

Lesson 6 Review of fundamentals: Fluid flow

The specific objective of this lesson is to conduct a brief review of the fundamentals of fluid flow and present:

1. A general equation for conservation of mass and specific equations for steady and incompressible flows
2. A general equation for conservation of momentum in integral form and discuss simplifications
3. Bernoulli equation and introduce the concepts of total, static and velocity pressures
4. Modified Bernoulli equation and introduce expression for head loss and fan/pump power
5. Methods for evaluating friction pressure drops with suitable correlations for friction factor
6. The concept of minor losses

At the end of the lesson, the student should be able to:

1. Write the general equation of mass transfer and be able to reduce it for incompressible and steady flows
2. Write the general equation of momentum transfer and reduce it to incompressible, steady flows
3. Apply equations of conservation of mass and momentum to simple problems
4. Write Bernoulli equation and define static, velocity and datum pressures and heads
5. Write modified Bernoulli equation to account for frictional losses and presence of fan/pump
6. Apply Bernoulli and modified Bernoulli equations to simple fluid flow problems relevant to refrigeration and air conditioning
7. Estimate friction pressure drops and minor losses

6.1. Fluid flow

In refrigeration and air-conditioning systems various fluids such as air, water and refrigerants flow through pipes and ducts. The flow of these fluids is subjected to certain fundamental laws. The subject of “Fluid Mechanics” deals with these aspects. In the present lesson, fundamentals of fluid flow relevant to refrigeration and air conditioning is discussed. Fluid flow in general can be compressible, i.e., the density of the fluid may vary along the flow direction. However in most of the refrigeration and air conditioning applications the density variations may be assumed to be negligible. Hence, the fluid flow for such systems is treated as incompressible. This assumption simplifies the fluid flow problem considerably. This assumption is valid as long as the velocity fluid is considerably less than the velocity of sound (Mach number, ratio of fluid velocity to sonic velocity less than 0.3). To analyze the fluid flow problems, in addition to energy conservation (1st law of thermodynamics), one has to consider the conservation of mass and momentum.

6.1.1. Conservation of mass:

As the name implies, this law states that mass is a conserved parameter, i.e., it can neither be generated nor destroyed; it can only be transferred. Mathematically, the equation of conservation of mass for a control volume is given by:

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \quad (6.1)$$

The first term on the left represents the rate of change of mass within the control volume, while the second term represents the net rate of mass flux through the control surface. The above equation is also known as *continuity equation*.

In most of the refrigeration and air conditioning systems, the fluid flow is usually steady, i.e., the mass of the control volume does not change with time. For such a steady flow process, Eq. (6.1) becomes:

$$\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \quad (6.2)$$

If we apply the above steady flow equation to a duct shown in Fig. 6.1, we obtain:

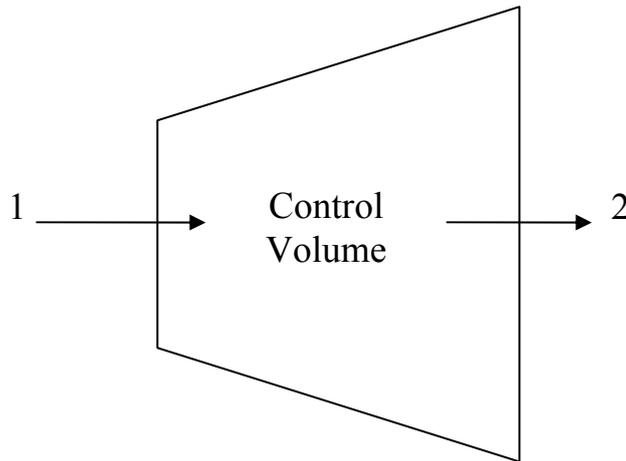


Fig. 6.1. Steady fluid flow through a duct

$$\dot{m}_1 = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m}_2 = \dot{m} \quad (6.3)$$

where \dot{m} is the mass flow rate of fluid through the control volume, ρ , A and V are the density, cross sectional area and velocity of the fluid respectively.

If we assume that the flow is incompressible ($\rho_1 = \rho_2 = \rho$), then the above equation reduces to:

$$A_1 V_1 = A_2 V_2 \quad (6.4)$$

The above equation implies that when $A_1 > A_2$, then $V_1 < V_2$, that is velocity increases in the direction of flow. Such a section is called a *nozzle*. On the other hand, if $A_1 <$

A_2 , then $V_1 > V_2$ and velocity reduces in the direction of flow, this type of section is called as *diffuser*.

