Sub Module 2.7

Systematic errors in temperature measurement

Systematic errors are situation dependent. We look at typical temperature measurement situations and discuss qualitatively the errors before we look at the estimation of these. The situations of interest are:

Measurement of temperature

- at a surface
- inside a solid
- of a flowing fluid

Surface temperature measurement using a compensated probe:

Consider the measurement of the temperature of a surface by attaching a thermocouple sensor normal to it, as shown in Figure 50.

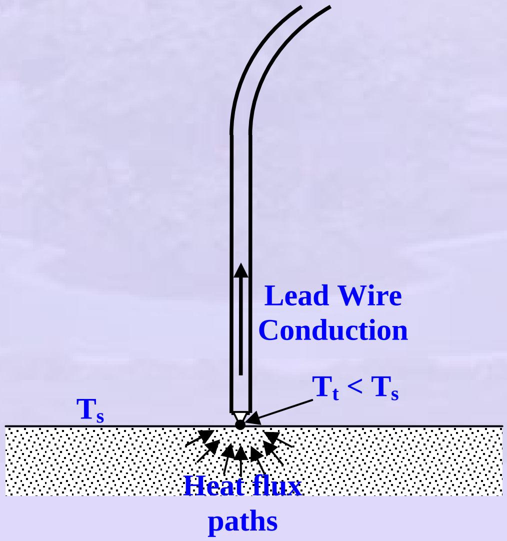


Figure 50 Temperature measurement of a surface

Lead wires conduct heat away from the surface and this is compensated by heat transfer to the surface as shown. This sets up a temperature field within the solid such that the temperature of the surface where the thermocouple is attached is depressed and hence less than the surface temperature elsewhere on the surface. This introduces an error in the surface temperature measurement. One way of reducing or altogether eliminating the conduction error is by the use of a compensated sensor as indicated in Figure 51.

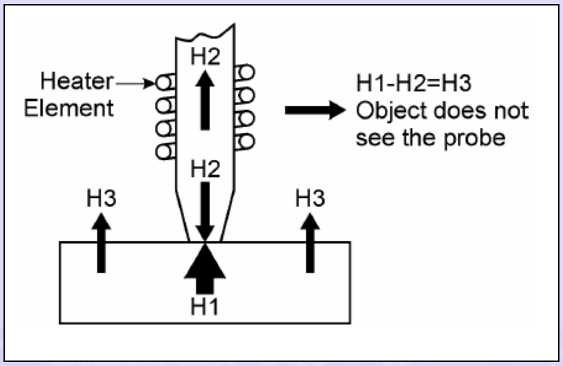


Figure 51 Schematic of a compensated probe

This figure is taken from “Industrial measurements with very short immersion & surface temperature measurements” by Tavener et al. The surface temperature, in the absence of the probe is at an equilibrium temperature under the influence of steady heat loss H_3 to an environment. The probe would involve as additional heat loss due to conduction. If we supply heat H_2 by heating the probe such that there is no temperature gradient along the thermocouple probe then H_1-H_2 =H_3 and the probe temperature is the same as the surface temperature. Figure 52 (taken from the same reference) demonstrates that the compensated probe indicates the actual surface temperature with negligible error.

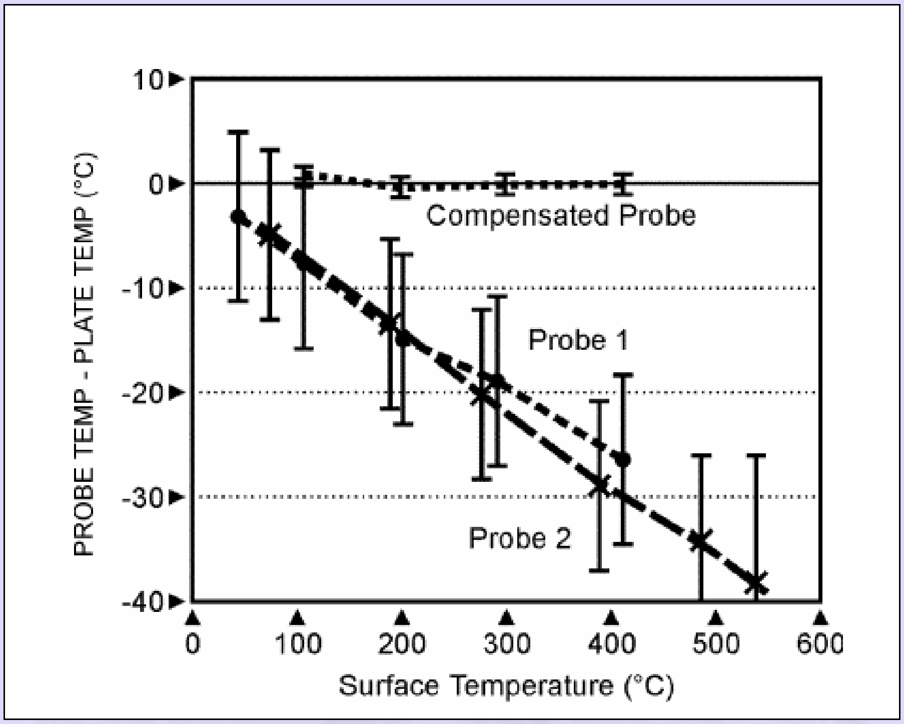


Figure 52 Comparison between the thermally compensated probe and two standard probes

Compensated probes as described above are commercially available from ISOTECH (Isothermal Technology Limited, Pine Grove, Southport, Merseyside, England) and described as 944 True Surface temperature measurement systems.

