# Lesson
# 32

# Cooling And Heating Load Calculations - Estimation Of Solar Radiation

## The specific objectives of this lecture are to:

1. Introduction to cooling and heating load calculations (*Section 32.1*)
2. Solar radiation, solar constant and solar irradiation (*Section 32.2*)
3. Solar geometry, latitude, declination, hour angles, local solar time and total sunshine hours (*Section 32.2.4*)
4. Derived solar angles (*Section 32.2.5*)
5. Angle of incidence for horizontal, vertical and tilted surfaces (*Section 32.2.6*)
6. Calculation of direct, diffuse and reflected radiation using ASHRAE solar radiation model (*Section 32.3*)
7. Effect of clouds (*Section 32.4*)

At the end of the lecture, the student should be able to:

1. Explain the need for cooling and heating load calculations
2. Explain the importance of solar radiation in air conditioning
3. Define solar angles namely, latitude, declination and hour angles and calculate the same and estimate the time of sunrise, sunset and total sunshine hours at a given location on a given day
4. Define derived solar angles and express them in terms of basic solar angles
5. Calculate the angle of incidence for surfaces of any orientation
6. Estimate direct, diffuse, reflected and total solar irradiation incident on surfaces of any orientation using ASHRAE models
7. Explain the effects of clouds on incident solar radiation

## 32.1 Introduction:

The primary function of an air conditioning system is to maintain the conditioned space at required temperature, moisture content with due attention towards the air motion, air quality and noise. The required conditions are decided by the end use of the conditioned space, e.g. for providing thermal comfort to the occupants as in comfort air conditioning applications, for providing suitable conditions for a process or for manufacturing a product as in industrial air conditioning applications etc. The reason behind carrying out cooling and heating load calculations is to ensure that the cooling and heating equipment designed or selected serves the intended purpose of maintaining the required conditions in the conditioned space. Design and/or selection of cooling and heating systems involve decisions regarding the required capacity of the equipment selected, type of the equipment etc. By carrying out cooling and heating load calculations one can estimate the capacity that will be required for various air conditioning equipment. For carrying out load calculations it is essential to have knowledge of various energy transfers that take place across the conditioned space, which will influence the required capacity of the air conditioning equipment. Cooling and heating load calculations involve a systematic step-wise procedure by following which one can estimate the various individual energy flows and finally the total energy flow across an air conditioned building.

## 32.2. Solar radiation:

In the study of air conditioning systems it is important to understand the various aspects of solar radiation because:

1. A major part of building heat gain is due to solar radiation, hence an estimate of the amount of solar radiation the building is subjected to is essential for estimating the cooling and heating loads on the buildings.
2. By proper design and orientation of the building, selection of suitable materials and landscaping it is possible to harness solar energy beneficially. This can reduce the overall cost (initial and operating) of the air conditioning system considerably by reducing the required capacity of the cooling and heating equipment.
3. It is possible, at least in certain instances to build heating and cooling systems that require only solar energy as the input. Since solar energy is available and is renewable, use of solar energy for applications such as cooling and heating is highly desirable.

For calculation purposes, the **sun** may be treated as a radiant energy source with surface temperature that is approximately equal to that of a blackbody at **6000 K**. The sprectum of wavelength of solar radiation stretches from 0.29 μm to about 4.75 μm, with the peak occurring at about 0.45 μm (the green portion of visible spectrum). Table 32.1 shows spectral distribution of solar radiation with percentage distribution of total energy in various bandwidths.

| Type of radiation | Wavelength band (μm) | % of total radiation |
|---|---|---|
| Invisible ultra-violet (UV) | 0.29 to 0.40 | **7** |
| Visible radiation | 0.40 to 0.70 | **39** |
| Near Infrared (IR) | 0.70 to 3.50 | **52** |
| Far infrared (FIR) | 4.00 to 4.75 | **2** |

***Table 32.1**. Spectral distribution of solar radiation*

### 32.2.1. Solar constant:

This is the flux of solar radiation on a surface normal to the sun's rays beyond the earth's atmosphere at the mean earth-sun distance. The currently accepted value of solar constant is 1370 W/m$^2$. Since the earth's orbit is slightly elliptical, the extra-terrestrial radiant flux varies from a maximum of 1418 W/m$^2$ on January 3$^{rd}$ to a minimum of 1325 W/m$^2$ on July 4$^{th}$.

### 32.2.2. Depletion of solar radiation due to earth's atmosphere:

In passing through the earth's atmosphere, which consists of dust particles, various gas molecules and water vapour, the solar radiation gets depleted due to reflection, scattering and absorption. The extent of this depletion at any given time depends on the atmospheric composition and length of travel of sun's rays through the atmosphere. The length of travel is expressed in terms of 'air mass, m' which is defined as the ratio of mass of atmosphere in the actual sun-earth path to that which would exist if the sun were directly overhead at sea level. As shown in Fig. 32.1, the air mass is given by:

$$\text{air mass, m} = \frac{\text{length OP}}{\text{length O'P}} = \frac{\sin 90^o}{\sin\beta} = \frac{1}{\sin\beta} \qquad (32.1)$$

where $\beta$ is called as altitude angle, which depends on the location, time of the day and day of the year. Thus smaller the altitude angle, larger will be the depletion of radiation.

### 32.2.3. Total solar irradiation:

In order to calculate the building heat gain due to solar radiation, one has to know the amount of solar radiation incident on various surfaces of the building. The rate at which solar radiation is striking a surface per unit area of the surface is called as the total solar irradiation on the surface. This is given by:

$$I_{i\theta} = I_{DN}\cos\theta + I_{d\theta} + I_{r\theta} \qquad (32.2)$$

where $I_{i\theta}$ = Total solar irradiation of a surface, W/m$^2$
$I_{DN}$ = Direct radiation from sun, W/m$^2$
$I_{d\theta}$ = Diffuse radiation from sky, W/m$^2$
$I_{r\theta}$ = Short wave radiation reflected from other surfaces, W/m$^2$
$\theta$ = Angle of incidence, degrees (Figure 32.2)

The first term on the RHS, i.e., $I_{DN} \cos \theta$, is the contribution of direct normal radiation to total irradiation. On a clear, cloudless day, it constitutes about 85 percent of the total solar radiation incident on a surface. However, on cloudy days the percentage of diffuse and reflected radiation components is higher. The objective of solar radiation calculations is to estimate the direct, diffuse and reflected radiations incident on a given surface. These radiations and the angle of incidence are affected by solar geometry.

