# Heat flux

## 1. Measurement of heat flux

Heat flux is defined as the amount of heat transferred per unit area per unit time from or to a surface. In a basic sense it is a derived quantity since it involves, in principle, two quantities viz. the amount of heat transfer per unit time and the area from/to which this heat transfer takes place. In practice, the heat flux is measured by the change in temperature brought about by its effect on a sensor of known area. The incident heat flux may set up either a steady state temperature field or a transient temperature field within the sensor. The temperature field set up may either be perpendicular to the direction of heat flux or parallel to the direction of heat flux. We study the various types of heat flux gages in what follows.

## 2. Foil type heat flux gage:

The foil type heat flux gage (also known as the Gardon gage after its inventor) consists of a thin circular foil of constantan stretched tightly over a cooled copper annulus as shown in Figure 1. One surface of the foil is exposed to the heat flux that is to be measured while the other surface may be taken as insulated. A copper wire is attached at the geometric center of the foil as indicated in the figure. A second copper wire is attached to the cooled copper annulus. The constantan foil forms two junctions with copper, the first one at its center and the second one at its periphery. Under steady state, the thermoelectric voltage across the two copper leads is a direct measure of the temperature difference set up between the center and the periphery of the constantan disk. The temperature difference is obtained by performing the following analysis.

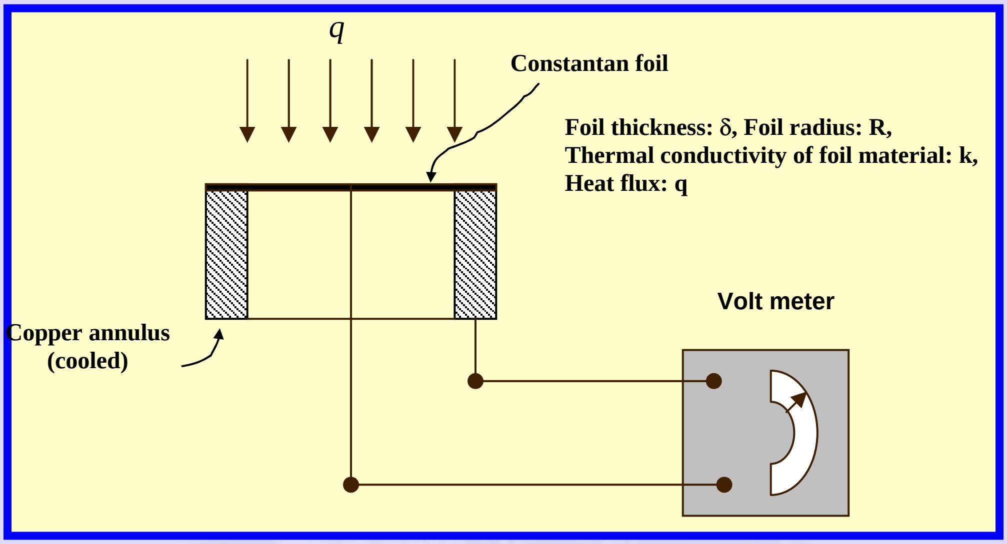


**Figure 1 Schematic of a foil type heat flux gage**

Heat balance for an annular element of the foil shown in Figure 2 is made as follows:

Heat gained by the foil element is $(q)(2\pi r dr)$

Net heat conducted in to the foil element $(2\pi k\delta)\frac{d}{dr}\left(r\frac{dT}{dr}\right)dr$

Sum of these should be zero. We cancel the common factor $2\pi dr$ to get

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{q}{k\delta}r=0 \quad \textbf{(1)}$$

The boundary conditions are

$$T \text{ is finite at } r=0;\ T=T_R \text{ at } r=R \quad \textbf{(2)}$$

Integrate equation 1 once with respect to *r* to get $r\frac{dT}{dr}+\frac{q}{k\delta}\frac{r^2}{2}=A$, where A is a constant of integration. This may be rearranged to get $\frac{dT}{dr}+\frac{q}{2k\delta}r=\frac{A}{r}$. Integrate this once more with respect to r to get $T+\frac{qr^2}{4k\delta}=A\ln(r)+B$ where *B* is a second constant of integration. The constant *A* has to be chosen equal to zero in order that the solution does not diverge at $r=0$. The constant *B* is obtained from the boundary condition at r = R as $B=T_R+\frac{qR^2}{4k\delta}$. With this the solution for the temperature is obtained as