Figure 53 shows how one can arrange a thermocouple to measure the temperature inside a solid. The thermocouple junction is placed at the bottom of a blind hole drilled into the solid. The gap between the thermocouple lead wires and the hole is filled with a **heat conducting cement**. The lead wire is exposed to the ambient as it emerges from the hole. It is easy to visualize that the lead wire conduction must be compensated by heat conduction into the junction from within the solid. Hence we expect the solid temperature to be greater than the junction temperature (this is the temperature that is indicated) which is greater than the ambient temperature. This assumes that the solid is at a temperature higher than the ambient.

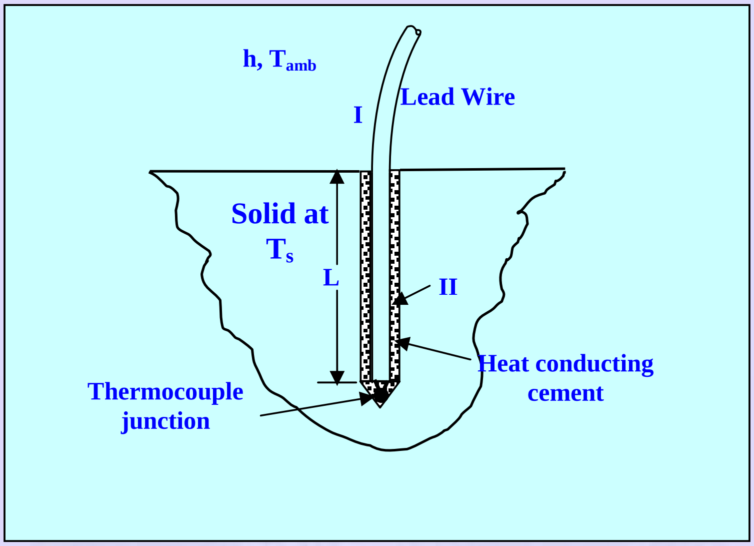


Figure 53 Measurement of temperature within a solid

Often it is necessary to measure the temperature of a **fluid flowing through a duct**. In order to prevent leakage of the fluid or prevent direct contact between the fluid and the temperature sensor, a thermometer well is provided as shown in Figure 54. The sensor is attached to the bottom of the **well** as indicated. The measured temperature is the temperature of the bottom of the well and what is desired to be measured is the fluid temperature.

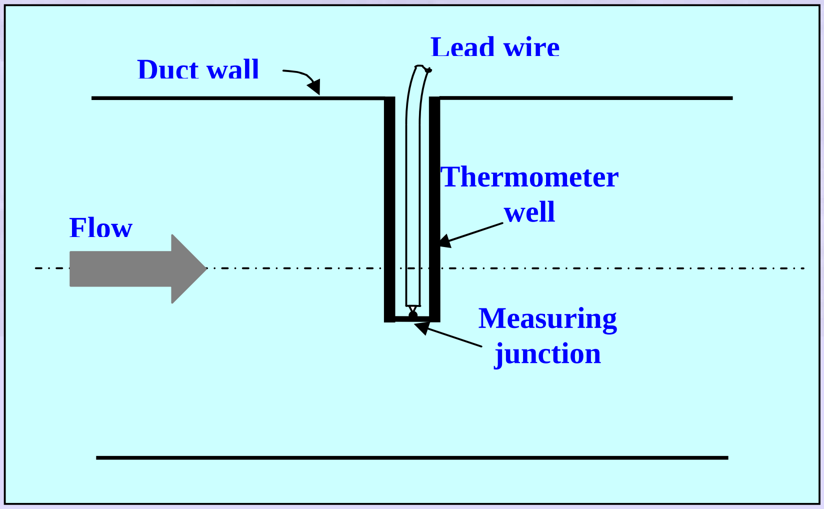


Figure 54 Measurement of temperature of a moving fluid

If the duct wall is different from the fluid temperature, heat transfer takes place by **conduction** between the fluid and the duct wall and hence the well bottom temperature will be at a value in between that of the fluid and the wall. There may also be **radiation** heat transfer between surfaces, further introducing errors. If the fluid flows at high speed (typical of supersonic flow of air) **viscous dissipation** – conversion of kinetic to internal energy – may also be important. With this background we generalize the thermometer error problem in the case of measurement of temperature of a gas flow as indicated in Figure 55.

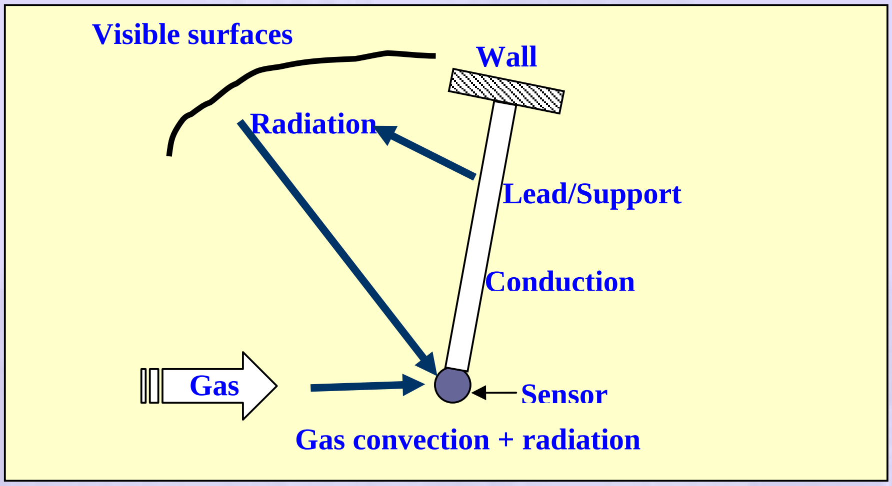


Figure 55 Heat transfer paths for a sensor in gas flow

The temperature of the sensor is determined, under the steady state, by a **balance** of the different heat transfer processes that take place, as indicated in Figure 55. Not all the heat transfer processes may be active in a particular case. The thermometer error is simply the difference between the **gas temperature** and **the sensor temperature**. Estimation of the error will be made later on.