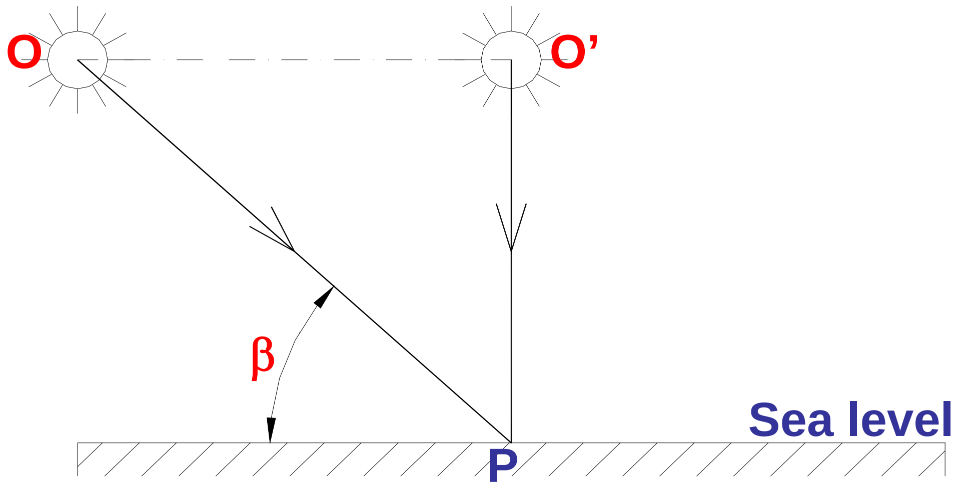


***Fig.32.1**: Depletion of solar radiation due to earth's atmosphere*

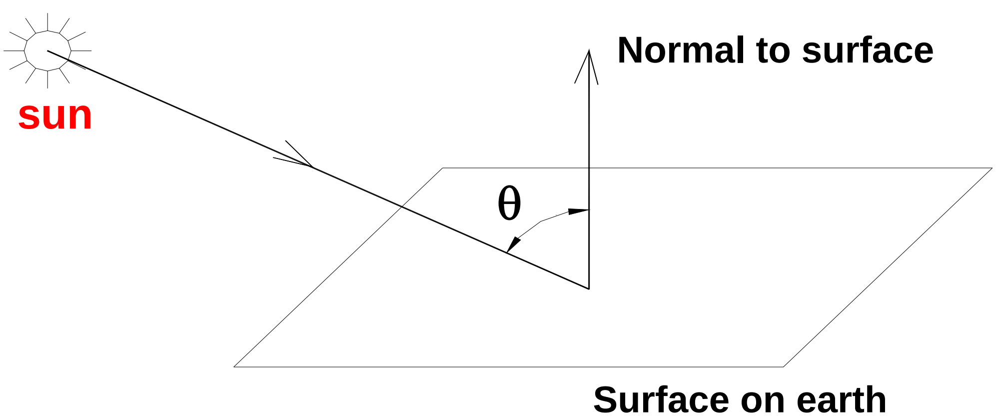


***Fig.32.2**: Definition of angle of incidence*

### 32.2.4. Solar geometry:

The angle of incidence $\theta$ depends upon:

i. Location on earth

ii. Time of the day, and

iii. Day of the year

The above three parameters are defined in terms of latitude, hour angle and declination, respectively.

The planet earth makes one rotation about its axis every 24 hours and one revolution about the sun in a period of about 365 ⌒ days. The earth's equatorial plane is tilted at an angle of about $23.5^o$ with respect to its orbital plane. The earth's rotation is responsible for day and night, while its tilt is responsible for change of seasons. Figure 32.3 shows the position of the earth at the start of each season as it revolves in its orbit around the sun. As shown in Fig.32.4, during summer solstice (June $21^{st}$) the sun's rays strike the northern hemisphere more directly than they do the southern hemisphere. As a result, the northern hemisphere experiences summer while the southern hemisphere experiences winter during this time. The reverse happens during winter solstice (December $21^{st}$).