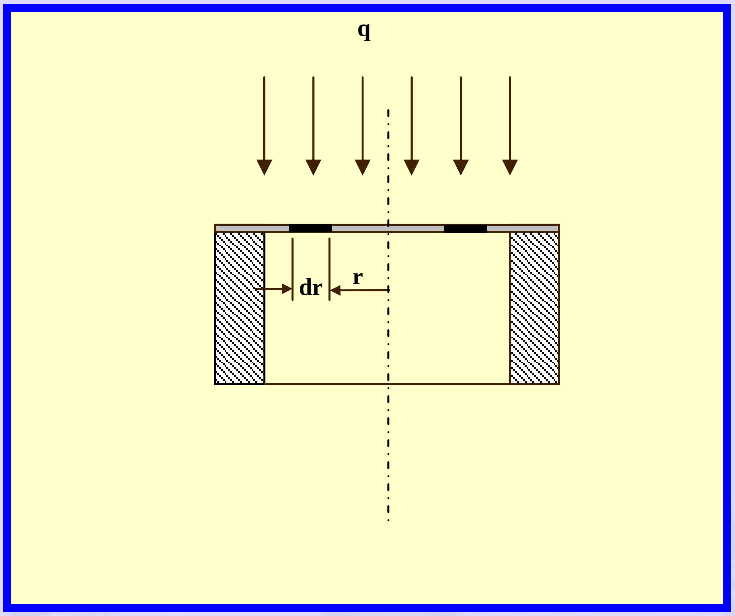


**Figure2 Energy balance over a foil element in the form of an annular ring**

$$T = T_R + \frac{q}{4k\delta}\left(R^2 - r^2\right) \qquad \textbf{(3)}$$

It may be noted that the constant *B* is nothing but the temperature at the center of the constantan disk. In view of this, equation 3 may be recast as

$$q = \frac{T_0 - T_R}{\left(R^2 / 4k\delta\right)} = K\left(T_0 - T_R\right) = K\Delta T \qquad \textbf{(4)}$$

In the above $T_0$ is the temperature at the center of the disk and the coefficient *K* is the gage constant given by $K = \frac{4k\delta}{R^2}\ W/m^2K$. The temperature difference between the center of the disk and the periphery is the output that appears as a proportional voltage $\Delta V$ across the terminals of the differential thermocouple. There is thus a linear relationship between the heat flux and the output of the heat flux gage.

# Example 1

A typical gage may be constructed using a $6\ \text{mm}$ diameter foil of $50\,\mu\text{m}$ thickness. The thermal conductivity of the foil material is typically $k = 20\ \text{W/m°C}$. Copper Constantan thermocouple pair gives an output of about $40\ \mu\text{V/°C}$. The given data corresponds to. $\delta = 50\,\mu\text{m} = 50 \times 10^{-6}\ \text{m},\ R = 3\,\text{mm} = 0.003\ \text{m},\ k = 20\ \text{W/m°C}$

The gage constant then works out to $K = \dfrac{4 \times 20 \times 5 \times 10^{-5}}{0.003^2} = 444.4\ \text{W/m}^2\text{°C}$. This may be rewritten in terms of the thermocouple output as $K = \dfrac{444.44}{40} = 11.1\ \text{W/m}^2\mu\text{V}$.