Summary of sources of error in temperature measurement:

- **Sensor interferes with the process**
  - **Conduction error in surface temperature measurement**
- **Sensor interferes with the process as well as other environments**
  - **Radiation error**
-

- **While measuring temperature of moving fluids convection and conduction processes interact and lead to error**
- **In case of high speed flow, viscous dissipation effects may be important**

Conduction error in thermocouple temperature measurement:

Lead wire model

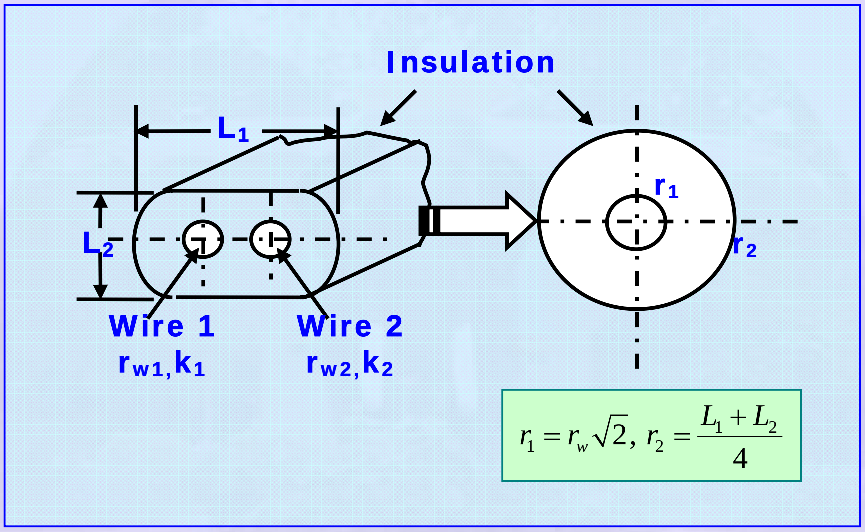


Figure 56 Single wire equivalent of a thermocouple

Heat transfer through the lead wires of a thermocouple leads to error in the measured temperature. Since a thermocouple consists of two wires of different materials covered with insulation, and since the **error estimation** should involve a **simple** procedure, we replace the actual thermocouple by a single wire thermal equivalent. How this is accomplished is indicated by referring to Figure 56. The cross section of an actual thermocouple is shown at the left in Figure 56. It consists of two wires of different materials with the indicated radii and thermal conductivity values. The insulation layer encloses the two wires as indicated. We replace the **two wires and the insulation** by a **single wire** of radius r_1 and a **coaxial insulation** layer of outer radius r_2.

The single wire model:

1) The area thermal conductivity product must be the same for the two wires and the single wire. Thus $(kA)_{two\ wires} = (kA)_{one\ wire} = k_1 A_{w1} + k_2 A_{w2}$. If the two wires have the same diameter (this is usually the case) we may replace this by $(kA)_{one\ wire} = k_1 A_{w1} + k_2 A_{w2} = \frac{(k_1 + k_2)}{2}(2A_{w1})$. Thus the thermal conductivity of the single wire equivalent is equal to the mean of the thermal conductivities of the two wires and the area of cross section of the single wire is twice the area of cross section of either wire. Hence the radius of the single wire equivalent is given by $r_1 = \sqrt{2} r_w$ as indicated in Figure 56.

2) The insulation layer is to be replaced by a coaxial cylinder of inner radius r_1 and the outer radius r_2. The outer radius is taken as $r_2 = \frac{L_1 + L_2}{4}$. Note that if $L_1 = L_2 = 2r$ (true for a circle of radius *r*), this formula gives $r_2 = r$, as it should.

3) Since the insulation layer is of low thermal conductivity while the wire materials have high thermal conductivities, it is adequate to consider heat conduction to take place along the single wire and radially across the insulation, as indicated in Figure 57.

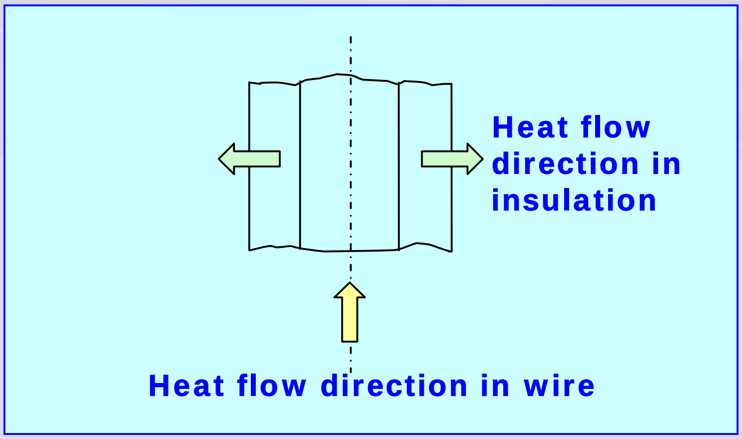


Figure 57 Heat flow directions

Figure 58 shows a typical application where the temperature of a surface exposed to a moving fluid is being measured. The solid is made of a low thermal conductivity plastic.

