For case (2) the mass ratios of the two stages are the same. The mass ratio ($1/\mathbf{MR}$) was defined by

$$m_0/m_f = (m_0)_1/[(m_0)_1 - [(m_p)_1] = (m_0)_2/[(m_0)_2 - (m_p)_2]$$
$$(m_0)_1 = 4500 = (m_i)_1 + (m_i)_2 + m_{pl}$$
$$e^{\Delta u/c} = 7.6968 = \{4500/[4500 - (m_p)_1]\}^2$$

Solving for the first-stage propellant mass gives $(m_p)_1 = 2878$ kg.

$$(m_i)_1 = (m_p)_1/0.88 = 3270 \text{ kg}$$
$$(m_0)_2 = (m_i)_2 + m_{pl} = 4500 - 3270 = 1230 \text{ kg}$$
$$e^{\Delta u/c} = 7.6968 = \{1230/[1230 - (m_p)_2]\}^2; \quad (m_p)_2 = 786.6 \text{ kg}$$
$$(m_i)_2 = (m_p)_2/0.88 = 894 \text{ kg}$$

The payload m_{pl} is $1230 - 894 = 336$ kg. This is about 22% larger than the payload of 275 kg in the first case. When the mass ratios of the stages are equal, the payload is a maximum for gravity-free vacuum flight and the distribution of the masses between the stages is optimum. For a single-stage vehicle with the same take-off mass and same propellant fraction, the payload is substantially less. See Problem 4–13.

If a three-stage vehicle had been used in Example 4–3 instead of a two-stage version, the payload would have been even larger. However, the theoretical payload increase will only be about 8 or 10%. A fourth stage gives an even smaller theoretical improvement; it would add only 3 to 5% to the payload. The amount of potential performance improvement diminishes with each added stage. Each additional stage means extra complications in an actual vehicle (such as a reliable separation mechanism, an interstage structure, joints or couplings in a connecting pipes and cables, etc.), requires additional inert mass (increasing the mass ratio **MR**), and compromises the overall reliability. Therefore, the minimum number of stages that will meet the payload and the Δu requirements is usually selected.

The flight paths taken by the vehicles in the two simplified cases of Example 4–3 are different, since the time of flight and the acceleration histories are different. One conclusion from this example applies to all multistage rocket-propelled vehicles; for each mission there is an optimum number of stages, an optimum distribution of the mass between the stages, and there is usually also an optimum flight path for each design, where a key vehicle parameter such as payload, velocity increment, or range is a maximum.

Launch Vehicles

Usually the *first or lowest stage*, often called a *booster stage*, is the largest and it requires the largest thrust and largest total impulse. All stages need chemical propulsion to achieve the desired thrust-to-weight ratio. These thrusts usually become smaller with each subsequent stage, also known as *upper stage* or *sustainer stage*. The thrust magnitudes depend on the mass of the vehicle, which in turn depends on the mass of the payload and the mission. Typical actual configurations are shown by simple sketches in Fig. 4–14. There is an optimum size and thrust value for each stage in a multistage vehicle and the analysis to determine these optima can be quite complex.

Many heavy launch vehicles have two to six *strap-on solid propellant motor boosters*, which together form a supplementary first stage strapped on or mounted to the first stage of the launch vehicle (Space Shuttle, Titan, Delta, Atlas, Ariane). This is shown in the third sketch of Fig. 4–14. The Russians have used liquid propellant strap-on boosters on several vehicles, because they give better performance. Boosters operate simultaneously with the first stage and, after they burn out, they are usually separated and dropped off before completion of the first stage's propulsive operation. This has also been called a *half stage* or *zero stage*, as in Table 1–3.

There is a variety of existing launch vehicles. The smaller ones are for low payloads and low orbits; the larger ones usually have more stages, are heavier, more expensive, have larger payloads, or higher mission velocities. The vehicle cost increases with the number of stages and the initial vehicle launch mass. Once a particular launch vehicle has been proven to be reliable, it is usually modified and uprated to allow improvements in its capability or mission flexibility. Each of the stages of a space launch vehicle can have several rocket engines, each with specific missions or maneuvers. The Space Shuttle system has 67 different rockets which are shown schematically in Fig. 1–13. In most cases each rocket engine is used for a specific maneuver, but in many cases the same engine is used for more than one specific purpose; the small reaction control thrusters in the Shuttle serve, for example, to give attitude control (pitch, yaw, and roll) during orbit insertion and reentry, for counteracting internal shifting of masses (astronaut movement, extendible arm), small trajectory corrections, minor flight path adjustments, docking, and precise pointing of scientific instruments.

The *spacecraft* is that part of a launch vehicle that carries the payload. It is the only part of the vehicle that goes into orbit or deep space and some are designed to return to earth. The final major space maneuver, such as orbit injection or planetary landing, often requires a substantial velocity increment; the propulsion system, which provides the force for this maneuver, may be integrated with the spacecraft, or it may be part of a discardable stage, just below the spacecraft. Several of the maneuvers described in Section 4–6 can often be accomplished by propulsion systems located in two different stages of a multistage vehicle. The selection of the most desirable propulsion systems, and the decision on which of the several propulsion systems will perform specific maneuvers, will depend on optimizing performance, cost, reliability, schedule, and mission flexibility as described in Chapter 17.