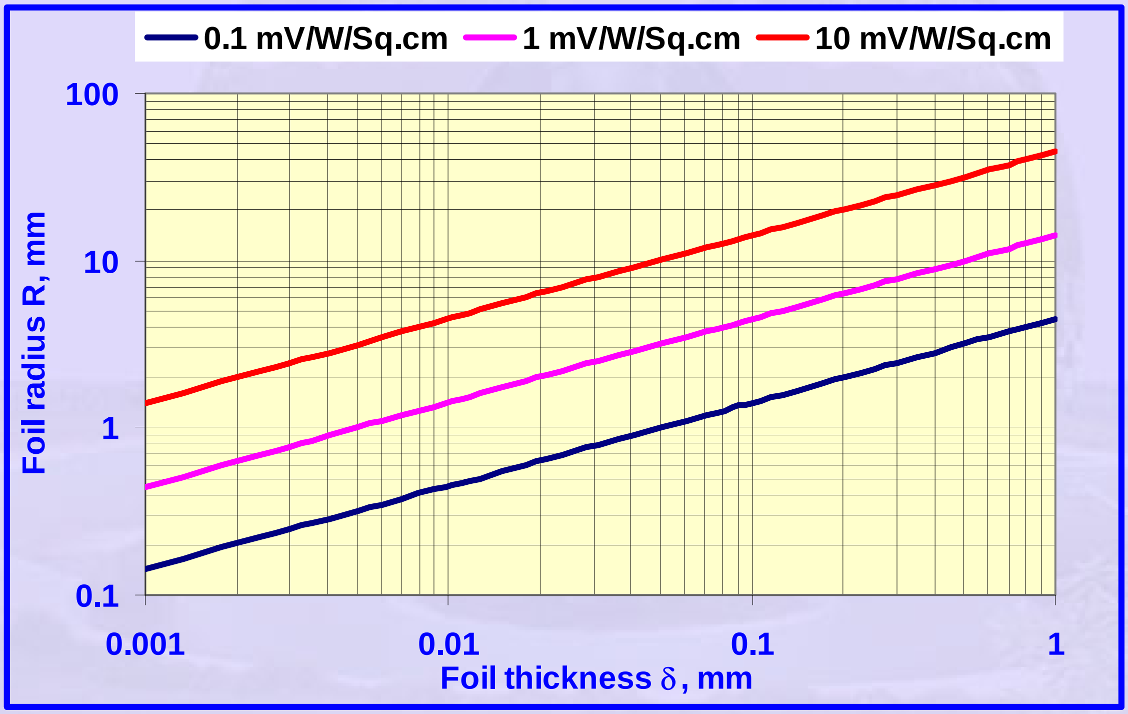


**Figure 3 Proportions of foil gage**

Figure 3 gives a plot that is useful in finding the R, □ combinations that will have a given sensitivity. The sensitivity (1/K) is specified in units of $\dfrac{\text{mV}}{\left(\dfrac{\text{W}}{\text{cm}^2}\right)}$ with both R and □ being in mm.

### Transient Analysis of the Foil Gage:

Under steady state we have seen that the temperature distribution in the foil is given by a quadratic variation with respect to r.

In fact we have:

$$T - T_R = \frac{q}{4k\delta}\left(R^2 - r^2\right) \quad (5)$$

With
$$T_0 - T_R = \frac{qR^2}{4k\delta} \quad (6)$$

From these, by division we have

$$\frac{(T - T_R)}{(To - T_R)} = \left[1 - \left(\frac{r}{R}\right)^2\right]. \quad (7)$$

In the steady sate the energy stored in the foil is given by

$$E = \int_0^R \left[\rho C_p \delta\, 2\pi r dr\right]\left[T - T_R\right]$$

Using equation (5) this is recast as

$$E = 2\rho C_p \delta\, \pi\left[T_o - T_R\right]\int_0^R r\left[1 - \left(\frac{r}{R}\right)^2\right] dr \quad (8)$$

But $\int_0^R r\left[1 - \left(\frac{r}{R}\right)^2\right] dr = \left(\frac{R^2}{2} - \frac{R^2}{4}\right) = \frac{R^2}{4}.$

$$\therefore E = \frac{2\rho C_p \delta R^2\left[T_o - T_R\right]}{2} = \rho C_p \delta R^2\left[T_o - T_R\right] \quad (9)$$

Consider now the unsteady state heat transfer in the foil. The input heat flux is partially stored in the foil and partially removed by the coolant at the foil periphery. The stored energy is the change in E with respect to time given by the above expression.

