CHAPTER 2

DEFINITIONS AND FUNDAMENTALS

Rocket propulsion is an exact but not a fundamental subject, and there are no basic scientific laws of nature peculiar to propulsion. The basic principles are essentially those of mechanics, thermodynamics, and chemistry.

Propulsion is achieved by applying a force to a vehicle, that is, accelerating the vehicle or, alternatively, maintaining a given velocity against a resisting force. This propulsive force is obtained by ejecting propellant at high velocity. This chapter deals with the definitions and the basic relations of this propulsive force, the exhaust velocity, and the efficiencies of creating and converting the energy and other basic parameters. The symbols used in the equations are defined at the end of the chapter. Wherever possible the American Standard letter symbols for rocket propulsion (as given in Ref. 2–1) are used.

2.1. DEFINITIONS

The *total impulse* I_t is the thrust force F (which can vary with time) integrated over the burning time t.

$$I_t = \int_0^t F\, dt \qquad (2\text{–}1)$$

For constant thrust and negligible start and stop transients this reduces to

$$I_t = Ft \qquad (2\text{–}2)$$

I_t is proportional to the total energy released by all the propellant in a propulsion system.

The *specific impulse* I_s is the total impulse per unit weight of propellant. It is an important figure of merit of the performance of a rocket propulsion system, similar in concept to the miles per gallon parameter used with automobiles. A higher number means better performance. Values of I_s are given in many chapters of this book and the concept of an optimum specific impulse for a particular mission is introduced later. If the total mass flow rate of propellant is $\dot{m}$ and the standard acceleration of gravity at sealevel g_0 is 9.8066 m/sec^2 or 32.174 ft/sec^2, then

$$I_s = \frac{\int_0^t F\, dt}{g_0 \int \dot{m}\, dt} \tag{2–3}$$

This equation will give a time-averaged specific impulse value for any rocket propulsion system, particularly where the thrust varies with time. During transient conditions (during start or the thrust buildup period, the shutdown period, or during a change of flow or thrust levels) values of I_s can be obtained by integration or by determining average values for F and $\dot{m}$ for short time intervals. For constant thrust and propellant flow this equation can be simplified; below, m_p is the total effective propellant mass.

$$I_s = I_t/(m_p g_0) \tag{2–4}$$

In Chapter 3 there is further discussion of the specific impulse. For constant propellant mass flow $\dot{m}$, constant thrust F, and negligibly short start or stop transients:

$$\begin{aligned} I_s &= F/(\dot{m} g_0) = F/\dot{w} \\ I_t/(m_p g_0) &= I_t/w \end{aligned} \tag{2–5}$$

The product $m_p g_0$ is the total effective propellant weight w and the weight flow rate is $\dot{w}$. The concept of weight relates to the gravitational attraction at or near sea level, but in space or outer satellite orbits, "weight" signifies the mass multiplied by an arbitrary constant, namely g_0. In the *Système International* (SI) or metric system of units I_s can be expressed simply in "seconds," because of the use of the constant g_0. In the USA today we still use the English Engineering (EE) system of units (foot, pound, second) in many of the chemical propulsion engineering, manufacturing, and test operations. In many past and current US publications, data and contracts, the specific impulse has units of thrust (lbf) divided by weight flow rate of propellants (lbf/sec), simplified as seconds. The numerical value of I_s is the same in the EE and the SI system of units. However, the units of I_s do not represent a measure of elapsed time, but a thrust force per unit "weight"-flow-rate. In this book the symbol I_s is used for the specific impulse, as listed in Ref. 2–1. For solid propellant systems the symbol I_{sp} is sometimes used, as listed in Ref. 2–2.

In a rocket nozzle the actual exhaust velocity is not uniform over the entire exit cross-section and does not represent the entire thrust magnitude. The velocity profile is difficult to measure accurately. For convenience a uniform axial velocity c is assumed which allows a one-dimensional description of the problem. This *effective exhaust velocity* c is the average equivalent velocity at which propellant is ejected from the vehicle. It is defined as

$$c = I_s g_0 = F/\dot{m} \tag{2–6}$$

It is given either in meters per second or feet per second. Since c and I_s differ only by an arbitrary constant, either one can be used as a measure of rocket performance. In the Russian literature c is generally used.

In solid propellant rockets it is difficult to measure the propellant flow rate accurately. Therefore, the specific impulse is often calculated from total impulse and the propellant weight (using the difference between initial and final motor weights and Eq. 2–5). In turn the total impulse is obtained from the integral of the measured thrust with time, using Eq. 2–1. In liquid propellant units it is possible to measure thrust and instantaneous propellant flow rate and thus to use Eq. 2–3 for calculation of specific impulse. Eq. 2–4 allows another definition for specific impulse, namely, the amount of impulse imparted to a vehicle per unit sea-level weight of propellant expended.

The term *specific propellant consumption* refers to the reciprocal of the specific impulse and is not commonly used in rocket propulsion. It is used in automotive and duct propulsion systems. Typical values are listed in Table 1–2.