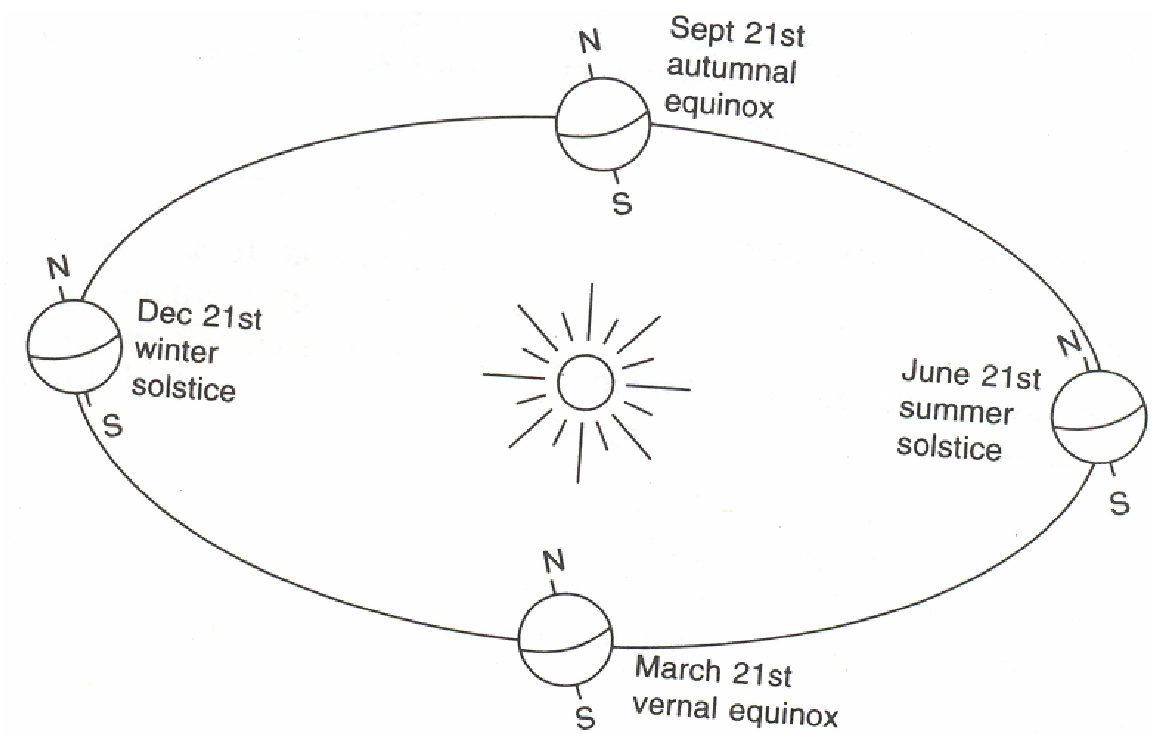


***Fig.32.3***: *Position of earth with respect to sun for different seasons*

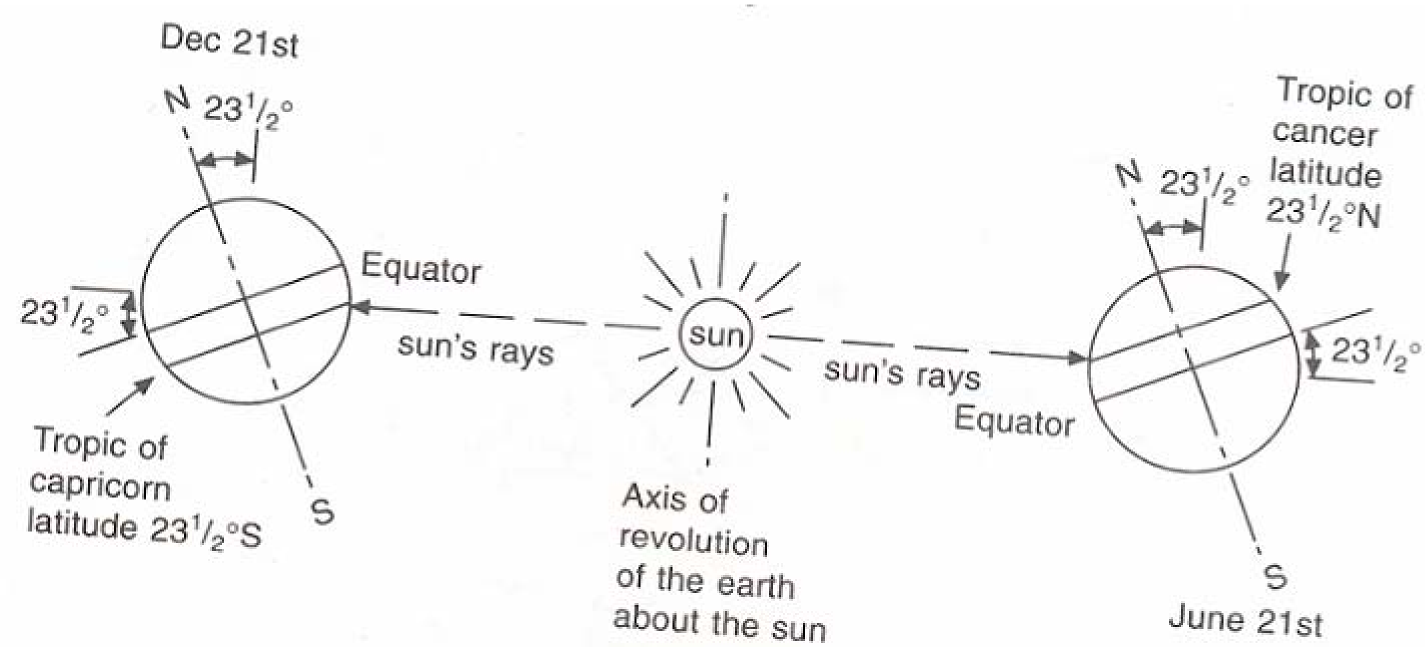


***Fig.32.4***: *Direction of sun's rays during summer and winter solstice*

Figure 32.5 shows the position of a point P on the northern hemisphere of the earth, whose center is at point O. Since the distance between earth and sun is very large, for all practical purposes it can be considered that the sun's rays are parallel to each other when they reach the earth.

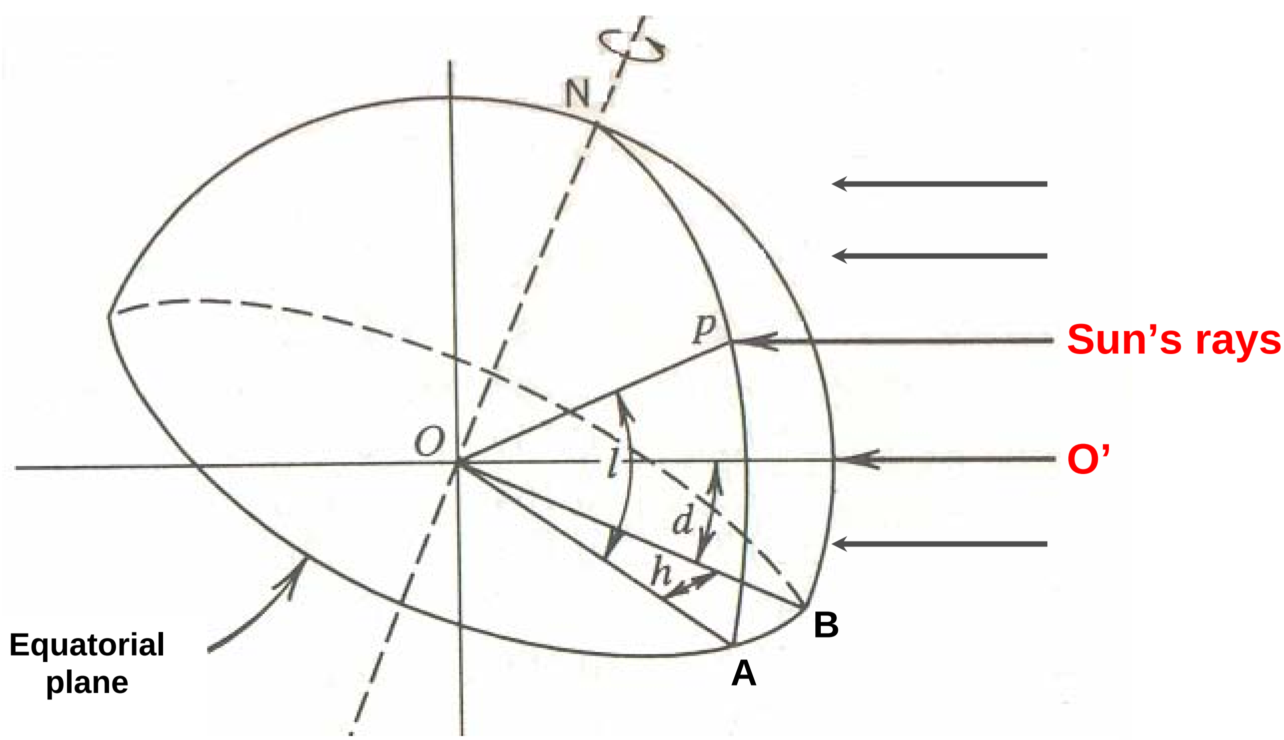


***Fig.32.5***: *Definition of latitude (l), declination (d) and hour angles (h)*

With reference to Fig.32.5, the various solar angles are defined as follows:

**Latitude, l:** It is the angle between the lines joining O and P and the projection of OP on the equatorial plane, i.e.,

$$\textbf{latitude, l} = \textbf{angle} \angle\textbf{POA}$$

Thus the latitude along with the longitude indicates the position of any point on earth and it varies from $0^o$ at equator to $90^o$ at the poles.

**Hour angle, h:** It is the angle between the projection of OP on the equatorial plane i.e., the line OA and the projection of the line joining the center of the earth to the center of the sun, i.e., the line OB. Therefore,

$$\textbf{hour angle, h} = \textbf{angle} \angle\textbf{AOB}$$

The hour angle is a measure of the time of the day with respect to solar noon. Solar noon occurs when the sun is at the highest point in the sky, and hour angles are symmetrical with respect to solar noon. This implies that the hour angles of sunrise and sunset on any given day are identical. The hour angle is $0^o$ at solar noon and varies from $0^o$ to $360^o$ in one rotation. Since it takes 24 clock hours for one rotation, each clock hour of time is equal to $15^o$ of hour angle. For example, at 10 A.M. (solar time) the hour angle is $330^o$, while at 4 P.M. it is $60^o$.

**Solar time:** Solar radiation calculations such as the hour angle are based on local solar time (LST). Since the earth's orbital velocity varies throughout the year, the local solar time as measured by a sundial varies slightly from the mean time kept by a clock running at uniform rate. A civil day is exactly equal to 24 hours, whereas a solar day is approximately equal to 24 hours. This variation is called as Equation of Time (EOT) and is available as average values for different months of the year. The EOT may be considered as constant for a given day. An approximate equation for calculating EOT given by Spencer (1971) is:

$$\mathbf{EOT = 0.2292(0.075 + 1.868\cos N - 32.077\sin N - 4.615\cos 2N - 40.89\sin 2N)} \qquad (32.3)$$

where $\mathbf{N = (n-1)\left(\frac{360}{365}\right)}$ **; n is the day of the year (counted from January 1$^{st}$)**

At any location, the local solar time is given by:

$$\mathbf{LST = LStT + EOT + 4(LON - LSM)} \qquad (32.4)$$

in the above equation LStT is the local standard time, LSM is the local standard time meridien and LON is the local longitude. In the above equation '+' sign is used if LON is to the east of LSM and '-' sign should be used if LON is to the west of LSM.

**Declination, d:** The declination is the angle between the line joining the center of the earth and sun and its projection on the equatorial plane, the angle between line OO' and line OB;

$$\textbf{declination, d} = \textbf{angle} \angle \textbf{O'OB}$$

For northern hemisphere, the declination varies from about +23.5$^o$ on June 21$^{st}$ (summer solstice) to -23.5$^o$ on December 21$^{st}$ (December 21$^{st}$). At equinoxes, i.e., on March 21$^{st}$ and September 21$^{st}$ the declination is 0$^o$ for northern hemisphere. The declination varies approximately in a sinusoidal form, and on any particular day the declination can be calculated approximately using the following equation:

$$\textbf{declination, d} = 23.47 \sin \frac{360(284 + N)}{365} \qquad (32.5)$$

where N is the day of the year numbered from January 1$^{st}$. Thus on March 6$^{th}$, N is 65 (65$^{th}$ day of the year) and from the above equation, declination on March 6$^{th}$ is equal to –6.4$^o$.

### 32.2.5. Derived solar angles:

In addition to the three basic solar angles, i.e., the latitude, hour angle and declination, several other angles have been defined (in terms of the basic angles), which are required in the solar radiation calculations. Figure 32.6 shows a schematic of one apparent solar path and defines the altitude angle ($\beta$), zenith angle ($\psi$) and solar azimuth angle ($\gamma$). It can be shown by analytical geometry that these angles are given by:

**Altitude angle, $\beta$:** It is the angle between the sun's rays and the projection of sun's rays onto a horizontal plane as shown in Fig.32.6. The expression for altitude angle is given by:

$$\textbf{Altitude angle}, \beta = \sin^{-1}(\cos l.\cos h.\cos d + \sin l.\sin d) \qquad (32.6)$$

**Zenith angle, $\psi$:** It is the angle between sun's rays and the surface normal to the horizontal plane at the position of the observer. It can be seen from Fig.32.6 that:

$$\textbf{Zenith angle}, \psi = \frac{\pi}{2} - \beta \qquad (32.7)$$

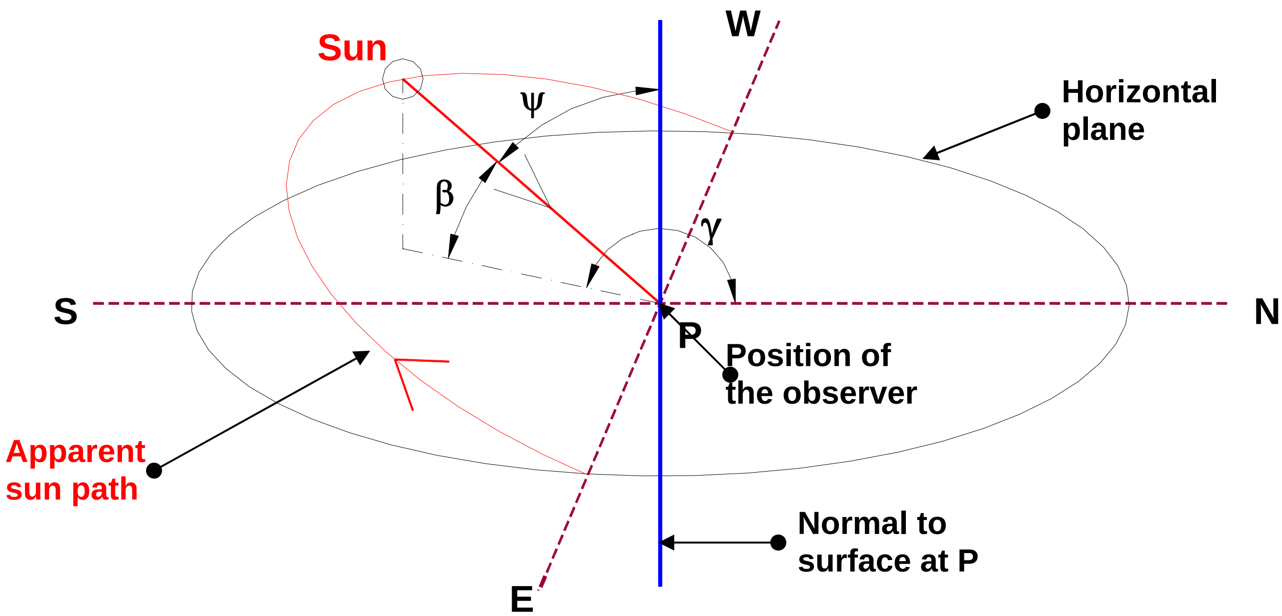


***Fig.32.6**: Definition of altitude angle, zenith angle and solar azimuth angle*

The altitude angle β is maximum at solar noon. Since the hour angle,h is $0^o$ at solar noon, the maximum altitude angle $\beta_{max}$ (solar noon) on any particular day for any particular location is given by substituting the value of h = $0^o$ in the expression for β given above (Eqn.(32.6)), thus it can be easily shown that:

$$\beta_{max} = \frac{\pi}{2} - |(l - d)| \qquad (32.8)$$

where $|(l - d)|$ is the absolute value of (l-d).

The equation for altitude angle can also be used for finding the time of sunrise, sunset and sunshine hours as the altitude angle is $0^o$ at both sunrise and sunset (Fig.32.6). Thus from the equation for β, at sunrise and sunset β = 0, hence the hour angle at sunrise and sunset is given by:

$$h_o = \cos^{-1}(-\tan l . \tan d) \qquad (32.9)$$

From the hour angle one can calculate the sunrise, sunset and total sunshine hours as the sunrise and sunset are symmetrical about the solar noon.

**Solar azimuth angle, γ:** As shown in Fig.32.6, the solar azimuth angle is the angle in the horizontal plane measured from north to the horizontal projection of the sun's rays. It can be shown that the solar azimuth angle is given by:

$$\gamma = \cos^{-1}\left(\frac{\cos l.\sin d - \cos d.\cos h.\sin l}{\cos \beta}\right) = \sin^{-1}\left(\frac{\cos d.\sin h}{\cos \beta}\right) \qquad (32.10)$$

At solar noon when the hour angle is zero, the solar azimuth angle is equal to $180^o$, if the latitude, l is greater than declination, d, and it is equal to $0^o$ if $l < d$. The solar azimuth angle at solar noon is not defined for l = d.

### 32.2.6. Incident angle of sun's rays, θ:

The incident angle of sun's rays θ, is the angle between sun's rays and the normal to the surface under consideration. The angle of incidence depends on the solar geometry and also the orientation of the surface.

**For horizontal surfaces:** For horizontal surfaces (Fig.32.7) the angle of incidence $\theta_{hor}$ is equal to the zenith angle, ψ, i.e.,

$$\theta_{hor} = \psi = \frac{\pi}{2} - \beta \qquad (32.11)$$

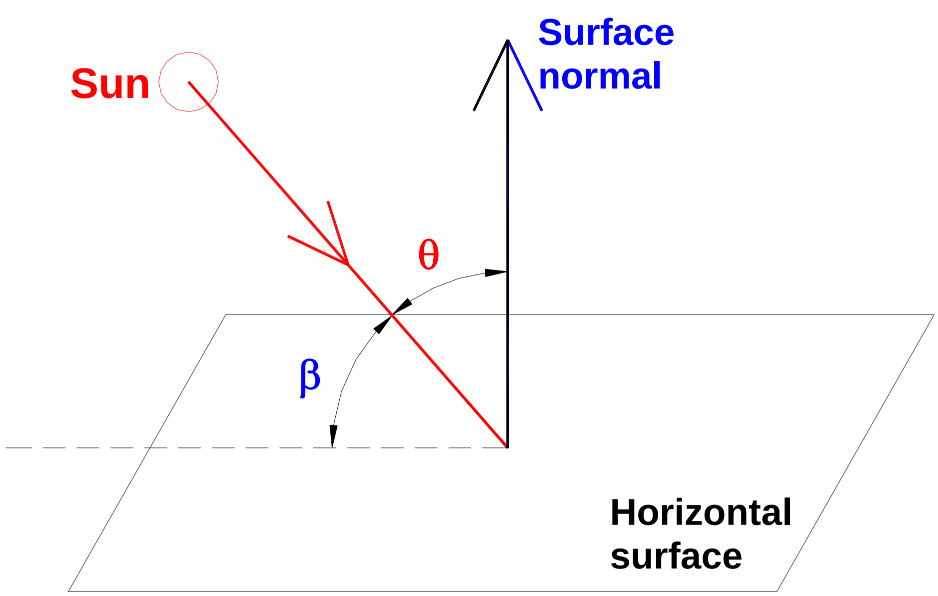


***Fig.32.7**: Incident angle for a horizontal surface*

**For vertical surfaces:** Figure 32.8 shows an arbitrarily orientated vertical surface (shaded) that is exposed to solar radiation. The angle of incidence of solar radiation on the vertical surface depends upon the orientation of the wall, i.e, east facing, west facing etc. Additional angles have to be defined to find the angle of incidence on the vertical walls.