6.1.2. Conservation of momentum:

The momentum equation is mathematical expression for the Newton's second law applied to a control volume. Newton's second law for fluid flow relative to an inertial coordinate system (control volume) is given as:

$$\left. \frac{d\vec{P}}{dt} \right)_{\text{control volume}} = \frac{\partial}{\partial t} \int_{CV} v\rho dV + \int_{CS} v\rho\vec{V} \cdot d\vec{A} = \vec{F})_{\text{on control volume}}$$

and

$$\vec{F})_{\text{on control volume}} = \sum \vec{F}_S + \sum \vec{F}_B = \frac{\partial}{\partial t} \int_{CV} v\rho dV + \int_{CS} v\rho\vec{V} \cdot d\vec{A}$$
(6.5)

In the above equation, $\left. \frac{d\vec{P}}{dt} \right)_{\text{control volume}}$ is the rate of change of linear momentum of the control volume, $\vec{F})_{\text{on control volume}}$ is the summation of all the forces acting on the control volume, $\sum \vec{F}_S$ and $\sum \vec{F}_B$ are the net surface and body forces acting on the control volume, \vec{V} is the velocity vector with reference to the control volume and v is the velocity (momentum per unit mass) with reference to an inertial (non-accelerating) reference frame. When the control volume is not accelerating (i.e., when it is stationary or moving with a constant velocity), then \vec{V} and v refer to the same reference plane.

The above equation states that the sum of all forces (surface and body) acting on a non accelerating control volume is equal to the sum of the rate of change of momentum inside the control volume and the net rate of flux of momentum out through the control surface. For steady state the linear momentum equation reduces to:

$$\vec{F} = \vec{F}_S + \vec{F}_B = \int_{CS} \vec{V}\rho\vec{V} \cdot d\vec{A} \quad \text{for steady state} \quad (6.6)$$

The surface forces consist of all the forces transmitted across the control surface and may include pressure forces, force exerted by the physical boundary on the control surface etc. The most common body force encountered in most of the fluid flow problems is the gravity force acting on the mass inside the control volume.

The linear momentum equation discussed above is very useful in the solution of many fluid flow problems. Some of the applications of this equation are: force exerted by the fluid flow on nozzles, bends in a pipe, motion of rockets, water hammers etc. [Example](#) shows the application of linear momentum equation.

The moment-of-momentum equation is the equation of conservation of angular momentum. It states that the net moment applied to a system is equal to the rate of

change of angular momentum of the system. This equation is applied for hydraulic machines such as pumps, turbines, compressors etc.

6.1.3. Bernoulli's equation:

The Bernoulli's equation is one of the most useful equations that is applied in a wide variety of fluid flow related problems. This equation can be derived in different ways, e.g. by integrating Euler's equation along a [streamline](#), by applying first and second laws of thermodynamics to [steady, irrotational, inviscid](#) and incompressible flows etc. In simple form the Bernoulli's equation relates the pressure, velocity and elevation between any two points in the flow field. It is a scalar equation and is given by:

$$\begin{array}{ccccccc} \frac{p}{\rho g} & + & \frac{V^2}{2g} & + & z & = & H = \text{constant} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{pressure} & & \text{velocity} & & \text{static} & & \text{total} \\ \text{head} & & \text{head} & & \text{head} & & \text{head} \end{array} \quad (6.7)$$

Each term in the above equation has dimensions of length (i.e., meters in SI units) hence these terms are called as pressure head, velocity head, static head and total heads respectively. Bernoulli's equation can also be written in terms of pressures (i.e., Pascals in SI units) as:

$$\begin{array}{ccccccc} p & + & \rho \frac{V^2}{2} & + & \rho g z & = & p_T \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{static} & & \text{velocity} & & \text{pressure due} & & \text{total} \\ \text{pressure} & & \text{pressure} & & \text{to datum} & & \text{pressure} \end{array} \quad (6.8)$$