Thus, $\frac{dE}{dt} = \rho C_p \delta \pi R^2 \frac{d\left(T_o - T_R\right)}{dt}$

The heat loss at r = R, is in fact given by the instantaneous conductive heat transfer at the periphery,

$$-2\pi Rk\delta \left.\frac{dT}{dr}\right|_{r=R} = \frac{q}{4k\delta}(-2R).(-2\pi Rk\delta) = \pi R^2 q$$
$$= \pi R^2 4k\delta \frac{(T_o - T_R)}{R^2} = \pi 4k\delta(T_o - T_R)$$

Thus we have

$$\rho C_p \delta \pi R^2.\frac{d(T_o - T_R)}{dt} + 4\pi\delta k(T_o - T_R) - \pi R^2 q = 0.$$

Or

$$\underbrace{\frac{d(T_o - T_R)}{dt} + \frac{4k(T_o - T_R)}{\rho C_p R^2}}_{\substack{\text{These two terms determine} \\ \text{first order system behaviour}}} - \frac{2q}{\rho C_p f} = 0 \qquad \textbf{(10)}$$

The time constant is identified as

$$\tau = \frac{\rho C_p . R^2}{4k} = \frac{R^2}{4\alpha} \qquad \textbf{(11)}$$

Where α is the thermal diffusivity of the foil material.

Figure 4 shows the relationship (11) between the foil radius and the first order time constant of the sensor in graphical form. This figure may be used in tandem with Figure 3 for determining the proportions and the time constant for the chosen dimensions of the gage. Figure 5 shows the actual view of a commercially available heat flux gage which has arrangement for cooling of the cylinder by water. The figure also gives the typical ranges of gage available and their characteristics, the sensitivity and time constant. The illustration has been taken from the manufacturer's web site indicated therein.