When a space vehicle is launched from the earth's surface into an orbit, it flies through three distinct trajectory phases. (1) Most are usually launched vertically and then undergo a turning maneuver while under rocket power to point the flight velocity vector into the desired direction. (2) The vehicle then follows a free-flight (unpowered) ballistic trajectory (usually elliptical), up to its apex. Finally (3) a satellite needs an extra push from a chemical rocket system up to add enough total impulse or energy to accelerate it to orbital velocity. This last maneuver is also known as *orbit insertion*. During the initial powered flight the trajectory angle and the thrust cut-off velocity of the last stage are adjusted by the guidance system to a velocity vector in space that will allow the vehicle to reach the apogee of its elliptic path exactly at the desired orbit altitude. As shown in Fig. 4–9, a *multistage ballistic missile* follows the same two ascent flight phases mentioned above, but it then continues its elliptical ballistic trajectory all the way down to the target.

Historically successful launch vehicles have been modified, enlarged, and improved in performance. The newer versions retain most of the old, proven, reliable components, materials, and subsystems. This reduces development effort and cost. Upgrading a vehicle allows an increase in mission energy (more ambitious mission) or payload. Typically, it is done by one or more of these types of improvement: increasing the mass of propellant without an undue increase in tank or case mass; uprating the thrust and strengthening the engine; more specific impulse; or adding successively more or bigger strap-on boosters. It also usually includes a strengthening of the structure to accept higher loads.

Figure 4–15 and Table 4–7 illustrate the growth of payload and mission capability for the early Titan family of space launch vehicles and the effect of the orbit on the payload. The figure shows the evolution of four different multistage configurations of the launch vehicle and their principal propulsion systems; the table defines the increase in payload for the four vehicle configurations and also how the payload is reduced as more ambitious orbits are flown. When each of these vehicles is equipped with an additional third stage, it is able to launch substantial payloads into earth escape or synchronous orbit. The table describes the propulsion for each of the several stages used on those vehicles and the payload for several arbitrarily selected orbits.

Table 4–7 shows the effects of orbit inclination and altitude on the payload. The inclination is the angle between the equatorial plane of the earth and the trajectory. An equatorial orbit has zero inclination and a polar orbit has 90° inclination. Since the earth's rotation gives the vehicle an initial velocity, a

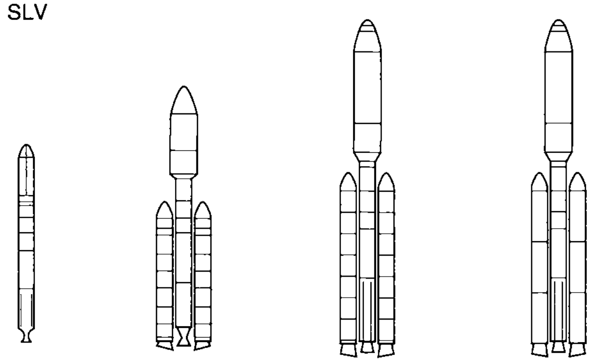
Launch vehicle	Titan II SLV	Titan III	Titan IV	Titan IVB
Configuration				
Major configuration modifications	Modified Titan II (ICBM)	Added two 5$\frac{1}{2}$ segment rocket boosters; more liquid propellant	Larger solid 7 segment rocket boosters, higher liquid rocket engine thrust, longer duration	New, 12% larger, 3 segment solid boosters with reinforced plastic cases
First flight	1988	1989	1990	1997

FIGURE 4–15. Upgrading methods are illustrated by these four related configurations in the evolution of the Titan Space Launch Vehicle family. Source: Lockheed-Martin Corp.

TABLE 4–7. Payload Capabilities and Rocket Propulsion Systems of Four Titan Space Launch Vehicle Configurations

Space Launch Vehicle	Titan II SLV	Titan III	Titan IV	Titan IV B
		Payloads (lbm) in Low Earth Orbits for 2-Stage Configurations		
100 mi circular orbit, 28.6° inclination from Cape Canaveral	5000	31,000	39,000	47,800
Same, but 99° launch from Vandenberg AFB	4200	26,800	32,000	38,800
Elliptic orbit, 100 mi → 1000 mi, 28.6° inclination	3000	25,000	~ 30,000	~ 34,000
		Payloads (lbm) in Synchronous Earth Orbit, 3-Stage Configurations		
Payload for third-stage propulsion system, optional (see below)	2200	4000	10,000	12,700
		Rocket Propulsion Systems in Titan Launch Vehicles		
Solid rocket boosters (United Technologies/CSD)	None	2 units, each metal case $5\frac{1}{2}$ segments $I_t = 123 \times 10^6$ lbf-sec	Same, but 7 segments $I_t = 159.7 \times 10^6$ lbf-sec	12% more propellant, 3 segments $I_t = 179 \times 10^6$ lbf-sec
Stage I, Aerojet LR 87-AJ-11 engine, N_2O_4 with 50% N_2H_4/50% UDMH	2 thrust chambers 430,000 lbf thrust at SL	Same, 529,000 lbf thrust (vacuum)	Same, but uprated to 550,000 lbf thrust in a vacuum	Same
Stage II, Aerojet LR 91-AJ-11 engine N_2O_4 with 50% N_2H_4 50% UDMH	101,000 lbf thrust in vacuum	Same	Uprated to 106,000 lbf thrust in vacuum	Same
Stage III has several alternative systems for each vehicle; only one is listed here	SSPS with Aerojet liquid storable propellant engine AJ 10-118 K (9800 lbf thrust)	United Technologies/CSD, Interim Upper Stage (IUS) solid propellant rocket motor (see Table 11-3)	Centaur; 2 Pratt & Whitney RL 10A-3-3A rocket engines, 33,000 lbf thrust, H_2/O_2	Same

Source: Lockheed-Martin Astronautics, Aerojet Propulsion Company, and Pratt & Whitney Division of United Technologies Corp.

launch from the equator in a eastward direction will give the highest payload. For the same orbit altitude other trajectory inclinations have a lower payload. For the same inclination the payload decreases with orbit altitude, since more energy has to be expended to overcome gravitational attraction.