The mass ratio **MR** of a vehicle or a particular vehicle stage is defined to be the final mass m_f (after rocket operation has consumed all usable propellant) divided by m_0 (before rocket operation). The various terms are depicted in Fig. 4–1.

$$\mathbf{MR} = m_f/m_0 \tag{2–7}$$

This applies to a single or a multi-stage vehicle; for the latter, the overall mass ratio is the product of the individual vehicle stage mass ratios. The final mass m_f is the mass of the vehicle after the rocket has ceased to operate when all the useful propellant mass m_p has been consumed and ejected. The final vehicle mass m_f includes all those components that are not useful propellant and may include guidance devices, navigation gear, payload (e.g., scientific instruments or a military warhead), flight control systems, communication devices, power supplies, tank structure, residual or unusable propellant, and all the propulsion hardware. In some vehicles it can also include wings, fins, a crew, life support systems, reentry shields, landing gears, etc. Typical values of **MR** can range from 60% for some tactical missiles to less than 10% for some unmanned launch vehicle stages. This mass ratio is an important parameter in analyzing flight performance, as explained in Chapter 4. When **MR** is applied to a single stage, then its upper stages become the "payload."

The *propellant mass fraction* ζ indicates the fraction of propellant mass m_p in an initial mass m_0. It can be applied to a vehicle, a stage of a vehicle or to a rocket propulsion system.

$$\zeta = m_p/m_0 \tag{2–8}$$

$$\zeta = (m_0 - m_f)/m_0 = m_p/(m_p + m_f) \tag{2–9}$$

$$m_0 = m_f + m_p \tag{2–10}$$

When applied to a rocket propulsion system, the mass ratio **MR** and propellant fraction ζ are different from those that apply to a vehicle as described above. Here the initial or loaded mass m_0 consists of the inert propulsion mass (the hardware necessary to burn and store the propellant) and the effective propellant mass. It would exclude masses of nonpropulsive components, such as payload or guidance devices. For example, in a liquid propellant rocket engine the final or inert propulsion mass m_f would include the propellant feed tanks, the pressurization system (with turbopump and/or gas pressure system), one or more thrust chambers, various piping, fittings and valves, an engine mount or engine structure, filters and some sensors. The *residual or unusable remaining propellant* is usually considered to be part of the final inert mass m_f, as it will be in this book. However, some rocket propulsion manufacturers and some literature assign residuals to be part of the propellant mass m_p. When applied to a rocket propulsion system, the value of the propellant mass fraction ζ indicates the quality of the design; a value of, say, 0.91 means that only 9% of the mass is inert rocket hardware and this small fraction contains, feeds, and burns a substantially larger mass of propellant. A high value of ζ is desirable.

The *impulse-to-weight* ratio of a complete propulsion system is defined as the total impulse I_t divided by the initial or propellant-loaded vehicle weight w_0. A high value indicates an efficient design. Under our assumptions of constant thrust and negligible start and stop transients, it can be expressed as

$$\frac{I_t}{w_0} = \frac{I_t}{(m_f + m_p)g_0} \tag{2–11}$$

$$= \frac{I_s}{m_f/m_p + 1} \tag{2–12}$$

The *thrust to weight* ratio F/w_0 expresses the acceleration (in multiples of the earth's surface acceleration of gravity) that the engine is capable of giving to its own loaded propulsion system mass. For constant thrust the maximum value of the thrust to weight ratio, or maximum acceleration, occurs just before termination or burnout because the vehicle mass has been diminished by the mass of useful propellant. Values of F/w are given in Table 2–1. The *thrust to weight ratio* is useful to compare different types of rocket systems.

Example 2–1. A rocket projectile has the following characteristics:

Initial mass	200 kg
Mass after rocket operation	130 kg
Payload, nonpropulsive structure, etc.	110 kg
Rocket operating duration	3.0 sec
Average specific impulse of propellant	240 sec

Determine the vehicle's mass ratio, propellant mass fraction, propellant flow rate, thrust, thrust-to-weight ratio, acceleration of vehicle, effective exhaust velocity, total impulse, and the impulse-to-weight ratio.

SOLUTION. Mass ratio of vehicle (Eq. 2–8) $\mathbf{MR} = m_f/m_0 = 130/200 = 0.65$; mass ratio of rocket system $\mathbf{MR} = m_f/m_0 = (130 - 110)/(200 - 110) = 0.222$. Note that the empty and initial masses of the propulsion system are 20 and 90 kg, respectively.

The propellant mass fraction (Eq. 2–9) is

$$\zeta = (m_0 - m_f)/m_0 = (90 - 20)/90 = 0.778$$

The propellant mass is $200 - 130 = 70$ kg. The propellant mass flow rate is $\dot{m} = 70/3 = 23.3$ kg/sec,

The thrust (Eq. 2–5) is

$$F = I_s \dot{w} = 240 \times 23.3 \times 9.81 = 54{,}857 \text{ N}$$

The thrust-to-weight ratio of the vehicle is

$$\text{initial value } F/w_0 = 54{,}857/(200 \times 9.81) = 28$$
$$\text{final value } 54{,}857/(130 \times 9.81) = 43$$