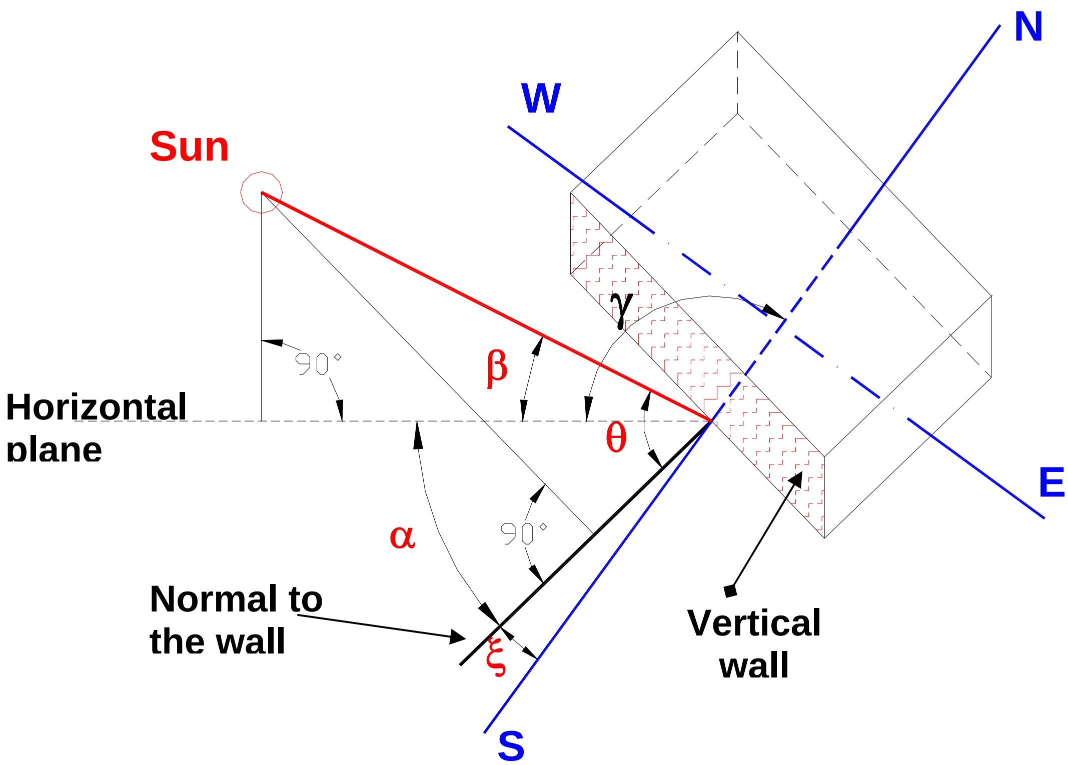


***Fig.32.8:*** *Calculation of incident solar angle for vertical surfaces*

Referring to Fig.32.8, the following additional angles are defined:

**Wall solar azimuth angle, $\alpha$:** This is the angle between normal to the wall and the projection of sun's rays on to a horizontal plane.

**Surface azimuth angle, $\xi$:** This is the angle between the normal to the wall and south. Thus when the wall is facing south, then the surface azimuth angle is zero and when it faces west, then the surface azimuth angle is $90^o$ and so on. The angle is taken as +ve if the normal to the surface is to the west of south and –ve if it is to the east of south.

From Fig.32.8 it can be seen that the wall solar azimuth angle, is given by:

$$\alpha = [\pi - (\gamma + \xi)].F \qquad (32.12)$$

The factor F is -1 for forenoon and +1 for afternoon.

Now it can be shown that the angle of incidence on the vertical surface, $\theta_{ver}$ is given by:

$$\theta_{ver} = \cos^{-1}(\cos\beta.\cos\alpha) \qquad (32.13)$$

**For an arbitrarily oriented surfaces:** For any surface that is tilted at an angle $\Sigma$ from the horizontal as shown in Fig.32.9, the incident angle $\theta$ is given by:

$$\theta = \cos^{-1}(\sin\beta.\cos\Sigma + \cos\beta.\cos\alpha.\sin\Sigma) \qquad (32.14)$$

This equation is a general equation and can be used for any arbitrarily oriented surface. For example, for a horizontal surface, $\Sigma$ is $0^o$, hence $\theta_{hor}$ is equal to $(90-\beta)$, as shown earlier. Similarly, for a vertical surface, $\Sigma$ is $90^o$, hence $\theta_{ver}$ is equal to $\cos^{-1}(\cos\beta.\cos\alpha)$, as shown before.

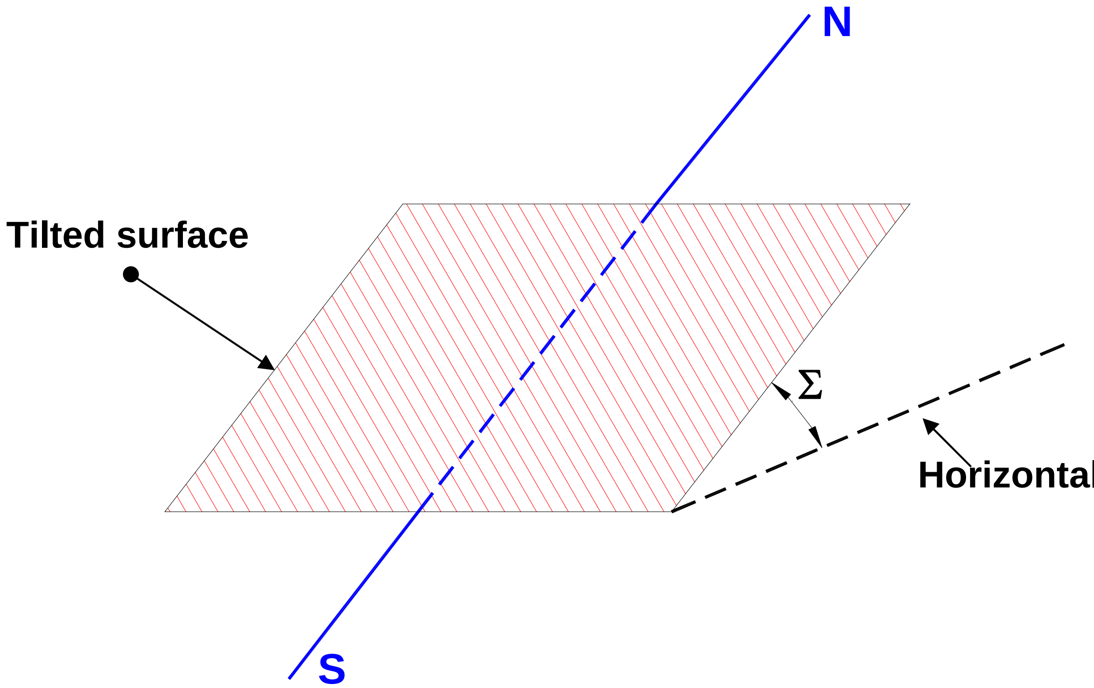


***Fig.32.9**: An arbitrarily oriented surface with a tilt angle $\Sigma$*

## 32.3. Calculation of direct, diffuse and reflected radiations:

### 32.3.1. Direct radiation from sun ($I_{DN}$):

Several solar radiation models are available for calculation of direct radiation from sun. One of the commonly used models for air conditioning calculations is the one suggested by ASHRAE. According to this model, the direct radiation $I_{DN}$ is given by:

$$I_{DN} = A.\exp\left(-\frac{B}{\sin\beta}\right) \quad (W/m^2) \qquad (32.15)$$

where A is the apparent solar irradiation which is taken as 1230 W/$m^2$ for the months of December and January and 1080 W/$m^2$ for mid-summer. Constant B is called as atmospheric extinction coefficient, which takes a value of 0.14 in winter and 0.21 in summer. The values of A and B for 21st day of each month have been computed are available either in the form of tables or empirical equations.