The Space Shuttle has its maximum payload when launched due east into an orbit with 28.5° inclination from Kennedy Space Flight Center in Florida, namely about 56,000 lb (or 25,455 kg) at a 100 nautical mile (185 km) orbit altitude. The payload decreases by about 100 lb (45.4 kg) for every nautical mile increase in altitude. If the inclination is 57°, the payload diminishes to about 42,000 lb (or 19,090 kg). If launched in a southerly direction from Vandenberg Air Force Base on the west coast in a 98° inclination into a circular, nearly polar orbit, the payload will be only about 30,600 lb or 13,909 kg.

The dramatic decrease of payload with circular orbits of increasing altitude and with different inclination is shown for the Pegasus, a relatively small, air-launched, space launch vehicle, in Fig. 4–16. The payload is a maximum when launching from the earth equator in the east direction, that is at 0° inclination.

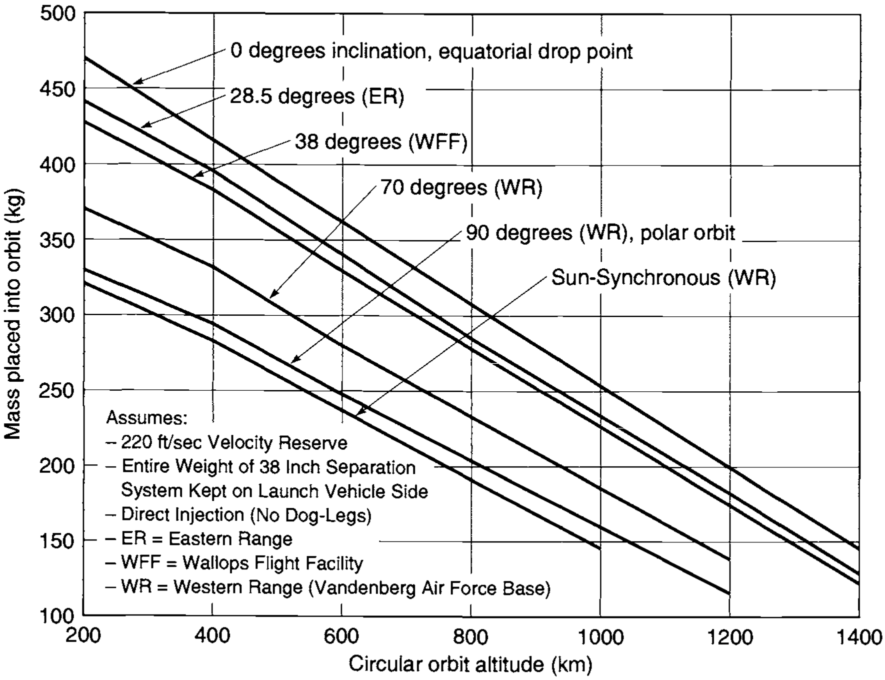


FIGURE 4–16. Decrease of payload with circular orbit altitude and orbit inclination for the Pegasus launch vehicle. This is an air-launched, relatively simple, three-stage launch vehicle of 50 in. diameter driven by a solid propellant rocket motor in each stage. (Courtesy Orbital Sciences Corporation)

The figure shows that a practical payload becomes too small for orbits higher than about 1200 km. To lift heavier payloads and to go to higher orbits requires a larger launch vehicle than the Pegasus. Figure 4–16 is based on the assumption of a particular payload separation mechanism (38 in.) and a specific Δu vehicle velocity reserve (220 ft/sec), for items such as the normal changes in atmospheric density (which can double the drag) or mass tolerances of the propulsion systems. Similar curves can be provided by the makers of all launch vehicles.

4.8. MILITARY MISSILES

The majority of all rocket propulsion systems built today are for military purposes. There is a large variety of missiles and military missions and therefore many different propulsion systems. All are chemical propulsion systems. They range from simple, small, unguided, fin-stabilized single-stage rocket projectiles (used in air-to-surface missions and surface-to-surface bombardment) up to complex, sophisticated, expensive, long-range, multistage ballistic missiles, which are intended for faraway military or strategic targets. The term "surface" means either land surface (ground launch or ground target), ocean surface (ship launched), or below the ocean surface (submarine launched). A *tactical missile* is used for attacking or defending ground troops, nearby military or strategic installations, military aircraft, or war missiles. The armed forces also use *military satellites* for missions such as reconnaissance, early warning of impending attack, secure communication, or navigation.