Bernoulli's equation is valid between any two points in the flow field when the flow is *steady, irrotational, inviscid and incompressible*. The equation is valid along a *streamline for rotational, steady and incompressible flows*. Between any two points 1 and 2 in the flow field for irrotational flows, the Bernoulli's equation is written as:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad (6.9)$$

Bernoulli's equation can also be considered to be an alternate statement of conservation of energy (1st law of thermodynamics). The equation also implies the possibility of conversion of one form of pressure into other. For example, neglecting the pressure changes due to datum, it can be concluded from Bernoulli's equation that the static pressure rises in the direction of flow in a diffuser while it drops in the direction of flow in case of nozzle due to conversion of velocity pressure into static pressure and vice versa. Figure 6.2 shows the variation of total, static and velocity pressure for steady, incompressible and inviscid, fluid flow through a pipe of uniform cross-section.

Since all real fluids have finite viscosity, i.e. in all actual fluid flows, some energy will be lost in overcoming friction. This is referred to as head loss, i.e. if the fluid

were to rise in a vertical pipe it will rise to a lower height than predicted by Bernoulli's equation. The head loss will cause the pressure to decrease in the flow direction. If the head loss is denoted by H_l , then Bernoulli's equation can be modified to:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_l \quad (6.10)$$

Figure 6.2 shows the variation of total, static and velocity pressure for steady, incompressible fluid flow through a pipe of uniform cross-section without viscous effects (solid line) and with viscous effects (dashed lines).

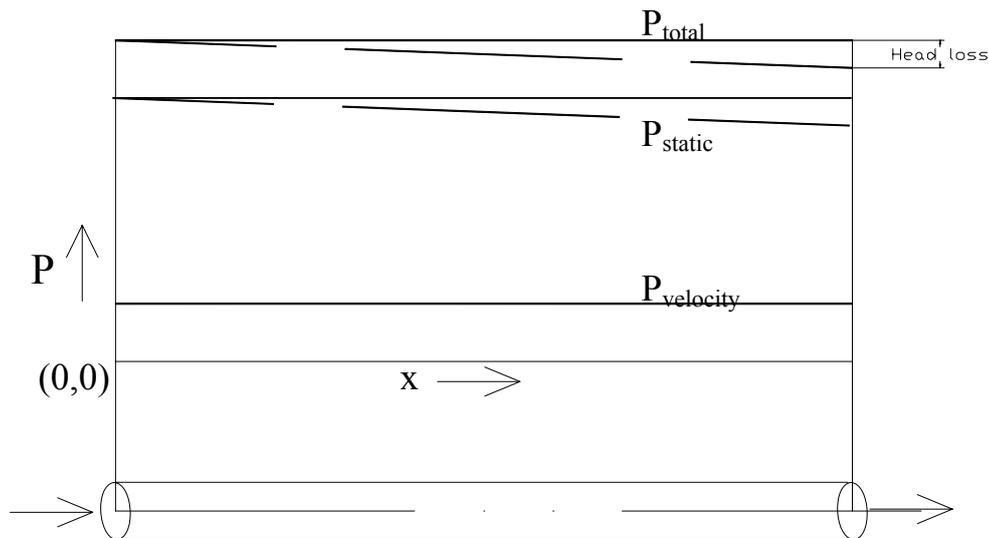


Fig. 6.2. Application of Bernoulli equation to pipe flow

Since the total pressure reduces in the direction of flow, sometimes it becomes necessary to use a pump or a fan to maintain the fluid flow as shown in Fig. 6.3.

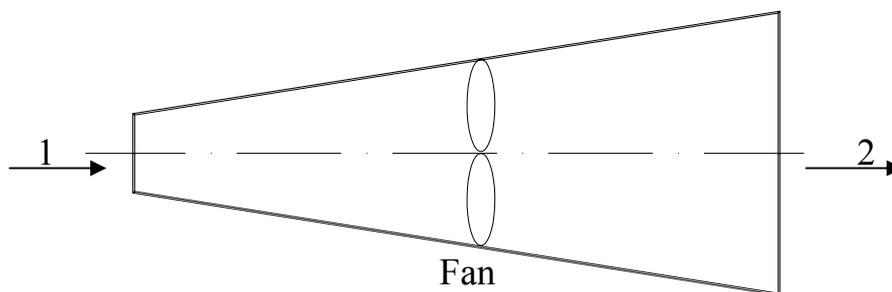


Fig. 6.3. Air flow through a duct with a fan

Energy is added to the fluid when fan or pump is used in the fluid flow conduit (Fig. 6.3), then the modified Bernoulli equation is written as:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_l \quad (6.11)$$