### 32.3.2. Diffuse radiation from sky, $I_d$:

According to the ASHRAE model, the diffuse radiation from a cloudless sky is given by:

$$\mathbf{I_d = C.I_{DN}.F_{WS} \qquad (W/m^2)} \tag{32.16}$$

The value of C is assumed to be constant for a cloudless sky for an average day of a month. Its average monthly values have been computed and are available in tabular form. The value of C can be taken as 0.135 for mid-summer and as 0.058 for winter. The factor $F_{WS}$ is called as **view factor or configuration factor** and is equal to the fraction of the diffuse radiation that is incident on the surface. For diffuse radiation, $F_{WS}$ is a function of the orientation of the surface only. It can be easily shown that this is equal to:

$$\mathbf{F_{WS} = \frac{(1+\cos\Sigma)}{2}} \tag{32.17}$$

where $\Sigma$ is the tilt angle. Obviously for horizontal surfaces ($\Sigma = 0^o$) the factor $F_{WS}$ is equal to 1, whereas it is equal to 0.5 for a vertical surface ($\Sigma = 90^o$). The above model is strictly true for a cloudless sky only as it assumes that the diffuse radiation from the sky falls uniformly on the surface. The diffuse radiation will not be uniform when the sky is cloudy.

### 32.3.3. Reflected, short-wave (solar) radiation, $I_r$:

The amount of solar radiation reflected from the ground onto a surface is given by:

$$\mathbf{I_r = (I_{DN} + I_d)\rho_g F_{WG}} \tag{32.18}$$

where $\rho_g$ is the reflectivity of the ground or a horizontal surface from where the solar radiation is reflected on to a given surface and $F_{WG}$ is view factor from ground to the surface. The value of reflectivity obviously depends on the surface property of the ground. The value of the angle factor $F_{WG}$ in terms of the tilt angle is given by:

$$\mathbf{F_{WG} = \frac{(1-\cos\Sigma)}{2}} \tag{32.19}$$

Thus for horizontal surfaces ($\Sigma = 0^o$) the factor $F_{WG}$ is equal to 0, whereas it is equal to 0.5 for a vertical surface ($\Sigma = 90^o$).

Though the ASHRAE clear sky model is widely used for solar radiation calculations in air conditioning, more accurate, but more involved models have also been proposed for various solar energy applications.

**Example:** Calculate the total solar radiation incident on a south facing, vertical surface at solar noon on June 21st and December 21st using the data given below:

Latitude = $23^o$
Reflectivity of the ground = 0.6
Assume the sky to be cloudless

**Ans.:**

| | | |
|---|---|---|
| Given: | Latitude angle, **l** | **= $23^o$** |
| | Hour angle, **h** | **= $0^o$** (solar noon) |
| | Declination, **d** | **= +$23.5^o$ (on June 21st)** |
| | | **= -$23.5^o$ (on December 21st)** |
| | Tilt angle, $\Sigma$ | **= $90^o$ (Vertical surface)** |
| | Wall azimuth angle, $\xi$ | **= $0^o$** (south facing) |
| | Reflectivity, $\rho_g$ | **= 0.6** |

**<u>June 21st :</u>**

Altitude angle $\beta$ at solar noon $\beta_{max} = \frac{\pi}{2} - |(l - d)| = 89.53^o$

At solar noon, solar azimuth angle, $\gamma = 0^o$ as l < d

$\therefore$ wall solar azimuth angle, $\alpha = 180 - (\gamma + \xi) = 180^o$

Incidence angle $\theta_{ver} = \cos^{-1}(\cos\beta . \cos\alpha) = 89.53^o$

<u>Direct radiation, $I_{DN}$ Cos ($\theta$):</u>

$$I_{DN} = A.\exp\left(-\frac{B}{\sin\beta}\right) = 1080.\exp\left(-\frac{0.21}{\sin 89.3}\right) = 875.4\ W/m^2$$

$$I_{DN}\cos\theta = 875.4\ X\cos 89.53 = 7.18\ W/m^2$$

<u>Diffuse radiation, $I_d$:</u>

View factor $F_{WS} = \frac{(1+\cos\Sigma)}{2} = 0.5$

Diffuse radiation $I_d = C.I_{DN}.F_{WS} = 0.135\ X\ 875.4\ X\ 0.5 = 59.1\ W/m^2$

Reflected radiation from ground ($\rho_g$ = 0.6), $I_d$:

View factor $F_{WG} = \frac{(1-\cos\Sigma)}{2} = 0.5$

**Reflected radiation, $I_r$:**

$$I_r = (I_{DN} + I_d)\rho_g F_{WG} = (875.43 + 59.1)\,X\,0.6\,X\,0.5 = 280.36\ W/m^2$$

$$\therefore \text{total incident radiation } I_t = I_{DN}\cos\theta + I_d + I_r = 346.64\ W/m^2$$

Calculations similar to the can be carried out for December 21st (declination is –23.5°) . Table 35.2 shows a comparison between the solar radiation on the south facing wall during summer (June 21st) and winter (December 21st):

| Parameter | June 21st | December 21st |
|---|---|---|
| Incident angle, $\theta$ | **89.53°** | **43.53°** |
| Direct radiation, $I_{DN}$ | **875.4 W/m²** | **1003.75 W/m²** |
| Direct radiation incident on the wall, $I_{DN}\cos\theta$ | **7.18 W/m²** | **727.7 W/m²** |
| Diffuse radiation, $I_d$ | **59.1 W/m²** | **29.1 W/m²** |
| Reflected radiation, $I_r$ | **280.36 W/m²** | **309.9 W/m²** |
| Total incident radiation, $I_t$ | **346.64 W/m²** | **1066.7 W/m²** |

The above table reveals an interesting fact. It is seen that in northern hemisphere, a wall facing south receives much less radiation in summer compared to winter. This is mainly due to the value of incident angle, which is much larger in summer compared to winter for a south facing wall. This reduces the contribution of the direct radiation significantly in summer compared to winter. In fact if the ground has lower reflectivity than the value used in the example (0.6), then the radiation incident on the vertical wall in summer will be almost negligible, while it will be still very high in winter. This implies that from air conditioning point of view, buildings in northern hemisphere should have windows on the south facing wall so that the cooling load in summer and heating load in winter reduces considerably.