Strategic missiles with a range of 3000 km or more have been two- or three-stage surface-to-surface rocket-propelled missiles. Early designs used liquid propellant rocket engines and some are still in service. Beginning about 30 years ago, newer strategic missiles have used solid propellant rocket motors. Both types usually also have a liquid propellant reaction control system (RCS) for accurately adjusting the final payload flight velocity (in magnitude, direction, and position in space) at the cut-off of the propulsion system of the last stage. A solid propellant RCS version also exists. The flight analysis and ballistic trajectories of the long-range missiles are similar in many ways to those described for launch vehicles in this chapter. See Fig. 4–9.

Solid propellant rocket motors are preferred for most tactical missile missions, because they allow simple logistics and can be launched quickly (Ref. 4–15). If altitudes are low and flight durations are long, such as with a cruise missile, an air-breathing jet engine and a winged vehicle, which provides lift, will usually be more effective than a long-duration rocket. However, a large solid propellant rocket motor is still needed as a booster to launch the cruise missile and bring it up to speed. There are a variety of different tactical missions, resulting in different sized vehicles with different propulsion needs, as explained later in this section and in Ref. 4–15.

For each of the tactical missile applications, there is an optimum rocket propulsion system and almost all of them use solid propellant rocket motors. For each application there is an optimum total impulse, an optimum thrust–time profile, an optimum nozzle configuration (single or multiple nozzles, with or without thrust vector control, optimum area ratio), optimum chamber pressure, and a favored solid propellant grain configuration. Low exhaust plume gas radiation emissions in the visible, infrared or ultraviolet spectrum and certain safety features (making the system insensitive to energy stimuli) can be very important in some of the tactical missile applications; these are discussed in Chapters 12 and 18.

Short-range, uncontrolled, unguided, single-stage rocket vehicles, such as military rocket projectiles (ground and air launched) and rescue rockets, are usually quite simple in design. Their general equations of motion are derived in Section 4.3, and a detailed analysis is given in Ref. 4–1.

Unguided military rocket-propelled missiles are today produced in larger numbers than any other category of rocket-propelled vehicles. The 2.75 in. diameter, folding fin unguided solid propellant rocket missile has recently been produced in the United States in quantities of almost 250,000 per year. Guided missiles for anti-aircraft, anti-tank, or infantry support have been produced in annual quantities of hundreds and sometimes over a thousand. Table 1–6 lists several guided missiles.

Because these rocket projectiles are essentially unguided missiles, the accuracy of hitting a target depends on the initial aiming and the dispersion induced by uneven drag, wind forces, oscillations, and misalignment of nozzles, body, and fins. Deviations from the intended trajectory are amplified if the projectile is moving at a low initial velocity, because the aerodynamic stability of a projectile with fins is small at low flight speeds. When projectiles are launched from an aircraft at a relatively high initial velocity, or when projectiles are given stability by spinning them on their axis, their accuracy of reaching a target is increased two- to ten-fold, compared to a simple fin-stabilized rocket launched from rest.

In guided *air-to-air* and *surface-to-air rocket-propelled missiles* the time of flight to a given target, usually called the *time to target* t_t, is an important flight-performance parameter. With the aid of Fig. 4–17 it can be derived in a simplified form by considering the distance traversed by the rocket (called the range) to be the integrated area underneath the velocity–time curve. This simplification assumes no drag, no gravity effect, nearly horizontal flight, a relatively small distance traversed during powered flight compared to the total range, and a linear increase in velocity during powered flight.

$$t_t = \frac{S + \frac{1}{2} u_p t_p}{u_0 + u_p} \tag{4–37}$$

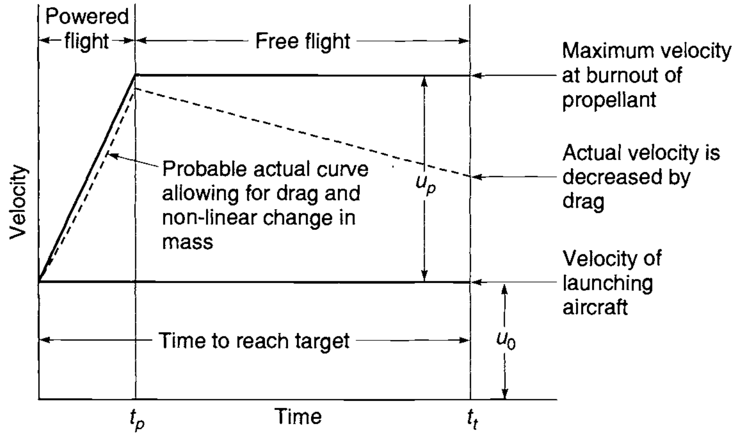


FIGURE 4-17. Simplified trajectory for an unguided, non-maneuvering, air-launched rocket projectile. Solid line shows flight velocity without drag or gravity and dashed curve shows likely actual flight.

Here S is the free-flight (unpowered) range, u_p is the velocity increase of the rocket during powered flight up to the time of burnout, t_p is the time of rocket burning, and u_0 is the initial velocity of the launching aircraft. For more accurate values, the velocity increase u_0 is the initial velocity of the launching aircraft. For more accurate values, the velocity increase u_p is given by Eq. 4–19. More accurate values can only be obtained through a detailed step-to-step trajectory analysis that considers the effects of drag and gravity.