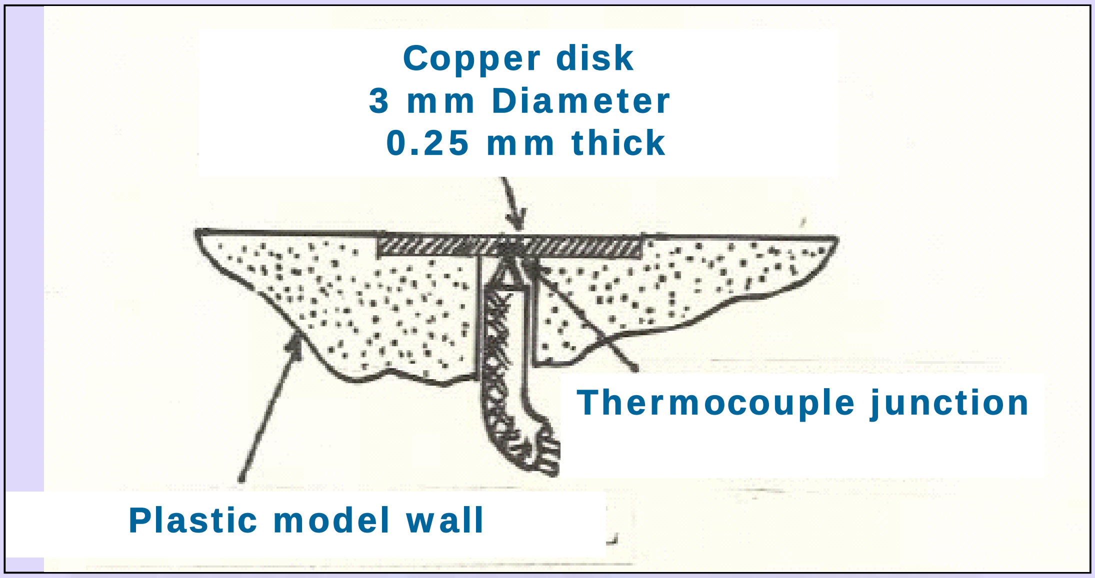


Figure 58 Surface temperature measurement

The thermocouple lead wires conduct away some heat that is gathered by the thermocouple in contact with the solid. This will tend to depress the temperature of the junction. In order to reduce the effect of this **thermocouple lead wire conduction**, the junction is attached to a heat collecting pad of copper as indicated in the figure.

Now consider the typical application presented in Figure 58. Figure 59 explains the nomenclature employed for the analysis of this case. The heat conducting pad receives heat from the front face of area S and loses heat only through the thermocouple due to lead wire conduction. The appropriate thermal parameters are as shown in Figure 59.

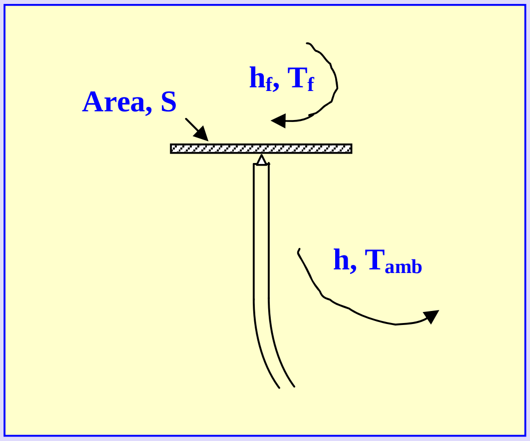


Figure 59 Nomenclature for lead wire conduction analysis

Heat loss through the lead wire is modeled by using fin type analysis, familiar to us from the study of heat transfer. Since the wire is usually very long, it may be assumed to infinitely long. The heat loss from the wire to the ambient is modeled as that due to an overall heat transfer coefficient given by

$$h_{Overall} = \frac{1}{\frac{1}{h} + \frac{r_2}{k_i}\ln\left(\frac{r_2}{r_1}\right)} \quad \textbf{(43)}$$

The perimeter of the wire is $P = 2\pi r_1$ and the area thermal conductivity product for the wire is $kA = \frac{(k_1 + k_2)}{2}\left(2\pi r_1^2\right) = \pi r_1^2 (k_1 + k_2)$. The appropriate fin parameter m is then given by

$$m = \sqrt{\frac{h_{Overall} P}{kA}} = \sqrt{\frac{h_{Overall} 2\pi r_1}{(k_1 + k_2)\pi r_1^2}} = \sqrt{\frac{2h_{Overall}}{(k_1 + k_2) r_1}} \quad \textbf{(44)}$$

Assuming the lead wire to be infinitely long, the heat loss through the lead wire is given by

$$Q_{Lead\ wire} = kAm\left(T_t - T_{amb}\right) \quad \textbf{(45)}$$

Under steady conditions this must equal the heat gained from the fluid by the pad given by

$$Q_{Gain\ by\ pad} = h_f S\left(T_f - T_t\right) \quad \textbf{(46)}$$

Equating (3) and (4) we solve for the sensor indicated temperature as

$$T_t = \frac{h_f S T_f + kAmT_{amb}}{h_f S + kAm} \qquad \textbf{(47)}$$

Equation 47 shows that the sensor temperature is a weighted mean of the fluid temperature and the ambient temperature. It is clear that the smaller the weight on the ambient side better it is from the point of view of temperature measurement. This is a general feature, as we shall see later, in all cases involving temperature measurement. The thermometric error is then given by

$$T_{Error} = T_t - T_f \qquad \textbf{(48)}$$

Example 18 below demonstrates the use of the above analysis in a typical situation.

Example 18

- *A copper constantan thermocouple of wire diameter each of 0.25 mm is used for measuring the temperature of a surface which is convectively heated by a fluid with a heat transfer coefficient of 67 W/m^2 °C. The area of the surface exposed to the fluid is 10 cm^2. The thermocouple has an insulation of thickness 1 mm all round and the overall size is 5 mm x 2.5 mm. The thermal conductivity of the insulation is 1 W/m°C. The thermocouple is exposed to an ambient at a temperature of 30°C subject to a heat transfer coefficient of 5 W/m^2 °C. If the fluid temperature is 200°C what is the temperature indicated by the thermocouple? Take thermal conductivity of copper as 386 W/m°C and the thermal conductivity of constantan as 22.7 W/m°C.*

- *Wire side calculation:*

  Diameter of each thermocouple wire $\mathrm{d} = 0.00025\ \mathrm{m}$

  Thermal conductivities of the thermocouple wires $k_1 = k_{Copper} = 386\ W/m°C,\ k_2 = k_{Constantan} = 22.7\ W/m°C$