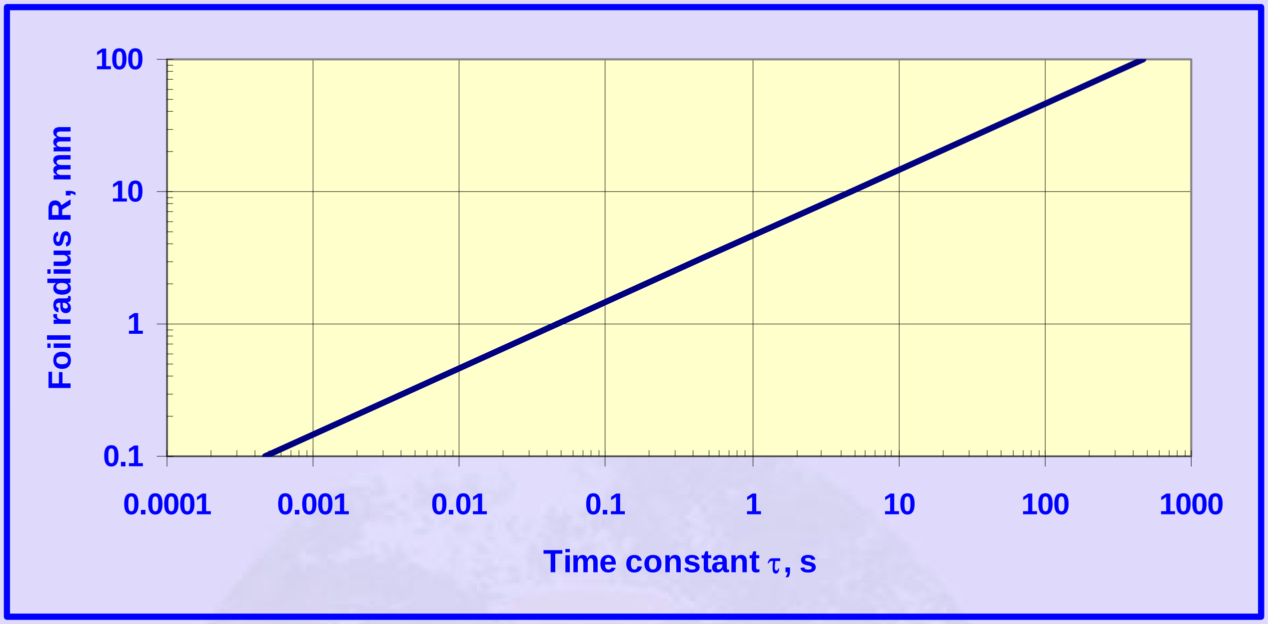


Figure 4 Time constant for a foil type heat flux gage


Figure 5 Commercial foil type heat flux gage

**Thin film sensors:**

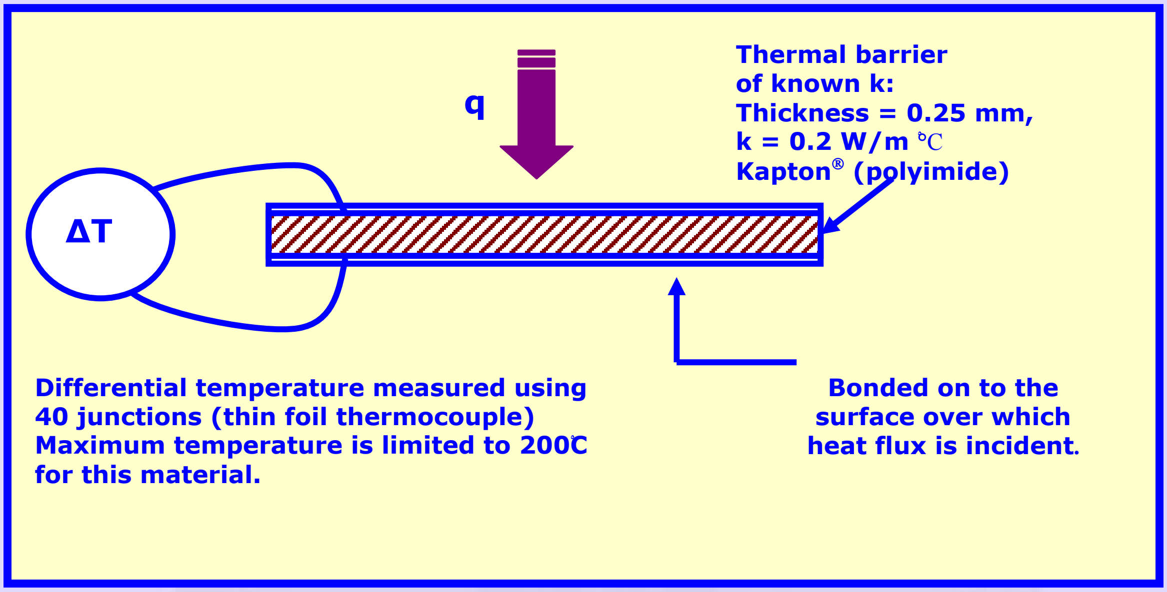


**Figure 6 Schematic of a thin film heat flux sensor**

The operation of a thin film heat flux sensor, shown schematically in Figure 6, is very simple. A thin barrier of known thermal conductivity is attached to a surface that is receiving the heat flux to be measured. The barrier imposes a thermal resistance parallel to the direction of the heat flux and the heat conduction in the barrier is one-dimensional. The temperature difference across the barrier is measured using a thermopile arrangement wherein several hot and cold junctions are connected in opposition. The output is proportional to the heat flux. Example 2 below brings out the typical characteristics of such a heat flux gage.

# Example 2

The geometrical description of a thin film heat flux sensor is given in Figure 5. The thermal conductivity of the barrier material is also given therein. Output of the gage is known to be V = 0.1 V (100 mV) at the maximum rated heat flux. Determine the maximum rated heat flux from this data.

Since there are 40 junctions, the total output corresponds to 0.1 / 40 = 0.0025 V per junction. Assuming that 40 μV corresponds to 1°C this translates to a temperature difference of

$$\Delta T = \frac{0.0025}{40\times 10^{-6}} = 62.5°C$$

Using known k and δ values we then have

$$q = k\frac{\Delta T}{\delta} = \frac{(0.2)(62.5)}{(0.25\times 10^{-3})}\frac{W}{m^2} = 50000\frac{W}{m^2}$$

Typically the solar heat flux is 1000 W / $m^2$. The above heat flux is some 50 times larger!