The maximum acceleration of the vehicle is $43 \times 9.81 = 421$ m/sec^2. The effective exhaust velocity (Eq. 2–6) is

$$c = I_s g_0 = 240 \times 9.81 = 2354 \text{ m/sec}$$

The total impulse (Eqs. 2–2 and 2–5) is

$$I_t = I_s w = 240 \times 70 \times 9.81 = 164{,}808 \text{ N-sec}$$

This result can also be obtained by multiplying the thrust by the duration. The impulse-to-weight ratio of the propulsion system (Eq. 2–11) is

$$I_t/w_0 = 164{,}808/[(200 - 110)9.81] = 187$$

2.2. THRUST

The thrust is the force produced by a rocket propulsion system acting upon a vehicle. In a simplified way, it is the reaction experienced by its structure due to the ejection of matter at high velocity. It represents the same phenomenon that pushes a garden hose backwards or makes a gun recoil. In the latter case, the forward momentum of the bullet and the powder charge is equal to the recoil or rearward momentum of the gun barrel. Momentum is a vector quantity and is defined as the product of mass times velocity. All ship propellers and oars generate their forward push at the expense of the momentum of the water or air masses, which are accelerated towards the rear. Rocket propulsion differs from these devices primarily in the relative magnitude of the accelerated masses and velocities. In rocket propulsion relatively small masses are involved which are *carried within* the vehicle and ejected at high velocities.

The thrust, due to a change in momentum, is given below. A derivation can be found in earlier editions of this book. The thrust and the mass flow are constant and the gas exit velocity is uniform and axial.

$$F = \frac{dm}{dt} v_2 = \dot{m} v_2 = \frac{\dot{w}}{g_0} v_2 \qquad (2\text{–}13)$$

This force represents the total propulsion force when the nozzle exit pressure equals the ambient pressure.

The pressure of the surrounding fluid (i..e, the local atmosphere) gives rise to the second contribution that influences the thrust. Figure 2–1 shows schematically the external pressure acting uniformly on the outer surface of a rocket chamber and the gas pressures on the inside of a typical thermal rocket engine. The size of the arrows indicates the relative magnitude of the pressure forces. The axial thrust can be determined by integrating all the pressures acting on areas that can be projected on a plane normal to the nozzle axis. The forces acting radially outward are appreciable, but do not contribute to the axial thrust because a rocket is typically an axially symmetric chamber. The conditions prior to entering the nozzle are essentially stagnation conditions.

Because of a fixed nozzle geometry and changes in ambient pressure due to variations in altitude, there can be an imbalance of the external environment or atmospheric pressure p_3 and the local pressure p_2 of the hot gas jet at the exit plane of the nozzle. Thus, for a steadily operating rocket propulsion system moving through a homogeneous atmosphere, the total thrust is equal to

$$F = \dot{m} v_2 + (p_2 - p_3) A_2 \qquad (2\text{–}14)$$

The first term is the *momentum thrust* represented by the product of the propellant mass flow rate and its exhaust velocity relative to the vehicle. The second term represents the *pressure thrust* consisting of the product of the cross-sectional area at the nozzle exit A_2 (where the exhaust jet leaves the

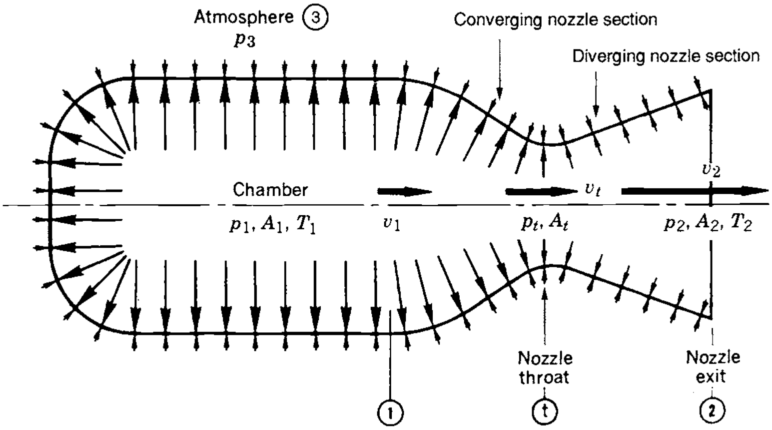


FIGURE 2–1. Pressure balance on chamber and nozzle interior walls is not uniform. The internal gas pressure (indicated by length of arrows) is highest in the chamber (p_1) and decreases steadily in the nozzle until it reaches the nozzle exit pressure p_2. The external or atmospheric pressure p_3 is uniform. At the throat the pressure is p_t. The four subscripts (shown inside circles) refer to the quantities A, v, T, and p at specific locations.

vehicle) and the difference between the exhaust gas pressure at the exit and the ambient fluid pressure. If the exhaust pressure is less than the surrounding fluid pressure, the pressure thrust is negative. Because this condition gives a low thrust and is undesirable, the rocket nozzle is usually so designed that the exhaust pressure is equal or slightly higher than the ambient fluid pressure.

When the ambient atmosphere pressure is equal to the exhaust pressure, the pressure term is zero and the thrust is the same as in Eq. 2–13. In the vacuum of space $p_3 = 0$ and the thrust becomes

$$F = \dot{m}v_2 + p_2A_2 \tag{2–15}$$

The pressure condition in which the exhaust pressure is exactly matched to the surrounding fluid pressure ($p_2 = p_3$) is referred to as the rocket nozzle with *optimum expansion ratio*. This is further elaborated upon in Chapter 3.