where H_p is the gain in head due to fan or pump and H_l is the loss in head due to friction. When fan or pump is used, the power required (W) to drive the fan/pump is given by:

$$W = \left(\frac{\dot{m}}{\eta_{\text{fan}}} \right) \left(\frac{(p_2 - p_1)}{\rho} + \frac{(V_2^2 - V_1^2)}{2} + g(z_2 - z_1) + \frac{gH_l}{\rho} \right) \quad (6.12)$$

where \dot{m} is the mass flow rate of the fluid and η_{fan} is the energy efficiency of the fan/pump. Some of the terms in the above equation can be negligibly small, for example, for air flow the potential energy term $g(z_2 - z_1)$ is quite small compared to the other terms. For liquids, the kinetic energy term $(v_2^2 - v_1^2)/2$ is relatively small. If there is no fan or pump then W is zero.

6.1.4. Pressure loss during fluid flow:

The loss in pressure during fluid flow is due to:

- a) Fluid friction and turbulence
- b) Change in fluid flow cross sectional area, and
- c) Abrupt change in the fluid flow direction

Normally pressure drop due to fluid friction is called as major loss or frictional pressure drop Δp_f and pressure drop due to change in flow area and direction is called as minor loss Δp_m . The total pressure drop is the summation of frictional pressure drop and minor loss. In most of the situations, the temperature of the fluid does not change appreciably along the flow direction due to pressure drop. This is due to the fact that the temperature tends to rise due to energy dissipation by fluid friction and turbulence, at the same time temperature tends to drop due to pressure drop. These two opposing effects more or less cancel each other and hence the temperature remains almost constant (assuming no heat transfer to or from the surroundings).

Evaluation of frictional pressure drop:

When a fluid flows through a pipe or a duct, the relative velocity of the fluid at the wall of the pipe/duct will be zero, and this condition is known as a *no-slip condition*. The no-slip condition is met in most of the common fluid flow problems (however, there are special circumstances under which the no-slip condition is not satisfied). As a result of this a velocity gradient develops inside the pipe/duct beginning with zero at the wall to a maximum, normally at the axis of the conduit. The velocity profile at any cross section depends on several factors such as the type of fluid flow (i.e. [laminar or](#)

[turbulent](#)), condition of the walls (e.g. adiabatic or non-adiabatic) etc. This velocity gradient gives rise to shear stresses ultimately resulting in frictional pressure drop.

The Darcy-Weisbach equation is one of the most commonly used equations for estimating frictional pressure drops in internal flows. This equation is given by:

$$\Delta p_f = f \frac{L}{D} \left(\frac{\rho V^2}{2} \right) \quad (6.13)$$

where f is the dimensionless friction factor, L is the length of the pipe/duct and D is the diameter in case of a circular duct and hydraulic diameter in case of a noncircular duct. The friction factor is a function of Reynolds number, $Re_D = \left(\frac{\rho V D}{\mu} \right)$ and the relative surface of the pipe or duct surface in contact with the fluid.

For steady, [fully developed](#), laminar, incompressible flows, the Darcy friction factor f (which is independent of surface roughness) is given by:

$$f = \frac{64}{Re_D} \quad (6.14)$$

For turbulent flow, the friction factor can be evaluated using the empirical correlation suggested by Colebrook and White is used, the correlation is given by:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[\frac{k_s}{3.7D} + \frac{2.51}{(Re_D)\sqrt{f}} \right] \quad (6.15)$$

Where k_s is the average roughness of inner pipe wall expressed in same units as the diameter D . Evaluation of f from the above equation requires iteration since f occurs on both the sides of it.