**Cooled thin wafer heat flux gage:**

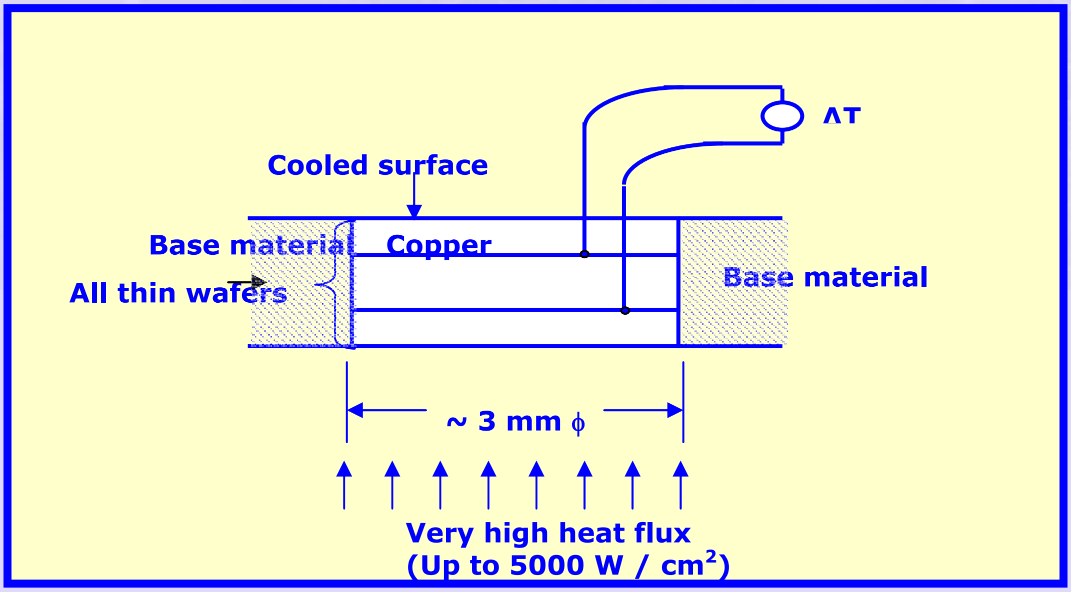


**Figure 7 Thin wafer type cooled heat flux gage**

The operational principle of the thin wafer type cooled heat flux gage is the same as the thin film gage above. Temperature drop across the constantan wafer is measured by the differential thermocouple arrangement shown in Figure 7. There are two T type junctions formed by the constantan wafer sandwiched between the two copper wafers

## Axial conduction guarded probe:

This probe (see Figure 8) is based on conduction through the probe in a direction parallel to the heat flux that is being measured. The gage consists of a cylinder of known thermal conductivity with an annular guard. The guard consists of outer annular cylinder made of the same material as that of the gage. It is exposed to the same heat flux and cooled at the back by the same coolant that is also used to cool the probe itself. Since the outer annulus experiences roughly the same axial temperature gradient as the probe, one dimensional conduction is achieved in the probe. The temperatures are measured by two embedded thermocouples as indicated in the figure. Fourier law is used to derive the heat flux from the measured temperature difference, the distance between the thermocouples and the known thermal conductivity of the probe material.