## 32.4. Effect of clouds:

It is mentioned earlier that on a clear day, almost 85% of the incident radiation is due to direct radiation and the contribution of diffuse radiation is much smaller. However, when the sky is cloudy, the contribution of diffuse radiation increases significantly. Though it is frequently assumed that the diffuse radiation from the sky reaches the earth uniformly on clear days, studies show that this is far from true. The diffuse radiation is even more non-uniform on cloudy days. Due to the presence of the clouds (which is extremely difficult to predict), the available solar radiation in an actual situation is highly variable. Clouds not only block the short-wave radiation from the sun, but they also block the long-wave radiation from earth. Thus it is very difficult to accurately account for the effect of clouds on solar and terrestrial radiation. Sometimes, in solar energy calculations a clearness index is used to take into account the effect of clouds. Arbitrarily a value of 1.0 is assigned for clearness index for a perfectly clear and cloudless sky. A clearness index value of less than 1.0 indicates the presence of clouds. The calculations are carried out for a clear sky and the resulting direct radiation is multiplied by the clearness index to calculate the contribution of direct radiation in the presence of clouds. Data on the value of the clearness index are available for a few select countries.

## Questions and answers:

**1.** Calculate the local solar time and the corresponding hour angle at 9 A.M (local standard time, L.St.T) on October $21^{st}$, for the Indian city of Kolkata located at $22^o82'$N and $88^o20'$E, the LSM for India is $82^o30'$. the EOT for Kolkata on October $21^{st}$ is 15 minutes.

**Ans.:** Local solar time, LST is given by the expression,

LST = LStT+15 +4(88.33-82.5) = **9 hours 38.32 minutes A.M.** (Ans.)

**the corresponding hour angle is $324.6^o$** (Ans.)

**2.** Find the maximum altitude angle for Kolkata ($l = 22^o82'$N) on June $21^{st}$.

**Ans.:** On June $21^{st}$, the declination angle,d is **$23.5^o$**.

The maximum altitude angle occurs at solar noon at which the hour angle is zero. Hence, Maximum altitude angle, $\beta_{max}$ is given by:

$$\beta_{max} = \frac{\pi}{2} - |(l - d)| = 90 - (22.82 - 23.5) = 89.3^o \quad \textbf{(Ans.)}$$

**3.** Find the sunrise, sunset and total sunshine hours at IIT Kharagpur ($\approx 22^oN$) on September 9th.

**Ans.:** On September 9th, N = 252, hence the declination, d is equal to $4.62^o$.

The hour angle at sunrise and sunset is given by,

$$h_o = \cos^{-1}(-\tan 22.\tan 4.62) = 91.87^o$$

Since each $15^o$ is equal to 1 hour, $91.87^o$ is equal to 6 hours and 8 minutes. Hence,

Sunrise takes place at (12.00 - 6.08) = **5.52 A.M (solar time)** **(Ans.)**

Sunset takes place at (12.00+6.08) = **6.08 P.M. (solar time)**

Total sunshine hours are 2 X 6.08 = **12 hours and 16 minutes**

**4.** What is the angle of incidence at 3 P.M. (solar time) of a north-facing roof that is tilted at an angle of $15^o$ with respect to the horizontal. Location: $22^oN$, and date September 9th.

**Ans.:** Given: Latitude, **l = 22$^o$** (N)
Solar time = 3 P.M. ⇒ hour angle, **h = 45$^o$**
Date = September 9th ⇒ declination, **d** = **4.62$^o$** (from earlier example)

**Altitude angle,** $\beta = \sin^{-1}(\cos l.\cos d.\cos h + \sin l.\sin d)$ = **43.13$^o$**

Since the roof is north facing, the surface azimuth angle $\xi$ is equal to $180^o$.

The solar azimuth angle $\gamma$ is given by:

$$\gamma = \sin^{-1}\left(\frac{\cos d.\sin h}{\cos \beta}\right) = \sin - 1\left(\frac{\cos 4.62 \sin 45}{\cos 43.13}\right) = 74.96^o$$

The wall solar azimuth angle $\alpha = 180 - (\gamma+\xi) = 180 - (74.96-180) = 285^o$

Hence the angle of incidence is

$$\theta = \cos^{-1}(\sin 43.13 \cos 15 - \cos 43.13.\cos 285.\sin 15) = 52.3^o$$

**5.** Find the direct normal radiation at Kolkata on June 21st at solar noon?

**Ans.:** From the earlier example, on June 21st at solar noon, the altitude angle for Kolkata is 89.3°. Hence the direct solar radiation is given by:

$$\mathbf{I_{DN} = A.exp\left(-\frac{B}{\sin\beta}\right) = 1080.exp\left(-\frac{0.21}{\sin 89.3}\right) = 875.4\ W/m^2} \text{ (Ans.)}$$

**6.** Find the diffuse and total solar radiation incident on a horizontal surface located at Kolkata on June 21st at solar noon?

**Ans.:** From the earlier example, on June 21st at solar noon, the direct solar radiation is equal to **875.4 W/m²**. Since the surface is horizontal, the view factor for diffuse radiation, $F_{WS}$ is equal to 1, whereas it is 0 for reflected radiation.

Hence, the diffuse solar radiation is given by:

$$\mathbf{I_d = C.I_{DN}.F_{WS} = 0.135\,X\,875.4 = 118.18\ W/m^2} \qquad \textbf{(Ans.)}$$

Since the surface is horizontal, the reflected solar radiation is zero. The angle of incidence, is given by:

$$\mathbf{\theta_{hor} = (90 - \beta_{noon}) = 90 - 89.3 = 0.7^o}$$

Hence the total incident solar radiation is given by:

$$\mathbf{I_t = I_{DN}.cos(\theta) + I_d = 875.4 \times cos(0.7) + 118.18 = 993.5\ W/m^2} \qquad \textbf{(Ans.)}$$