In unguided air-launched air-to-air or air-to-surface projectiles the aiming is done by orienting the launching aircraft. In guided missiles (air-to-air, air-to-ground, ground-to-air, or ground-to-incoming-missile) the rocket's thrust direction, thrust magnitude, or thrust pulse timing can be commanded by an intelligent guidance and control system to chase a maneuvering moving target. The guidance system senses the flight path of the target, calculates a predicted impact point, and then controls the flight path of the guided missile to achieve an impact (or near-impact if a proximity fuse is used) with the target. It can also apply to a ground-launched or a satellite-launched antiballistic missile. In both the unguided projectile and the guided missile the hit probability increases as the time to target t_t is reduced. In one particular air-to-air combat situation, the effectiveness of the rocket projectile varied approximately inversely as the cube of the time to target. The best results (e.g., best hit probability) are usually achieved when the time to target is as small as practically possible.

The analysis of the missile and propulsion configuration that gives the minimum time to target over all the likely flight scenarios can be complex. The following rocket propulsion features and parameters will help to reduce the time to target, but their effectiveness will depend on the specific mission, range, guidance and control system, and the particular flight conditions.

1. *High initial thrust* or *high initial acceleration* for the missile to quickly reach a high-initial-powered flight velocity.
2. Application of additional *lower thrust* to counteract drag and gravity losses and thus maintain a high flight velocity. This can be a single rocket propulsion system that has a short high initial thrust and a smaller (10 to 25%) sustaining thrust of lower duration. It can also be a system that applies discrete pulses of thrust to increase vehicle velocity after drag forces have caused it to diminish, thus maintaining a higher average flight velocity.
3. For higher supersonic flight speeds, a *two-stage missile* can be more effective. Here the first stage is dropped off after its propellant has been consumed, thus reducing the inert mass of the next stage, and improving its mass ratio and thus its flight velocity increase.
4. If the target is highly maneuverable and if the closing velocity between missile and target is large, it may be necessary not only to provide an axial thrust, but also to apply large *side forces* or side accelerations to a tactical missile. This can be accomplished either by aerodynamic forces (lifting surfaces or flying at an angle of attack) or by multiple nozzle propulsion systems with variable or pulsing thrusts; the rocket engine then has an axial thruster and several side thrusters. The thrusters have to be so located that all the thrust forces are essentially directed through the center of gravity of the vehicle. The thrusters that provide the side accelerations have also been called *divert* thrusters, since they divert the vehicle in a direction normal to the axis of flight direction.
5. *Drag losses* can be reduced if the missile has a large L/D ratio (or a small cross-sectional area) and if the propellant density is high, allowing a smaller missile volume. The drag forces can be high if the missile travels at low altitude and high speed.

A unique military application is *rocket assisted gun launched projectiles* for attaining longer artillery ranges. Their small rocket motors withstand very high accelerations in the gun barrel (5000 to 10,000 g_0 is typical). They are in production.

4.9. AERODYNAMIC EFFECT OF EXHAUST PLUMES

The effect of rocket exhaust jets or plumes on the aerodynamic characteristics of a missile is usually to decrease the vehicle drag at supersonic missile speeds and to increase it at subsonic speeds. On *subsonic* vehicles, a supersonic rocket plume acts very much like an ejector and sucks adjacent air into its path. This affects vehicles where the rocket is located on a tapering aft end. The ejector action of the flame accelerates the adjacent air, thereby increasing the skin friction locally and usually reducing the pressure on the vehicle aft body or base plate near the nozzle exit location.

At *supersonic* speeds there often is a turbulent wake area with a low local pressure at the aft end of projectile. With the action of a rocket plume, the void space is filled with rocket gases and the pressure on the aft portion of the body is increased. This increases the pressure thrust and thus reduces the base drag. Exhaust plume effects are discussed in Chapter 18. In fact, some artillery munitions and short-range rockets can achieve increased range (by 10 to 50%) by adding a small rocket-type gas generator; its plume fills the void at the base of the projectile with reaction gas at a finite pressure, thus increasing the base pressure of the projectile and reducing the base drag.

4.10. FLIGHT STABILITY

Stability of a vehicle is achieved when the vehicle does not rotate or oscillate in flight. Unstable flights are undesirable, because pitch or yaw oscillations increase drag (flying at an angle of attack most of the time) and cause problems with instruments and sensors (target seekers, horizon scanners, sun sensors, or radar). Instability often leads to tumbling (uncontrolled turning) of vehicles, which causes missing of orbit insertion, missing targets, or sloshing of liquid propellant in tanks.

Stability can be built in by proper design so that the flying vehicle will be inherently stable, or stability can be obtained by appropriate controls, such as the aerodynamic control surfaces on an airplane, a reaction control system, or hinged multiple rocket nozzles.

Flight stability exists when the overturning moments (e.g., those due to a wind gust, thrust misalignment, or wing misalignment) are smaller than the stabilizing moments induced by thrust vector controls or by aerodynamic control surfaces. When the destabilizing moments exceed the stabilizing moments about the center of gravity, the vehicle turns or tumbles. In unguided vehicles, such as low-altitude rocket projectiles, stability of flight in a rectilinear motion is achieved by giving a large stability margin to the vehicle by using tail fins and by locating the center of gravity ahead of the center of aerodynamic pressure. In a vehicle with an active stability control system, a nearly neutral inherent stability is desired, so that the applied control forces are small, thus requiring small control devices, small RCS thrusters, small actuating mechanisms, and structural mass. Neutral stability is achieved by locating aerodynamic surfaces and the mass distribution of the components within the vehicle in such a manner that the center of gravity is only slightly above the center of aerodynamic pressure. Because the aerodynamic moments change with Mach number, the center of pressure does not stay fixed during accelerating flight but shifts, usually along the vehicle axis. The center of gravity also changes its position as propellant is consumed and the vehicle mass decreases. Thus it is usually very difficult to achieve neutral missile stability at all altitudes, speeds, and flight conditions.