ASHRAE suggests the following form for determination of friction factor,

$$f_1 = 0.11 \left(\frac{k_s}{D} + \frac{0.68}{Re_D} \right)^{0.25} \quad (6.16)$$

If f_1 determined from above equation equals or exceeds 0.018 then f is taken to be same as f_1 . If it is less than 0.018 then f is given by:

$$f = 0.85f_1 + 0.0028 \quad (6.17)$$

Another straightforward equation suggested by Haaland (1983) is as follows:

$$\frac{1}{f^{1/2}} \approx -1.8 \log_{10} \left[\frac{6.9}{Re_D} + \left(\frac{k_s/D}{3.7} \right)^{1.11} \right] \quad (6.18)$$

Evaluation of minor loss, Δp_m :

The process of converting static pressure into kinetic energy is quite efficient. However, the process of converting kinetic energy into pressure head involves losses. These losses, which occur in ducts because of bends, elbows, joints, valves etc. are called minor losses. This term could be a misnomer, since in many cases these are more significant than the losses due to friction. For almost all the cases, the minor losses are determined from experimental data. In turbulent flows, the loss is proportional to square of velocity. Hence these are expressed as:

$$\Delta p_m = K \frac{\rho V^2}{2} \quad (6.19)$$

Experimental values for the constant K are available for various valves, elbows, diffusers and nozzles and other fittings. These aspects will be discussed in a later chapter on distribution of air.

Questions:

1. Is the flow incompressible if the velocity field is given by $\vec{V} = 2x^3\vec{i} - 6x^2y\vec{j} + t\vec{k}$? [\(Answer\)](#)
2. Derive the expression of fully developed laminar flow velocity profile through a circular pipe using control volume approach. [\(Answer\)](#)
3. A Static-pitot (Fig. Q3) is used to measure the flow of an inviscid fluid having a density of 1000 kg/m^3 in a 100 mm diameter pipe. What is the flow rate through the duct assuming the flow to be steady and incompressible and mercury as the manometer fluid? [\(Solution\)](#)

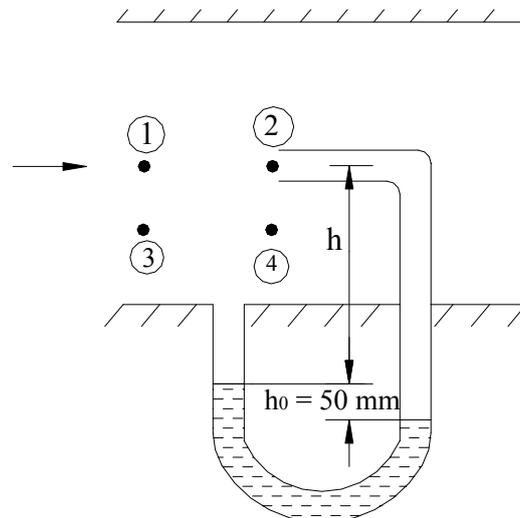


Fig. Q3. Figure of problem 3

4. Calculate the pressure drop in 30 m of a rectangular duct of cross section 12.5 mm X 25 mm when saturated water at 60°C flows at 5 cm/s? [\(Solution\)](#) Hint: Lundgren

determined that for rectangular ducts with ratio of sides 0.5 the product of $f.Re=62.19$.

5. A fluid is flowing through a pipeline having a diameter of 150 mm at 1 m/s. The pipe is 50 m long. Calculate the head loss due to friction? ([Solution](#)) (Density and viscosity of fluid are 850 kg/m^3 and 0.08 kg/m.s respectively)

6. A fluid flows from point 1 to 2 of a horizontal pipe having a diameter of 150 mm. The distance between the points is 100 m. The pressure at point 1 is 1 MPa and at point 2 is 0.9 MPa. What is the flow rate? ([Solution](#)) (Density and kinematic viscosity of fluid are 900 kg/m^3 and $400 \times 10^{-6} \text{ m}^2/\text{s}$ respectively)

7. Three pipes of 0.5 m, 0.3 m and 0.4 m diameters and having lengths of 100 m, 60 m and 80 m respectively are connected in series between two tanks whose difference in water levels is 10 m as shown in Fig. Q7. If the friction factor for all the pipes is equal to 0.05, calculate the flow rate through the pipes. ([Solution](#))