  The area of cross section of each wire is

  $$\mathrm{A} = \pi\frac{\mathrm{d}^2}{4} = \pi\frac{0.00025^2}{4} = 4.909\times10^{-8}\,\mathrm{m}^2$$

  Effective thermal conductivity area product for the thermocouple pair is

  $$kA = \frac{(k_1+k_2)}{2}(2A) = (386+22.7)\times4.909\times10^{-8} = 2.006\times10^{-5}\,W\,m/°C$$

  Overall heat transfer coefficient is now calculated:

  The overall heat transfer coefficient is calculated by combining the insulation and film resistances. We have $\mathrm{h} = 5\ \mathrm{W/m^2\,°C}$. *The radius of the single wire equivalent is* $r_1 = \frac{d}{\sqrt{2}} = \frac{0.00025}{\sqrt{2}} = 0.000177\ m$. *The outer* *radius of effective insulation layer is* $r_2 = \frac{0.005+0.0025}{4} = 0.001875\ m$.

Thermal conductivity of insulation material is $k_i = 1\ W/m°C$. *The overall heat transfer coefficient is*

$$h_{Overakl} = \frac{1}{\frac{1}{h}+\frac{r_2}{k_i}\ln\left(\frac{r_2}{r_1}\right)} = \frac{1}{\frac{1}{5}+\frac{0.001875}{1}\ln\left(\frac{0.001875}{0.000177}\right)} = 4.892\ W/m^2°C$$

The overall heat transfer coefficient perimeter product is thus given by

$$\left(h_{Overall}P\right) = 2\pi r_1 h_{Overall} = 2\times\pi\times 0.000177\times 4.892 = 0.005433\ W/m°C$$

The fin parameter is calculated as

$$m = \sqrt{\frac{h_{Overall}P}{kA}} = \sqrt{\frac{4.892}{2.006\times10^{-5}}} = 16.457\ m^{-1}$$

- *The surface temperature may now be calculated by equating the heat transfer from the fluid to surface to that lost through the thermocouple insulation. The appropriate data is:*

$$h_f = 100\ W/m^2°C,\ S = 10\ cm^2 = 10^{-3}m^2,\ T_f = 200°C \text{ and } T_{amb} = 30°C$$

- *From the material presented earlier, assuming the thermocouple wires to be very long, the surface temperature is given by*

$$T_t = \frac{h_f S T_f + kAm\ T_{amb}}{h_f S + kAm}$$

$$= \frac{100\times0.001\times200+2.006\times10^{-5}\times16.457\times30}{100\times0.001+2.006\times10^{-5}\times16.457} = 199.2°C$$

- *The thermometer error is* $T_t - T_f = 199.2 - 200 = -0.8°C$

Temperature error due to radiation:

Errors in temperature measurement may occur due to surface radiation, especially at elevated temperatures. We consider the same example that was considered above. Assume that the copper disk has a surface emissivity of . Let it also view a cold background at T_{bkg}. The heat loss is now due to lead wire conduction along with radiation to the ambient. Heat loss due to radiation is given by

$$Q_{Radiation} = \varepsilon\sigma S\left(T_t^4 - T_{bkg}^4\right)$$

Note that the temperatures are to be expressed in Kelvin in Equation 49 and is the Stefan Boltzmann constant. The temperature of the sensor is determined by equating heat gain by convection to heat loss by conduction and radiation. Thus

$$Q_{Lead\ wire} + Q_{Radiation} = Q_{Gain\ by\ pad} \qquad \textbf{(50)}$$

Using Equations 45, 46 and 49 we then have

$$kAm\left(T_t - T_{amb}\right) + \varepsilon\sigma S\left(T_t^4 - T_{bkg}^4\right) = h_f S\left(T_f - T_t\right) \qquad \textbf{(51)}$$

The above non-linear algebraic equation needs to be solved to arrive at the value of the measured temperature.

Example 19

- *Reconsider Example 1 with the following additional data:*

  The copper pad has a surface emissivity of 0.05 and views a cooler background at a temperature of 450 K. What is the thermometric error in this case?

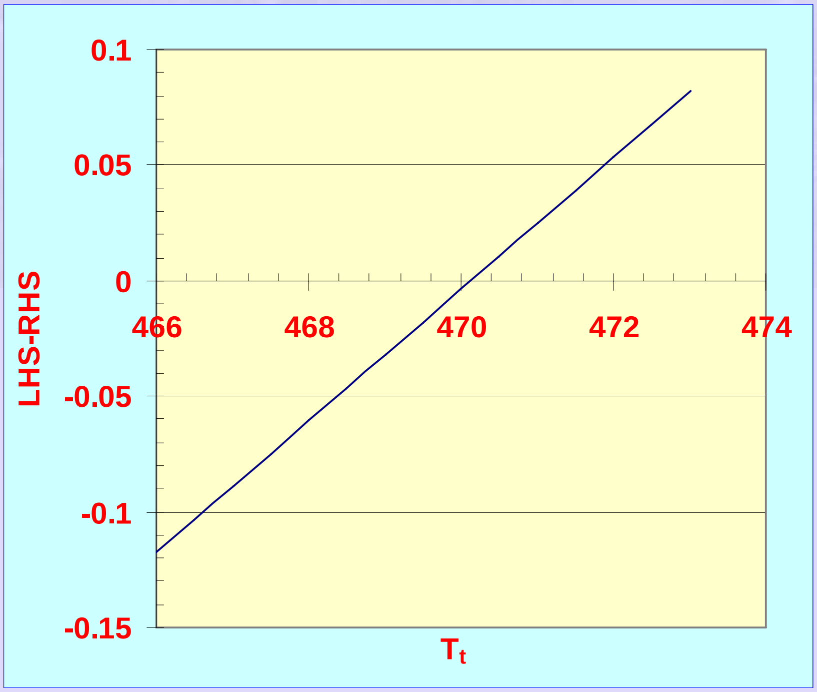
- *In addition to the heat loss by lead wire conduction we have to include that due to radiation. This is given by*

$$Q_{Radiation} = 0.05 \times 5.67 \times 10^{-8}(T_t^4 - 450^4) = 2.84 \times 10^{-12}(T_t^4 - 450^4)$$

- *Using the material already available in Example 1 the equation that governs the sensor temperature is given by*

$$0.00033(T_t - 30) + 2.84 \times 10^{-12}(T_t^4 - 450^4) = 0.067\left(473 - T_t\right)$$

This equation may be solved by Newton Raphson method. Alternately the solution may be obtained by making a plot of the difference between the left hand side and right hand side of this equation and locate the point where it crosses the temperature axis. Such a plot is shown below.