Equation 2–14 shows that the thrust of a rocket unit is independent of the flight velocity. Because changes in ambient pressure affect the pressure thrust, there is a variation of the rocket thrust with altitude. Because atmospheric pressure decreases with increasing altitude, the thrust and the specific impulse will increase as the vehicle is propelled to higher altitudes. This change in pressure thrust due to altitude changes can amount to between 10 and 30% of the overall thrust, as is shown for a typical rocket engine in Fig. 2–2. Table 8–1 shows the sea level and high altitude thrust for several rocket engines. Appendix 2 gives the properties of the Standard Atmosphere (ambient pressure).

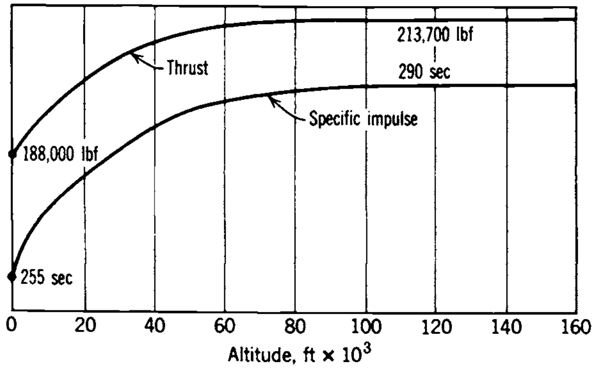


FIGURE 2–2. Altitude performance of RS 27 liquid propellant rocket engine used in early versions of the Delta launch vehicle.

2.3. EXHAUST VELOCITY

The *effective exhaust velocity* as defined by Eq. 2–6 applies to all rockets that thermodynamically expand hot gas in a nozzle and, indeed, to all mass expulsion systems. From Eq. 2–14 and for constant propellant mass flow this can be modified to

$$c = v_2 + (p_2 - p_3)A_2/\dot{m} \tag{2–16}$$

Equation 2–6 shows that c can be determined from thrust and propellant flow measurements. When $p_2 = p_3$, the effective exhaust velocity c is equal to the average actual exhaust velocity of the propellant gases v_2. When $p_2 \neq p_3$ then $c \neq v_2$. The second term of the right-hand side of Eq. 2–16 is usually small in relation to v_2; thus the effective exhaust velocity is usually close in value to the actual exhaust velocity. When $c = v_2$ the thrust (from Eq. 2–14) can be rewritten as

$$F = (\dot{w}/g_0)v_2 = \dot{m}c \tag{2–17}$$

The *characteristic velocity* has been used frequently in the rocket propulsion literature. Its symbol c^*, pronounced "cee-star," is defined as

$$c^* = p_1 A_t/\dot{m} \tag{2–18}$$

The characteristic velocity c^* is used in comparing the relative performance of different chemical rocket propulsion system designs and propellants; it is easily determined from measured data of $\dot{m}$, p_1, and A_t. It relates to the efficiency of the combustion and is essentially independent of nozzle characteristics.

However, the specific impulse I_s and the effective exhaust velocity c are functions of the nozzle geometry, such as the nozzle area ratio A_2/A_t, as shown in Chapter 3. Some values of I_s and c^* are given in Tables 5–4 and 5–5.

Example 2–2. The following measurements were made in a sea level test of a solid propellant rocket motor:

Burn duration	40 sec
Initial mass before test	1210 kg
Mass of rocket motor after test	215 kg
Average thrust	62,250 N
Chamber pressure	7.00 MPa
Nozzle exit pressure	0.070 MPa
Nozzle throat diameter	0.0855 m
Nozzle exit diameter	0.2703 m

Determine $\dot{m}$, v_2, c^*, c, and I_s at sea level, and c and I_s at 1000 and 25,000 m altitude. Assume an invariant thrust and mass flow rate and negligible short start and stop transients.

SOLUTION. The mass flow rate $\dot{m}$ is determined from the total propellant used (initial motor mass − final motor mass) and the burn time.

$$\dot{m} = (1210 - 215)/40 = 24.9 \text{ kg/sec}$$

The nozzle areas at the throat and exit are

$$A_t = \pi D^2/4 = \pi \times 0.0855^2/4 = 0.00574 \text{ m}^2$$
$$A_2 = \pi D^2/4 = \pi \times 0.2703^2/4 = 0.0574 \text{ m}^2$$

Equation 2–14 is to be solved for v_2, the actual average exhaust velocity.

$$\begin{aligned} v_2 &= F/\dot{m} - (p_2 - p_3)A_2/\dot{m} \\ &= 62{,}250/24.9 - (0.070 - 0.1013)10^6 \times 0.0574/24.9 \\ &= 2572 \text{ m/sec} \end{aligned}$$

The characteristic velocity and effective exhaust velocity are found from Eqs. 2–6 and 2–18 for sea level conditions.

$$c^* = p_1 A_t/\dot{m} = 7.00 \times 10^6 \times 0.00574/24.9 = 1613 \text{ m/sec}$$
$$I_s = F/\dot{m}g_0 = 62{,}250/(24.9 \times 9.81) = 255 \text{ sec}$$
$$c = I_s g_0 = 255 \times 9.81 = 2500 \text{ m/sec}$$