Stability considerations affect rocket propulsion system design in several ways. By careful nozzle design it is possible to minimize thrust misalignment and thus to minimize torques on the vehicle and the reaction control propellant consumption. It is possible to exercise control over the travel of the center of gravity by judicious design. In liquid propellant rockets, special design provisions, special tank shapes, and a careful selection of tank location in the vehicle afford this possibility. The designer generally has less freedom in controlling the travel of the center of gravity of solid propellant rockets. By using nozzles at the end of a blast tube, as shown in Fig. 14–6, it is possible to place the solid propellant mass close to the vehicle's center of gravity. Attitude control liquid propellant engines with multiple thrusters have been used satisfactorily to obtain control moments for turning vehicles in several ways, as described in Section 4.6 and in Chapter 6.

Unguided rocket projectiles and missiles are often given a roll or rotation by inclined aerodynamic fins or inclined multiple rocket exhaust gas nozzles to improve flight stability and accuracy. This is similar to the rotation given to bullets by spiral-grooved rifles. This *spin stability* is achieved by gyroscopic effects, where an inclination of the spin axis is resisted by torques. The centrifugal effects cause problems in emptying liquid propellant tanks and extra stresses on solid propellant grains. In some applications a low-speed roll is applied not for spin stability but to assure that any effects of thrust vector deviations or aerodynamic shape misalignments are minimized and canceled out.

PROBLEMS

1. For a vehicle in gravitationless space, determine the mass ratio necessary to boost the vehicle velocity by 1600 m/sec when the effective exhaust velocity is 2000 m/sec.
Answer: 0.449.

2. What is the mass ratio m_p/m_0 for a vehicle that has one-fifth its original takeoff mass at the time of the completion of rocket operation?
Answer: 0.80.

3. Determine the burnout velocity and burnout altitude for a dragless projectile with the following parameters for a simplified vertical trajectory: $\bar{c} = 2209$ m/sec; $m_p/m_0 = 0.57$; $t_p = 5.0$ sec; and $u_0 = h_0 = 0$.
Answers: $u_p = 1815$ m/sec; $h_p = 3.89 \times 10^3$ m.

4. Assume that this projectile had a drag coefficient essentially similar to the 0° curve in Fig. 4–3 and redetermine the answers of Problem 3 and the approximate percentage errors in u_p and h_p. Use a step-by-step method.

5. A research space vehicle in gravity-free and drag-free outer space launches a smaller spacecraft into a meteor shower region. The 2 kg instrument package of this spacecraft (25 kg total mass) limits the maximum acceleration to no more than 50 m/sec^2. It is launched by a solid propellant rocket motor ($I_s = 260$ sec and $\zeta = 0.88$). Determine

(a) the maximum allowable burn time, assuming steady propellant mass flow;

(b) the maximum velocity relative to the launch vehicle.

(c) Solve for (a) and (b) if half of the total impulse is delivered at the previous propellant mass flow rate, with the other half at 20% of this mass flow rate.

6. For a satellite cruising in a circular orbit at an altitude of 500 km, determine the period of revolution, the flight speed, and the energy expended to bring a unit mass into this orbit.
Answers: 1.58 hr, 7613 m/sec, 33.5 MJ/kg.

7. A large ballistic rocket vehicle has the following characteristics: propellant mass flow rate: 12 slugs/sec (1 slug = 32.2 lbm = 14.6 kg); nozzle exit velocity: 7100 ft/sec; nozzle exit pressure: 5 psia (assume no separation); atmospheric pressure: 14.7 psia (sea level); takeoff weight: 12.0 tons (1 ton = 2000 lbf); burning time: 50 sec; nozzle exit area: 400 in.^{2}. Determine (*a*) the sea-level thrust; (*b*) the sea-level effective exhaust velocity; (*c*) the initial thrust-to-weight ratio; (*d*) the initial acceleration; (*e*) the mass inverse ratio m_0/m_f.
Answers: 81,320 lbf; 6775 ft/sec; 3.38; 2.38g_0.

8. In Problem 7 compute the altitude and missile velocity at the time of power plant cutoff, neglecting the drag of the atmosphere and assuming a simple vertical trajectory.

9. A spherical satellite has 12 identical monopropellant thrust chambers for attitude control with the following performance characteristics: thrust (each unit): 5 lbf; I_s (steady state or more than 2 sec); 240 sec; I_s (pulsing duration 20 msec): 150 sec; I_s (pulsing duration 100 msec): 200 sec; satellite weight: 3500 lbf; satellite diameter: 8 ft; satellite internal density distribution is essentially uniform; disturbing torques, Y- and Z-axes: 0.00005 ft-lbf average; disturbing torque, for X-axis: 0.001 ft-lbf average; distance between thrust chamber axes: 8 ft; maximum allowable satellite pointing position error: $\pm 1°$. Time interval between pulses is 0.030 sec.

(a) What would be the maximum and minimum vehicle angular drift per hour if no correction torque were applied?
Answers: 0.466 and 0.093 rad.