It is clear that the sensor temperature is now 470.2 K or 197.2°C. The temperature error has changed to -2.8°C! Error due to radiation is, in fact, more than that due to lead wire conduction.

Measurement of temperature within a solid:

Now we shall look at the situation depicted in Figure 60. Temperature error is essentially due to conduction along the lead wires. However, the portion embedded within the solid (II) has a different environment as compared to the part that is outside (I). Both of these may be treated by the single wire model introduced earlier. Assume that the solid is a temperature higher than the ambient. The thermocouple junction will then be at an intermediate temperature between that of the solid and the ambient. Heat transfer to the embedded thermocouple is basically by conduction while the heat transfer away from the part outside the solid is by conduction and convection. The embedded part is of finite length L while the portion outside may be treated as having an infinite length.

Represent the temperature of the single wire equivalent as T_i in a plane coinciding with the surface of the solid. Let T_t be the temperature of the junction while T_s is the temperature of the solid. Let the ambient temperature be T_{amb}. The fin parameter for the embedded part may be calculated based on the overall heat transfer coefficient given by

$$h_{Overall,II} = \frac{1}{\frac{r_3}{k_c}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_2}{k_i}\ln\left(\frac{r_2}{r_1}\right)} \quad \textbf{(52)}$$

In the above, k_c is the thermal conductivity of the heat conducting cement, r_3 is the radius of the hole and the other symbols have the earlier meanings. Note that expression 52 is based on two conductive resistances in series. The corresponding fin parameter is $m_{II} = \sqrt{\frac{h_{Overall,II}P}{kA}}$. The overall heat transfer coefficient for the exposed part of the thermocouple is given by the expression given earlier, viz. $h_{Overakl,I} = \frac{1}{\frac{1}{h} + \frac{r_2}{k_i}\ln\left(\frac{r_2}{r_1}\right)}$. The corresponding fin parameter value is $m_I = \sqrt{\frac{h_{Overall,I}P}{kA}}$.

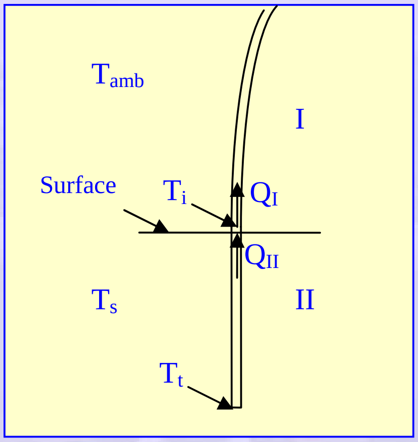


Figure 60 Nomenclature for thermal analysis

Referring now to Figure 60 we see that the heat transfer across the surface through the thermocouple should be the same i.e. $Q_{II} = Q_I$. Using familiar fin analysis, we have

$$Q_{II} = h_{Overall,II}P\left(T_s - T_i\right)\frac{\tanh\left(m_{II}L\right)}{m_{II}} \quad \textbf{(53)}$$

For the exposed part, we have

$$Q_I = kAm_I\left(T_i - T_{amb}\right) \quad \textbf{(54)}$$

Equating the above two expressions we solve for the unknown temperature T_i. Thus

$$T_i = \frac{w_1T_s + w_2T_{amb}}{w_1 + w_2} \ \ where$$
$$w_1 = h_{Overall,II}P\frac{\tanh\left(m_{II}L\right)}{m_{II}} \ \ and \ \ w_2 = kAm_I \quad \textbf{(55)}$$

Having found the unknown temperature T_i, we make use of fin analysis for the embedded part to get the temperature T_t. Using familiar fin analysis, we have

$$T_t = T_s + \frac{(T_i - T_s)}{\cosh(m_{II} L)} \quad (56)$$

Note that the fin analysis assumes negligible heat transfer near the bottom of the hole!

Following points may be made in summary:

1) **The longer the depth of embedding smaller the thermometric error**
2) **Higher the thermal conductivity of the epoxy filling the gap between the thermocouple and the hole the smaller the thermometric error**
3) **The smaller the diameter of the thermocouple wires smaller is the thermometric error**
4) **Smaller the thermal conductivity of the thermocouple wires smaller the thermometric error**
5) **If it is possible the insulation over the thermocouple wires should be as thin as possible in the embedded portion and as thick as possible in the portion that is outside the hole**

Example 20

- *Thermocouple described in Example 1 is used to measure the temperature of a solid by embedding it in a 6 mm diameter hole that is 15 mm deep. The space between the thermocouple and the hole is filled with a heat conducting epoxy that has a thermal conductivity of 10 W/m°C. The lead wires coming out of the hole are exposed to an ambient at 30°C with a heat transfer coefficient of 5 W/m^2°C. If the temperature of the solid is 80°C, estimate the temperature indicated by the thermocouple.*