For altitudes of 1000 and 25,000 m the ambient pressure (see Appendix 2) is 0.0898 and 0.00255 MPa. From Eq. 2–16 the altitude values of c can be obtained.

$$c = v_2 + (p_2 - p_3)A_2/\dot{m}$$

At 1000 m altitude,

$$c = 2572 + (0.070 - 0.0898) \times 10^6 \times 0.0574/24.9 = 2527 \text{ m/sec}$$
$$I_s = 2527/9.81 = 258 \text{ sec}$$

At 25,000 m altitude,

$$c = 2572 + (0.070 - 0.00255) \times 10^6 \times 0.0574/24.9 = 2727 \text{ m/sec}$$
$$I_s = 2727/9.80 = 278 \text{ sec}$$

2.4. ENERGY AND EFFICIENCIES

Although efficiencies are not commonly used directly in designing rocket units, they permit an understanding of the energy balance of a rocket system. Their definitions are arbitrary, depending on the losses considered, and any consistent set of efficiencies, such as the one presented in this section, is satisfactory in evaluating energy losses. As stated previously, two types of energy conversion processes occur in any propulsion system, namely, the generation of energy, which is really the conversion of stored energy into available energy and, subsequently, the conversion to the form in which a reaction thrust can be obtained. The kinetic energy of ejected matter is the form of energy useful for propulsion. The *power of the jet* P_{jet} is the time rate of expenditure of this energy, and for a constant gas ejection velocity v this is a function of I_s and F

$$P_{\text{jet}} = \tfrac{1}{2}\dot{m}v^2 = \tfrac{1}{2}\dot{w}g_0 I_s^2 = \tfrac{1}{2}Fg_0 I_s = \tfrac{1}{2}Fv_2 \qquad (2\text{–}19)$$

The term *specific power* is sometimes used as a measure of the utilization of the mass of the propulsion system including its power source; it is the jet power divided by the loaded propulsion system mass, P_{jet}/m_0. For electrical propulsion systems which carry a heavy, relatively inefficient energy source, the specific power can be much lower than that of chemical rockets. The energy input from the energy source to the rocket propulsion system has different forms in different rocket types. For chemical rockets the energy is created by combustion. The maximum energy available per unit mass of chemical propellants is the heat of the combustion reaction Q_R; the *power input to a chemical engine* is

$$P_{\text{chem}} = \dot{m}Q_R J \qquad (2\text{–}20)$$

where J is a conversion constant which depends on the units used. A large portion of the energy of the exhaust gases is unavailable for conversion into kinetic energy and leaves the nozzle as residual enthalpy. This is analogous to the energy lost in the high-temperature exhaust gases of internal combustion engines.

The *combustion efficiency* for chemical rockets is the ratio of the actual and the ideal heat of reaction per unit of propellant and is a measure of the source efficiency for creating energy. Its value is high (approximately 94 to 99%), and it is defined in Chapter 5. When the power input P_{chem} is multiplied by the combustion efficiency, it becomes the power available to the propulsive device, where it is converted into the kinetic power of the exhaust jet. In electric propulsion the analogous efficiency is the power conversion efficiency. For solar cells it has a low value; it is the efficiency for converting solar radiation energy into electric power (10 to 20%).

The *power transmitted to the vehicle* at any one time is defined in terms of the thrust of the propulsion system F and the vehicle velocity u:

$$P_{\text{vehicle}} = Fu \tag{2–21}$$

The *internal efficiency* of a rocket propulsion system is an indication of the effectiveness of converting the system's energy input to the propulsion device into the kinetic energy of the ejected matter; for example, for a chemical unit it is the ratio of the kinetic power of the ejected gases expressed by Eq. 2–19 divided by the power input of the chemical reaction as given in Eq. 2–20. Internal efficiencies are used in Example 2–3. The energy balance diagram for a chemical rocket (Fig. 2–3) shows typical losses. The internal efficiency can be expressed as

$$\eta_{\text{int}} = \frac{\text{kinetic power in jet}}{\text{available chemical power}} = \frac{\frac{1}{2}\dot{m}v^2}{\eta_{\text{comb}} P_{\text{chem}}} \tag{2–22}$$

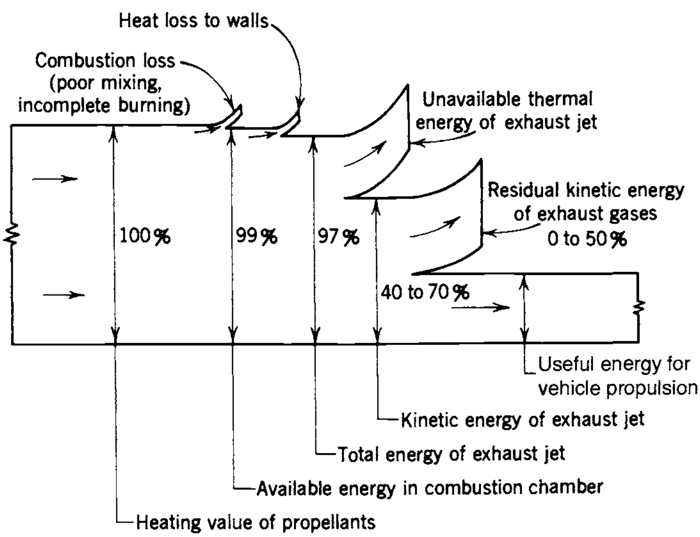


FIGURE 2–3. Typical energy balance diagram for a chemical rocket.