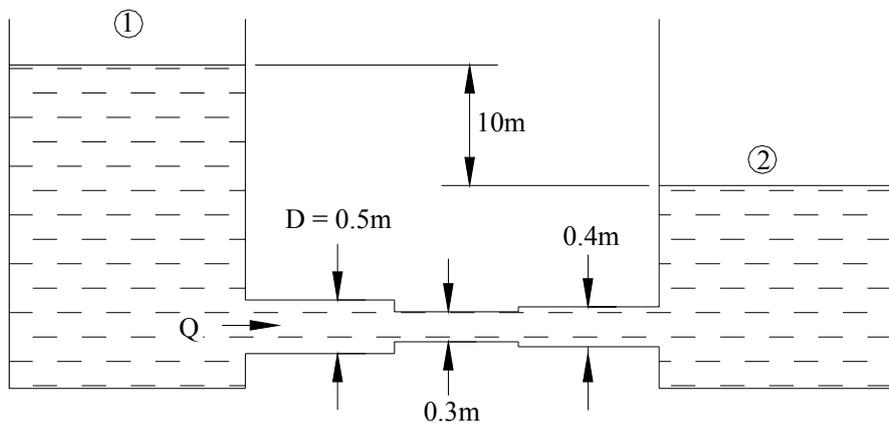


Fig. Q7. Figure of problem 7

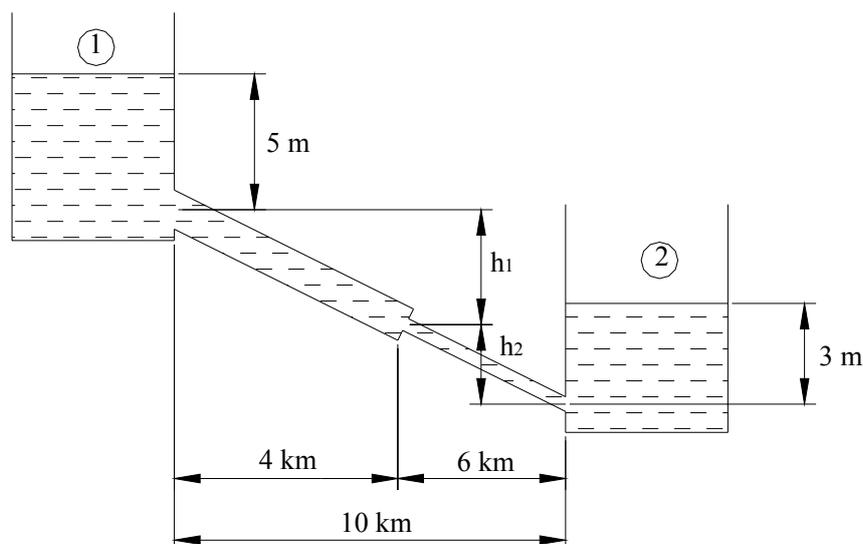


Fig. Q8. Figure of problem 8

8. Two reservoirs 10 kms apart is connected by a pipeline which is 0.25 m in diameter in the first 4 kms, sloping at 5 m per km, and the remaining by a 0.15 m diameter sloping at 2 m per km as is shown in Fig. Q8. The levels of water above the pipe openings are 5 m and 3 m in the upper and lower reservoirs respectively. Taking $f = 0.03$ for both pipes and neglecting contraction and exit losses at openings calculate the rate of discharge through the pipelines. ([Solution](#))

9. A 10 cm hose with 5 cm discharges water at $3 \text{ m}^3/\text{min}$ to the atmosphere as is shown in Fig. Q9. Assuming frictionless flow, calculate the force exerted on the flange bolts. ([Solution](#))

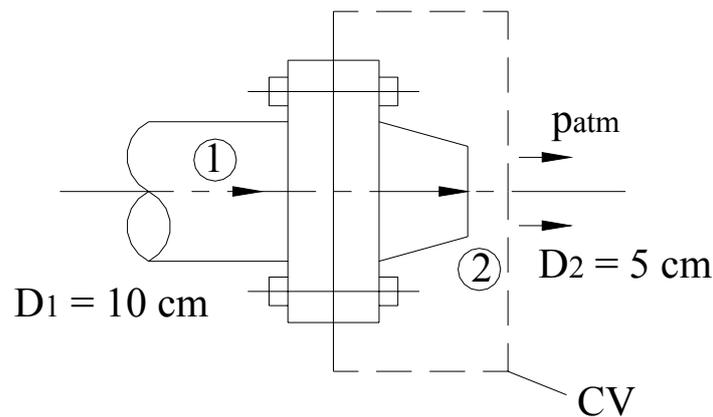


Fig. Q9. Figure of problem 9