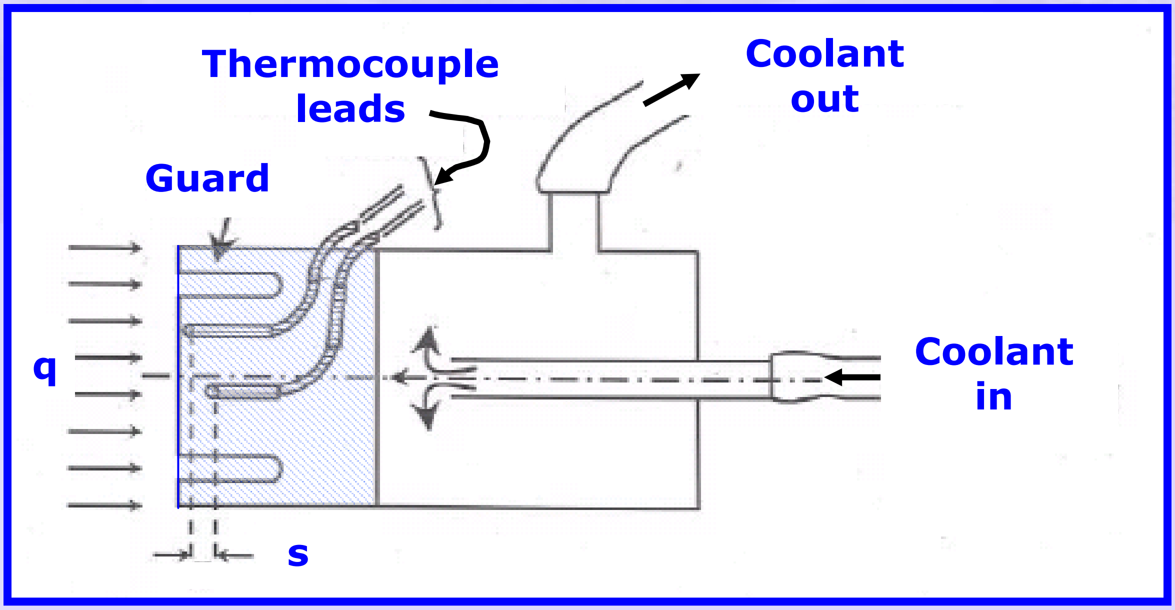

**Figure 8 Axial conduction guarded heat flux probe**

## Example 3

Consider an axial conductivity guarded heat flux probe made of an alloy material of thermal conductivity equal to 45 W/m°C. The two thermocouples are placed 1 cm apart. The incident heat flux is known to be $10^5$ W/m$^2$. The probe has a diameter of 25 mm. Determine the indicated temperature difference ΔT and the heat Q that needs to be removed from the back surface of the probe.

$$q = 100000\,\frac{W}{m^2} = k\frac{\Delta T}{s}\,(\text{by Fourier law})$$

$$\therefore \Delta T = \frac{100000 \times 0.01}{45} = 22.2°C$$

$$\text{Heat to be removed} = Q = qA = 100000 \times \pi \times \frac{0.025^2}{4} = 49.1\,W$$

This amount of heat is removable by an air stream.

**Slug type sensor:**

Schematic of a slug type heat flux sensor is shown in Figure 9. A mass M of a material of specific heat c is embedded in the substrate as shown. Frontal area A of the slug is exposed to the heat flux to be measured while all the other surfaces of the slug are thermally insulated as indicated. When the incident flux is absorbed at the surface of the slug, it heats the slug, and uniformly so if it is made of a material of high thermal conductivity.

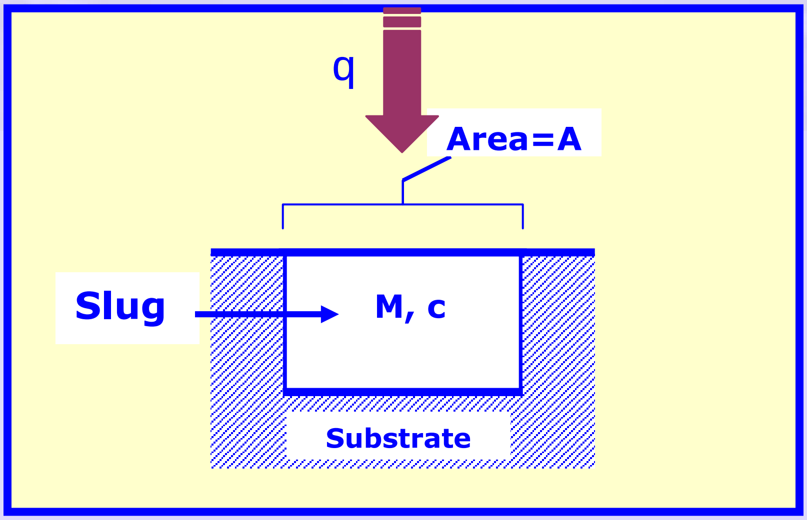

**Figure 9 Slug type heat flux sensor**

In the ideal case with no heat loss the equation for the temperature of the slug is given by

$$Mc\frac{dT}{dr} = qA \quad \textbf{(12)}$$

On integration this will yield the slug temperature as a linear function of time given by

$$T = T_0 + \frac{qA}{Mc}t \quad \textbf{(13)}$$

Obviously we have to allow the process of heating to terminate by stopping the exposure of the slug to the incident heat flux when the slug temperature reaches its maximum allowable temperature.