Typical values of η_{int} are listed later in Example 2–3.

The *propulsive efficiency* (Fig. 2–4) determines how much of the kinetic energy of the exhaust jet is useful for propelling a vehicle. It is also used often with duct jet engines and is defined as

$$\eta_P = \frac{\text{vehicle power}}{\text{vehicle power} + \text{residual kinetic jet power}}$$
$$= \frac{Fu}{Fu + \frac{1}{2}(\dot{w}/g_0)(c-u)^2} = \frac{2u/c}{1+(u/c)^2} \tag{2–23}$$

where F is the thrust, u the absolute vehicle velocity, c the effective rocket exhaust velocity with respect to the vehicle, $\dot{w}$ the propellant weight flow rate, and η_p the propulsive efficiency. The propulsive efficiency is a maximum when the forward vehicle velocity is exactly equal to the exhaust velocity. Then the residual kinetic energy and the absolute velocity of the jet are zero and the exhaust gases stand still in space.

While it is desirable to use energy economically and thus have high efficiencies, there is also the problem of minimizing the expenditure of ejected mass, which in many cases is more important than minimizing the energy. In nuclear reactor energy and some solar energy sources, for example, there is an almost unlimited amount of heat energy available; yet the vehicle can only carry a limited amount of working fluid. Economy of mass expenditures of working fluid can be obtained if the exhaust velocity is high. Because the specific impulse is proportional to the exhaust velocity, it is a measure of this propellant mass economy.

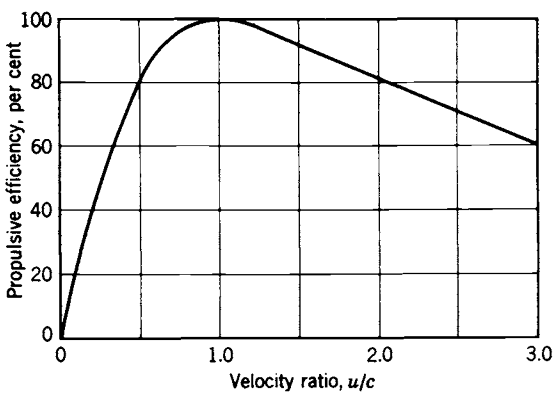


FIGURE 2–4. Propulsive efficiency at varying velocities.

2.5. TYPICAL PERFORMANCE VALUES

Typical values of representative performance parameters for different types of rocket propulsion are given in Table 2–1 and in Fig. 2–5.

Chemical rockets have relatively low values of specific impulse, relatively light machinery (i.e., low engine weight), a very high thrust capability, and therefore high acceleration and high specific power. At the other extreme, the ion propulsion devices have a very high specific impulse, but they must carry a heavy electrical power source with them to deliver the power necessary for high ejection velocities. The very low acceleration potential for the electrical propulsion units and those using solar radiation energy usually requires a long period for accelerating and thus these systems are best used for missions where the flight time is long. The low thrust values of electrical systems imply that they are not useful in fields of strong gravitational gradients (for takeoff or landing) but are best used in a true space flight mission.

The chemical systems (solid and liquid propellant rockets) are fully developed and widely used for many different vehicle applications. They are described in Chapters 5 to 15. Electrical propulsion has been in operation in many space flight applications (see Chapter 19). Some of the other types are still in their exploratory or development phase, but may become useful.

Example 2–3. As a comparison of different propulsion systems, compute the energy input and the propellant flow required for 100 N thrust with several types of propulsion systems.

SOLUTION. From Equations 2–13 and 2–19,

$$\dot{m} = F/(I_s g_0)$$
$$\text{power input} = P_{\text{jet}}/\eta_{\text{int}} = \tfrac{1}{2}\dot{m}v_2^2/\eta_{\text{int}}$$

From Table 2–1 typical values of I_s and from experience typical internal efficiencies were selected. Depending on the propellant and the design, these values may vary somewhat. The equations above were solved for $\dot{m}$ and the power input as indicated in the table below.

Engine Type	η_{int}	I_s	v_2 (m/sec)	$\dot{m}$ (kg/sec)	Power Input (kW)
Chemical rocket	0.50	300	2940	0.0340	294
Nuclear fission	0.50	800	7840	0.0128	787
Arc—electrothermal	0.50	600	5880	0.0170	588
Ion electrostatic	0.90	2000	19,600	0.0051	1959

More than half a megawatt of power is needed for the last three propulsion systems, but the propellant flows are small. The data for the last two types are illustrative, but hypothetical. To date the largest experimental units have been about 120 kW for arcjets and perhaps 10 kW with ion propulsion. Although thruster designs for megawatt-level units are feasible, it is unlikely that the needed flight-qualified electrical power generator would be available in the next decade.