(b) What is the frequency of pulsing action (how often does an engine pair operate?) at 20-msec, 100-msec, and 2-sec pulses in order to correct for angular drift? Discuss which pulsing mode is best and which is impractical.

(c) If the satellite was to remain in orbit for 1 year with these same disturbances and had to maintain the accurate positions for 24 hr each day, how much propellant would be required? Discuss the practicality of storing and feeding such propellant.

10. For an ideal multistage launch vehicle with several stages, discuss the following: (*a*) the effect on the ideal mission velocity if the second and third stages are not started immediately but are each allowed to coast for a short period after shutoff and separation of the prior stage before rocket engine start of the next stage; (*b*) the effect on the mission velocity if an engine malfunctions and delivers a few percent less than the intended thrust but for a longer duration and essentially the full total impulse of that stage.

11. Given a cylindrically shaped space vehicle (D = 1 m, height is 0.7 m, average density is 1.1 g/cm^3) with a flat solar cell panel on an arm (mass of 32 kg, effective moment arm is 1.5 m, effective average area facing normally toward sun is 0.6 m^2) in a set of essentially frictionless bearings and in a low orbit at 160 km altitude with sunlight being received, on the average, about 60% of the period:

(a) Compute the maximum solar pressure-caused torque and the angular displacement this would cause during 1 day if not corrected.

(b) Using the data from the atmospheric table in Appendix 2 and an arbitrary average drag coefficient of 1.0 for both the body and the flat plate, compute the drag force and torque.

(c) Using stored high-pressure air at 14×10^6 N/m^2 initial pressure as the propellant for attitude control, design an attitude control system to periodically correct for these two disturbances (F, I_s, t, I_t, etc.).

(d) If the vector of the main thrust rocket of the vehicle (total impulse of 67×10^3N-sec) is misaligned and misses the center of gravity by 2 mm, what correction would be required from the attitude control system? What would need to be done to the attitude control system in *c* above to correct for this error also?

12. A bullet-shaped toy rocket has a pressurized tank of volume V_0, and is partly filled with water (an incompressible liquid) and partly with compressed air at initial pressure of 50 psia and initial ambient temperature T_0. Assume no water losses during start. Also assume that the ambient air pressure is constant for the altitudes attained by this toy rocket. The empty weight of the toy is 0.30 lbf and it can carry 1.0 lbm of water when the V_0 is half-filled with water. Make other assumptions to suit the calculations.

(a) What type of nozzle is best for this application?
Answer: Converging nozzle.

(b) What are the desired nozzle dimensions to assure vertical takeoff with about 0.5 g acceleration?

(c) What is the specific impulse of the water at start and near propellant exhaustion?

(d) What happens if only 50 psia air (no water) is ejected?

(e) What is the approximate proportion of water to air volume for maximum altitude?

(f) Sketch a simple rocket release and thrust start device and comment on its design and potential problems.

(g) About how high will it fly vertically?

13. Determine the payload for a single-stage vehicle in Example 4–3. Compare it with the two-stage vehicle.
Answer: 50.7 kg, which is 18.4% of the payload for a two-stage vehicle.

14. Use the data given in Example 4–3, except that the payload is fixed at 250 kg and the Δu is not given but has to be determined for both cases, namely equal-sized stages and stages of equal mass ratio. What can be concluded from these results and the results in the example?

15. An airplane that is flying horizontally at a 7000 m altitude, at a speed of 700 km/hr over flat country, releases an unguided missile with four small tail fins for flight stability. Determine the impact location (relative to the release point as projected onto the earth surface), the impact angle, and the time from release to target. Assume that the drag causes an average of about 8% reduction in flight velocities.

16. An earth satellite is in an elliptical orbit with the perigee at 600 km altitude and an eccentricity of $e = 0.866$. Determine the parameters of the new satellite trajectory, if a rocket propulsion system is fired in the direction of flight giving an incremental velocity of 200 m/sec (a) when fired at apogee, (b) when fired at perigee, and (c) when fired at perigee, but in the opposite direction, reducing the velocity.

17. A sounding rocket (75 kg mass, 0.25 m diameter) is speeding vertically upward at an altitude of 5000 m and a velocity of 700 m/sec. What is the deceleration in multiples of g due to gravity and drag? (Use C_D from Fig. 4–3 and use Appendix 2).

18. A single-stage weather sounding rocket has a take-off mass of 1020 kg, a sea-level initial acceleration of 2.00 g, carries 799 kg of useful propellant, has an average specific gravity of 1.20, a burn duration of 42 sec, a vehicle body shaped like a cylinder with an L/D ratio of 5.00 with a nose cone having a half angle of 12 degrees. Assume the center of gravity does not change during the flight. The vehicle tumbled (rotated in an uncontrolled manner) during the flight and failed to reach its objective. Subsequent evaluation of the design and assembly processes showed that the maximum possible thrust misalignment was 1.05 degrees with a maximum lateral off-set of 1.85 mm. Assembly records show it was 0.7 degrees and 1.1 mm for this vehicle. Since the propellant flow rate was essentially constant, the thrust at altitude cutoff was 16.0% larger than at take-off. Determine the maximum torque applied by the thrust at start and at cutoff. Then determine the approximate maximum angle through which the vehicle will rotate during powered flight, assuming no drag. Discuss the result.