- *From the results in Example 1, the following are available:*

$$kA = 2.006\times10^{-5}\,\text{W m/°C},\; m_I = 16.457\ \text{m}^{-1}$$

- *The weight w_2 is then given by*

$$w_2 = kAm_I = 2.006\times10^{-5}\times16.457 = 0.00033\ \text{W m/°C}$$

- *For the embedded part, the following calculations are made:*

$$r_3 = \frac{0.006}{2} = 0.003\ \text{m},\; k_c = 10\,\text{W/m°C},$$

$$h_{Overall,II} = \frac{1}{\frac{0.003}{10}\ln\left(\frac{0.003}{0.001875}\right) + \frac{0.001875}{1}\ln\left(\frac{0.001875}{0.00177}\right)} = 218.88\ \text{W/m}^2\text{°C}$$

The fin parameter is then calculated as

$$m_{II} = \sqrt{\frac{218.8\times2\times\pi\times0.00177}{2.006\times10^{-5}}} = 110.08\ \text{m}^{-1}$$

With L = 0.015 m, we have $m_{II}L = 110.08\times0.015 = 1.651$

The weight w_1 is then given by

$$w_1 = 218.8\times2\times\pi\times0.00177\times\frac{\tanh(1.651)}{110.08} = 0.002052$$

- *The unknown temperature T_i is now calculated as*

$$T_i = \frac{0.002052\times80 + 0.00033\times30}{0.002052 + 0.00033} = 73.07°C$$

- *The sensor temperature is then calculated as*

$$T_t = 80 + \frac{(73.07 - 80)}{\cosh(1.651)} = 77.4°C$$

- *The thermometric error is thus equal to -2.6℃.*

The thermometer well problem

This is a fairly common situation as has been mentioned earlier. The well (shown schematically in Figure 61) acts as a protection for the temperature sensor but leads to error due to axial conduction along the well. It is easily recognized that the well may be treated as a fin and the analysis made earlier will be adequate to estimate the thermometric error.

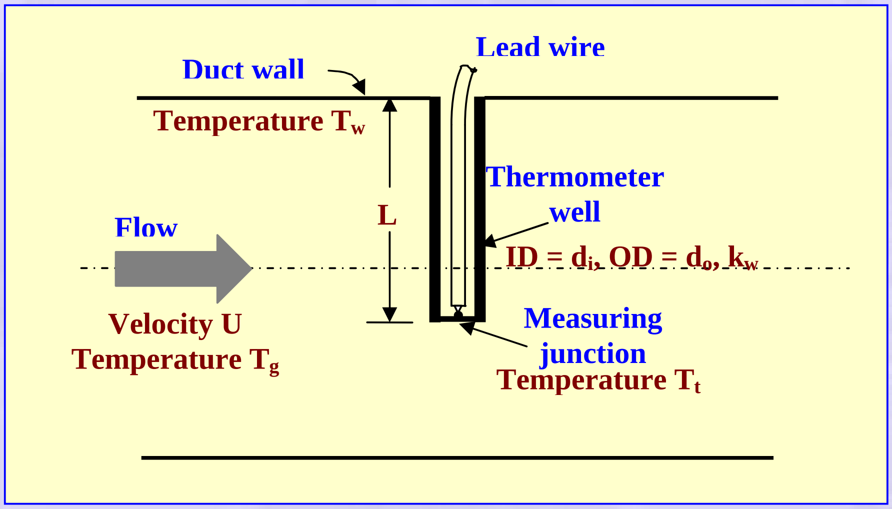


Figure 61 Nomenclature for the thermometer well problem

Assumptions:

1) Since the thermometer well has a much larger cross section area than the thermocouple wires conduction along the wire is ignored.

2) The thermometer well is heated by the gas while it cools by radiation to the walls of the duct (based on $T_g > T_t > T_w$).

3) Well is treated as a cylinder in cross flow for determining the convection heat transfer coefficient between the gas and the well surface.

The heat transfer coefficient is calculated based on the Zhukaskas correlation given by

$$Nu = C\,\mathrm{Re}^m\,\mathrm{Pr}^n \qquad \textbf{(57)}$$

In this relation Re, the Reynolds number is based on the outside diameter of the well and all the properties are evaluated at a suitable mean temperature. The constants C, m and n are given in Table 6.

Radiation heat transfer may be based on a linearised model if the gas and wall temperatures are close to each other. In that case the well temperature variation along its length is also not too big. Thus we approximate the radiant flux $q_R = \varepsilon\sigma\left(T^4 - T_w^4\right)$ by the relation $q_R \approx 4\varepsilon\sigma T_w^3\left(T - T_w\right) = h_R\left(T - T_w\right)$ where $h_R = 4\varepsilon\sigma T_w^3$ is referred to as the radiation heat transfer coefficient.

Table 12 Constants in the Zhukaskas correlation

Re	C	m
1-40	0.75	0.4
$40\text{-}10^3$	0.51	0.5
$10^3\text{-}2\times10^5$	0.26	0.6
$2\times10^5\text{-}10^6$	0.076	0.7
		m
Pr < 10		0.36
Pr > 10		0.37

Analysis:

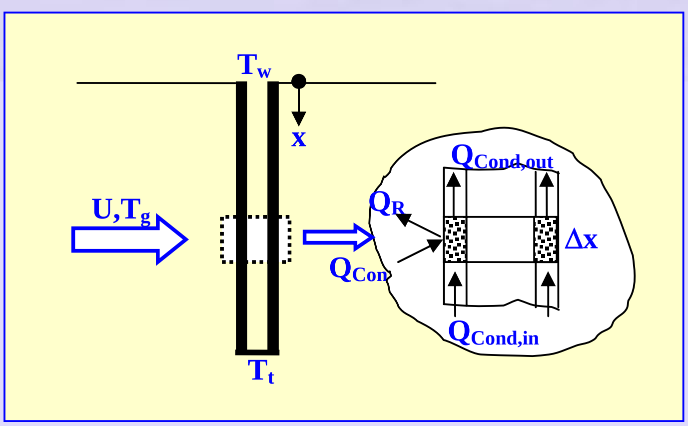


Figure 62 Thermometer well analysis schematic

Refer to Figure 62 and the inset that shows an expanded view of an elemental length of the well. Various fluxes crossing the boundaries of the element are:

1) $Q_{Con} = hP\Delta x\left(T_g - T\right)$ 2) $Q_R = h_R P\Delta x\left(T - T_w\right)$ 3) $Q_{Cond,in} = k_w A\left.\frac{dT}{dx}\right|_x$ and

4) $Q_{Cond,in} = k_w A\left.\frac{dT}{dx}\right|_{x-\Delta x}$

In the above the perimeter P is given by $P = \pi d_o$ and area of cross section A is given by $A = \pi\frac{\left(d_o^2 - d_i^2\right)}{4}$. Energy balance requires that $Q_{Cond,in} + Q_{con} = Q_{Cond,out} + Q_R$. Substituting the expressions for the fluxes and using Taylor expansion of the derivative, we have

$$k_w A\left.\frac{dT}{dx}\right|_x + hP\Delta x\left(T_g - T\right) = h_R P\Delta x(T - T_w) + k_w A\left.\frac{dT}{dx}\right|_x - k_w A\left.\frac{d^2T}{dx^2}\right|_x \Delta x$$

This equation may be rearranged as

$$\frac{d^2T}{dx^2} - \frac{(h + h_R)P}{k_w A}T + \frac{hP}{k_w A}T_g + \frac{h_R P}{k_w A}T_w = 0 \qquad \textbf{(58)}$$

Let $T_{ref} = \frac{hT_g + h_R T_w}{h + h_R}$ be a reference temperature. Then Equation 58 is rewritten as

$$\frac{d^2\theta}{dx^2} - m_{eff}^2\theta = 0 \qquad \textbf{(59)}$$

where $\theta = T - T_{ref}$ and $m_{eff} = \sqrt{\frac{\left(h + h_R\right)P}{k_w A}}$ is the effective fin parameter. Equation 59 is the familiar fin equation whose solution is well known. Assuming insulated boundary condition at the sensor location, the indicated sensor temperature is given by

$$\theta_t = \left(T_t - T_{ref}\right) = \frac{\left(T_w - T_{ref}\right)}{\cosh\left(m_{eff}L\right)} \qquad \textbf{(60)}$$

The thermometric error is thus given by

$$\left(T_t - T_g\right) = \left(T_{ref} - T_g\right) + \frac{\left(T_w - T_{ref}\right)}{\cosh\left(m_{eff} L\right)} \quad \textbf{(61)}$$

Following points may be made in summary:

1) **The longer the depth of immersion L smaller the thermometric error**
2) **Lower the thermal conductivity of the well material the smaller the thermometric error**
3) **Smaller the emissivity of the well and hence the h_R smaller the thermometric error**
4) **Larger the fluid velocity and hence the h smaller the thermometric error**

Example 21

- *Air at a temperature of 373 K is flowing in a tube of diameter 10 cm at an average velocity of 0.5 m/s. The tube walls are at a temperature of 353 K. A thermometer well of outer diameter 4 mm and wall thickness 1 mm made of iron is immersed to a depth of 5 cm, perpendicular to the axis. The iron tube is dirty because of usage and has a surface emissivity of 0.85. What will be the temperature indicated by a thermocouple that is attached to the bottom of the thermometer well? What is the consequence of ignoring radiation?*

- *Step wise calculations are shown below:*

- *Step 1.Well outside convective heat transfer coefficient:*

  Given data:

$$d_0 = 0.004\text{ m},\ U = 0.5\text{ m/s},\ T_f = 373\text{ K},\ T_w = 353\text{ K}$$

The fluid properties are taken at the mean temperature given

by $T_m = \dfrac{373+353}{2} = 363\text{ K}$

From table of properties for air the desired properties are:

$$\nu = 23.02\times10^{-6}\text{ m}^2/\text{s},\ k = 0.0313\text{ W/m K},\ Pr = 0.7$$

The Reynolds number based on outside diameter of thermometer well is

$$Re = \frac{Ud_0}{\nu} = \frac{0.5\times0.004}{23.02\times10^{-6}} = 86.9$$

Zhukaskas correlation is used now. For the above Reynolds number the appropriate constants in the Zhukaskas correlation are

$$C = 0.51,\ m = 0.5\text{ and }n = 0.37.$$

⊙ *Step 4 Well treated as a fin:*

Well material has a thermal conductivity of $k_w = 45\ W/mK$

Internal diameter of well is equal to outside diameter minus twice the wall thickness and is given by $d_i = d_0 - 2t = 0.004 - 2\times 0.001 = 0.002\ m$

The fin parameter

$$m_f = \sqrt{\frac{\pi d_0 (h + h_r)}{k_w \pi \frac{\left(d_0^2 - d_i^2\right)}{4}}} = \sqrt{\frac{0.004\times\left(32.6 + 8.5\right)}{45\times\frac{\left(0.004^2 - 0.002^2\right)}{4}}} = 34.87\ m^{-1}$$

Since the well length is L = 0.05 m the non-dimensional fin parameter is

$$\mu_f = m_f L = 34.87\times 0.05 = 1.74$$

⊙ *Step 5 Non-dimensional well bottom temperature*

It is given by $\theta_t = \frac{1}{\cosh(\mu_f)} = \frac{1}{\cosh(1.74)} = 0.339$

Hence the temperature indicated by the sensor attached to the well is

$$T_t = T_{ref} + \theta_t\left(T_w - T_{ref}\right) = 368.9 + 0.339\times\left(353 - 368.9\right) = 363.5\ K$$

The thermometric error is some 9.5°C.

⊙ *If radiation is ignored the above calculations should be done by taking* $h_r = 0$ *and* $T_{ref} = T_w$. *This is left as an exercise to the student.*