TABLE 2–1. Ranges of Typical Performance Parameters for Various Rocket Propulsion Systems

Engine Type	Specific Impulse[a] (sec)	Maximum Temperature (°C)	Thrust-to-Weight Ratio[b]	Propulsion Duration	Specific Power[c] (kW/kg)	Typical Working Fluid	Status of Technology
Chemical—solid or liquid bipropellant	200–410	2500–4100	10^{-2}–100	Seconds to a few minutes	10^{-1}–10^{3}	Liquid or solid propellants	Flight proven
Liquid monopropellant	180–223	600–800	10^{-1}–10^{-2}	Seconds to minutes	0.02–200	N_2H_4	Flight proven
Nuclear fission	500–860	2700	10^{-2}–30	Seconds to minutes	10^{-1}–10^{3}	H_2	Development was stopped
Resistojet	150–300	2900	10^{-2}–10^{-4}	Days	10^{-3}–10^{-1}	H_2, N_2H_4	Flight proven
Arc heating—electrothermal	280–1200	20,000	10^{-4}–10^{-2}	Days	10^{-3}–1	N_2H_4,H_2,NH_3	Flight proven
Electromagnetic including Pulsed Plasma (PP)	700–2500	—	10^{-6}–10^{-4}	Weeks	10^{-3}–1	H_2 Solid for PP	Flight proven
Hall effect	1000–1700	—	10^{-4}	Weeks	10^{-1}–5×10^{-1}	Xe	Flight proven
Ion—electrostatic	1200–5000	—	10^{-6}–10^{--4}	Months	10^{-3}–1	Xe	Several have flown
Solar heating	400–700	1300	10^{-3}–10^{-2}	Days	10^{-2}–1	H_2	In development

[a] At p_1 = 1000 psia and optimum gas expansion at sea level ($Ip_2 = p_3$ = 14.7 psia).
[b] Ratio of thrust force to full propulsion system sea level weight (with propellants, but without payload).
[c] Kinetic power per unit exhaust mass flow.

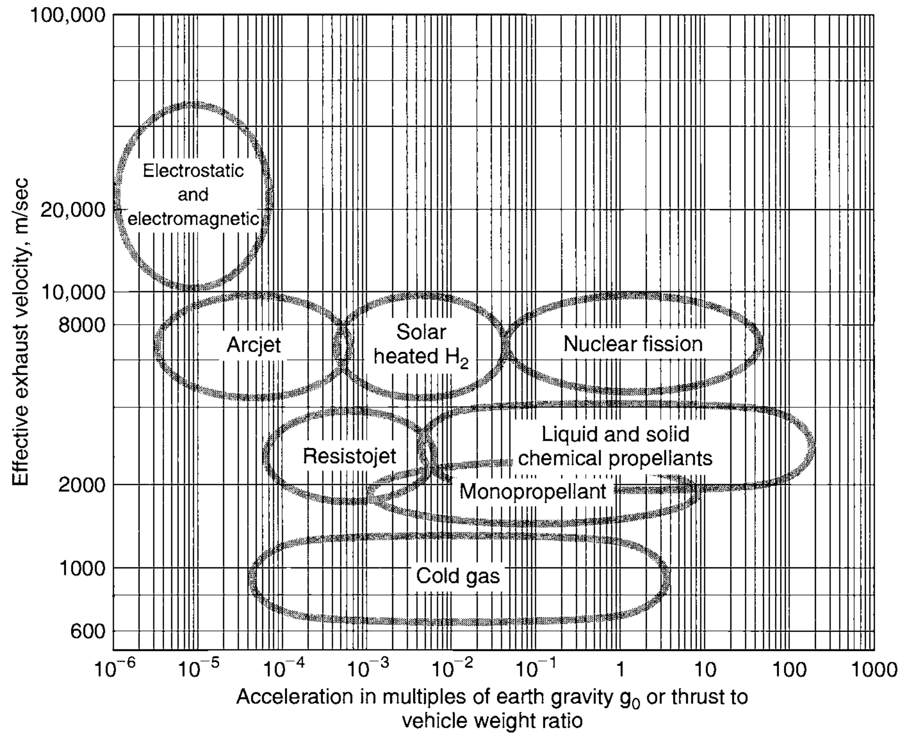


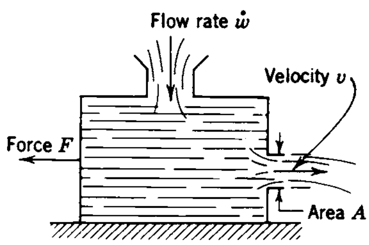
FIGURE 2–5. Exhaust velocities as a function of typical vehicle accelerations. Regions indicate approximate performance values for different types of propulsion systems. The mass of the vehicle includes the propulsion system, but the payload is assumed to be zero.

PROBLEMS

When solving problems, three appendixes (see end of book) may be helpful:

Appendix 1. Conversion Factors and Constants
Appendix 2. Properties of the Earth's Standard Atmosphere
Appendix 3. Summary of Key Equations

1. Prove that the value of the reaction thrust F equals twice the total dynamic pressure across the area A for an incompressible fluid as shown below.