SYMBOLS

a	major axis of ellipse, m, or acceleration, m/sec^2 (ft/sec^2)
A	area, m^2
b	minor axis of ellipse, m
B	numerical value of drag integral
c	effective exhaust velocity, m/sec (ft/sec)
$\bar{c}$	average effective exhaust velocity, m/sec
C_D	drag coefficient
C_L	lift coefficient
d	total derivative
D	drag force, N (lbf)
e	eccentricity of ellipse, $e = \sqrt{1 - b^2/a^2}$
e	base of natural logarithm (2.71828)
E	energy, J
F	thrust force, N (lbf)
F_f	final thrust, N
F_g	Gravitational attraction force, N
F_0	initial thrust force, N

g gravitational acceration, m/sec^2
g_0 gravitational acceleration at sea level, 9.8066 m/sec^2
$\bar{g}$ average gravitational attraction, m/sec^2
G universal or Newton's gravity constant, 6.6700 $\times 10^{11}$ m^3/kg-sec^2
h altitude, m
h_p altitude of rocket at power cutoff, m
I_s specific impulse, sec
k_d diffuse coefficient of reflectivity
k_s specular coefficient of reflectivity
l distance of moment arm, m
L lift force, N (lbf)
m instantaneous mass, kg (lbm)
m_f final mass after rocket operation, kg
m_p propellant mass, kg
m_0 initial launching mass, kg
$\dot{m}$ mass flow rate of propellant, kg/sec
M_a angular moment of inertia, kg-m^2
MR mass ratio of vehicle = m_f/m_0
n number of stages
p pressure, N/m^2 or Pa (psi)
r radius, m, or distance between the centers of two attracting masses, m
R instantaneous radius from vehicle to center of Earth, m
R_0 Effective earth radius, 6.3742 $\times$ 10^6 m
S range, m
t time, sec
t_p time from launching to power cutoff or time from propulsion start to thrust termination, sec
t_t time to target, sec
T torque, N-m (ft-lbf)
u vehicle flight velocity, m/sec (ft/sec)
u_a orbital velocity at apogee, m/sec
u_p velocity at power cutoff, m/sec, or orbital velocity at perigee, m/sec
u_0 initial or launching velocity, m/sec
w weight, N (in some problems, lbf)

Greek Letters

α angle of attack, or angular acceleration, angle/sec^2
ζ propellant mass fraction ($\zeta = m_p/m_0$)
θ angle between flight direction and horizontal, or angle of incident radiation, deg or rad
μ gravity constant for earth, 3.98600 $\times$ 10^{14} m^3/sec^2
ρ mass density, kg/m^3
τ period of revolution, sec

ψ	angle of thrust direction with horizontal
ω	angular speed, deg/sec (rad/sec)

Subscripts

e	escape condition
f	final condition at rocket thrust termination
max	maximum
p	power cutoff or propulsion termination
s	satellite
z	zenith
0	initial condition or takeoff condition

REFERENCES

4–1. J. B. Rosser, R. R. Newton, and G. L. Gross, *Mathematical Theory of Rocket Flight*, McGraw-Hill Book Company, 1947; or F. R. Gantmakher and L. M. Levin, *The Flight of Uncontrolled Rockets*, Macmillan, New York, 1964.

4–2. *Orbital Flight Handbook*, NASA SP33, 1963, Part 1: Basic Techniques and Data. Part 2: Mission Sequencing Problems. Part 3: Requirements.

4–3. V. A. Chobotov (Ed.) *Orbital Mechanics*, Educational Series, AIAA, 1991.

4–4. J. W. Cornelisse, H. F. R. Schöyer, and K. F. Wakker, *Rocket Propulsion and Space Flight Dynamics*, Pitman Publishing, London, 1979.

4–5. R. S. Wolf, "Development of a Handbook for Astrobee F Flight Performance Predictions," *Journal of Spacecraft and Rockets*, Vol. 24, No. 1, January–February 1987, pp. 5–6.

4–6. W. Hohmann, *Die Erreichbarkeit der Himmelskörper (Accessibility of Celestial Bodies)*, Oldenburg, Munich, 1925.

4–7. "Spacecraft Aerodynamic Torques," *NASA SP 8058*, January 1971 (N 71-25935).

4–8. "Spacecraft Radiation Torques," *NASA SP 8027*, October 1969 (N 71-24312).

4–9. "Spacecraft Gravitational Torques," *NASA SP 8024*, May 1964 (N 70–23418).

4–10. "Spacecraft Magnetic Torques," *NASA SP 8018*, March 1969 (N 69-30339).

4–11. W. J. Larson and J. R. Wertz, *Space Mission Analysis and Design*, Second edition, published jointly by Microcosm, Inc. and Kluwer Academic Press, 1992.

4–12. J. J. Pocha, *An Introduction to Mission Design for Geostationary Satellites*, Kluwer Academic Publishers, Hingham, MA 1987, 222 pages.

4–13. M. H. Kaplan, *Orbital Spacecraft Dynamics and Control*, John Wiley & Sons, New York, 1976.

4–14. J. R. Wertz (Ed.), *Spacecraft Attitude Determination and Control*, Kluwer Academic Publishers, Hingham, MA, 1980, 858 pages.

4–15. G. E. Jensen and D. W. Netzer, *Tactical Missile Propulsion*, Vol. 170, Progress in Astronautics and Aeronautics, AIAA, 1996.