## Response of a slug type sensor with a small heat loss:

Consider now the case when there is a small heat leak from the slug sensor. Let the loss be proportional to the temperature excess of the slug with respect to the casing or the substrate. The loss coefficient is given as $K_L$ W/°C. Equation (12) will now be replaced by

$$Mc\frac{dT}{dr} = qA - K_L\left(T - T_c\right) \quad \textbf{(14)}$$

Let $T - T_c = \theta$ and $\frac{K_L}{Mc} = \in$. With these equation (14) will be recast as

$$\frac{d\theta}{dt} - \frac{qA}{Mc} = -\in\theta \quad \textbf{(15)}$$

Since $K_L$ is expected to be small, the parameter $\in$ is small. The solution may be sought by expanding it in the form $\theta = \theta^{(1)} + \in\theta^{(2)} + ......$ Substituting this in equation (15) we have

$$\frac{d\theta^{(1)}}{dt} - \frac{qA}{Mc} + \in\frac{d\theta^{(2)}}{dt} + ..... = -\left[\in\theta^{(1)} + \in^2\theta^{(2)} + ......\right] \quad \textbf{(16)}$$

Collecting terms of same order, the above is replaced by

$$\frac{d\theta^{(1)}}{dt} = \frac{qA}{Mc} \quad \textbf{(17a)}$$

$$\frac{d\theta^{(2)}}{dt} = -\frac{qA}{Mc}\theta^{(1)} \quad \textbf{(17b)}$$

These equations are solved to get

$$\theta^{(1)} = \frac{qAt}{Mc};\ \theta^{(2)} = -\frac{qAt^2}{2Mc} \quad \textbf{(17c)}$$

Thus the response of the slug follows the relation

$$\theta \approx \frac{qAt}{Mc} - \in \frac{qA}{2Mc}.t^2 = \frac{qA}{Mc}\left[t - \frac{K_L}{Mc}.\frac{t^2}{2}....\right] \quad \textbf{(18)}$$

Thus a slight amount of nonlinearity is seen in the solution.

## Example 4

A slug type of sensor is made of a copper slug of 3 mm thickness. The specification is that the temperature of the slug should not increase by more than 40°C.

*a) What is the time for which the slug can be exposed to an incident heat flux of 10000 W/m$^2$ and there is negligible heat loss from the slug?*

*b) What is the time for which the slug can be exposed to an incident heat flux of 10000 W/m$^2$ and there is small heat loss from the slug specified by a loss coefficient of 50 W/°C?*

Note: We make all calculations based on unit area of slug exposed to the incident heat flux. Properties of Copper are taken from hand book

Density ρ = 8890 kg / m$^3$, specific heat C = 398 J/kg°C

The maximum temperature rise during operation is $\Delta T_{max}$ = 40°C

*a) If there is negligible heat loss the temperature increases linearly and the maximum exposure time is given by equation (13).*

$$t_{max} = \frac{3\times10^{-3}\times8890\times398\times40}{10000} = 42.46 \text{ s}$$

b) When the heat loss is taken into account, what happens is worked out below.We have, from the given data $\frac{K_L}{Mc} = \frac{50}{8890 \times 3 \times 10^{-3} \times 398} = 0.00047$ and $\frac{qA}{Mc} = \frac{q}{\rho\delta c} = \frac{10000}{8890 \times 3 \times 10^{-3} \times 398} = 0.942$. With these, we get, using equation xx $\theta \approx 0.942\left[t - 0.00047\frac{t^2}{2}\right]$.

With $\Delta T_{max} = 40^\circ C$ we have $\Delta T_{max} = 0.942\, t_{max}\left[1 - 0.000235\, t_{max}\right]$

or $t_{max} = \frac{\Delta T_{max}}{0.942\left[1 - 0.000235 t_{max}\right]}$. This gives the maximum exposure time

approximately as $t_{max} \approx \frac{40}{0.942\left[1 - 0.000235 \times 42.46\right]} = 47.2\text{s}$