2. The following data are given for a certain rocket unit: thrust, 8896 N; propellant consumption, 3.867 kg/sec; velocity of vehicle, 400 m/sec; energy content of propellant, 6.911 MJ/kg. Assume 100% combustion efficiency.

   Determine (*a*) the effective velocity; (*b*) the kinetic jet energy rate per unit flow of propellant; (c) the internal efficiency; (*d*) the propulsive efficiency; (*e*) the overall efficiency; (*f*) the specific impulse; (*g*) the specific propellant consumption.
   Answers: (*a*) 2300 m/sec; (*b*) 2.645 MJ-sec/kg; (*c*) 38.3%; (*d*) 33.7%; (*e*) 13.3%; (*f*) 234.7 sec; (*g*) 0.00426 sec^{-1}.

3. A certain rocket has an effective exhaust velocity of 7000 ft/sec; it consumes 280 lbm/sec of propellant mass, each of which liberates 2400 Btu/lbm. The unit operates for 65 sec. Construct a set of curves plotting the propulsive, internal, and overall efficiencies versus the velocity ratio u/c ($0 < u/c < 1.0$). The rated flight velocity equals 5000 ft/sec. Calculate (*a*) the specific impulse; (*b*) the total impulse; (*c*) the mass of propellants required; (*d*) the volume that the propellants occupy if their average specific gravity is 0.925.
   Answers: (*a*) 217.5 sec; (*b*) 3,960,000 lbf-sec; (*c*) 18,200 lbm; (*d*) 315 ft^3.

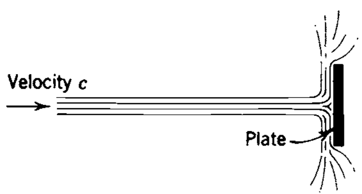
4. For the rocket in Problem 2, calculate the specific power, assuming a propulsion system dry mass of 80 kg and a duration of 3 min.

5. For the values given in Table 2–1 for the various propulsion systems, calculate the total impulse for a fixed propellant mass of 2000 kg.

6. A jet of fluid hits a stationary flat plate in the manner shown below.

   (a) If there is 50 kg of fluid flowing per minute at an abolute velocity of 200 m/sec, what will be the force on the plate?
   Answer: 167 N.

   (b) What will this force be when the plate moves in the direction of flow at $u = 50$ km/h?
   Answer: 144 N.



7. Plot the variation of the thrust and specific impulse against altitude, using the atmospheric pressure information given in Appendix 2, and the data for the Minuteman first-stage rocket thrust chamber in Table 11–3. Assume that $p_2 = 8.66$ psia.

8. Derive an equation relating the mass ratio **MR** and the propellant mass fraction.
   Answer: $\zeta = 1 - \mathbf{MR}$.

SYMBOLS (English engineering units are given in parentheses)

A area, m^2 (ft^2)
A_t nozzle throat area, m^2 (ft^2)
A_2 exist area of nozzle, m^2 (ft^2)
c effective velocity, m/sec (ft/sec)
c^* characteristic velocity, m/sec (ft/sec)
E energy, J (ft-lbf)
F thrust force, N (lbf)
g_0 standard sea level acceleration of gravity, 9.80665 m/sec^2 (32.174 ft/sec^2)
I_s specific impulse, sec
I_t impulse or total impulse, N-sec (lbf-sec)
J conversion factor or mechanical equivalent of heat, 4.184 J/cal or 1055 J/Btu or 778 ft-lbf/Btu.
m mass, kg (slugs) (1 slug = mass of 32.174 lb of weight at sea level)
$\dot{m}$ mass flow rate, kg/sec (lbm/sec)
m_f final mass (after rocket propellant is ejected), kg (lbm or slugs)
m_p propellant mass, kg (lbm or slugs)
m_0 initial mass (before rocket propellant is ejected), kg (lbm or slugs)
MR mass ratio (m_f/m_0)
p pressure, pascal [Pa] or N/m^2 (lbf/ft^2)
p_3 ambient or atmospheric pressure, Pa (lbf/ft^2)
p_2 rocket gas pressure at nozzle exit, Pa (lbf/ft^2)
p_1 chamber pressure, Pa (lbf/ft^2)
P power, J/sec (ft-lbf/sec)
P_s specific power, J/sec-kg (ft-lbf/sec-lbm)
Q_R heat of reaction per unit propellant, J/kg (Btu/lbm)
t time, sec
u vehicle velocity, m/sec (ft/sec)
v_2 gas velocity leaving the rocket, m/sec (ft/sec)
w weight, N or $kg\text{-}m/sec^2$ (lbf)
$\dot{w}$ weight flow rate, N/sec (lbf/sec)
w_0 initial weight, N or $kg\text{-}m/sec^2$ (lbf)

Greek Letters

ζ propellant mass fraction
η efficiency
η_{comb} combustion efficiency
η_{int} internal efficiency
η_p propulsive efficiency

REFERENCES

2–1. "American National Standard Letter Symbols for Rocket Propulsion," *ASME Publication Y 10.14*, 1959.

2–2. "Solid Propulsion Nomenclature Guide," *CPIA Publication 80*, Chemical Propulsion Information Agency, Johns Hopkins University, Laurel, MD., May 1965, 18 pages.