Sub Module 2.6

Measurement of transient temperature

Many processes of engineering relevance involve variations with respect to time. The system properties like temperature, pressure and flow rate vary with time. These are referred to as transients and the measurement of these transients is an important issue while designing or choosing the proper measurement technique and the probe. Here we look at the measurement of temperature transients.

Temperature sensor as a first order system - Electrical analogy

Let us look at a typical temperature measurement situation. We visualize the temperature probe as a system that is subject to the temperature transient. The probe is exposed to the environment whose temperature changes with time and it is desired to follow the temperature change as closely as possible. In Figure 44 we show the schematic of the thermal model appropriate for this study.

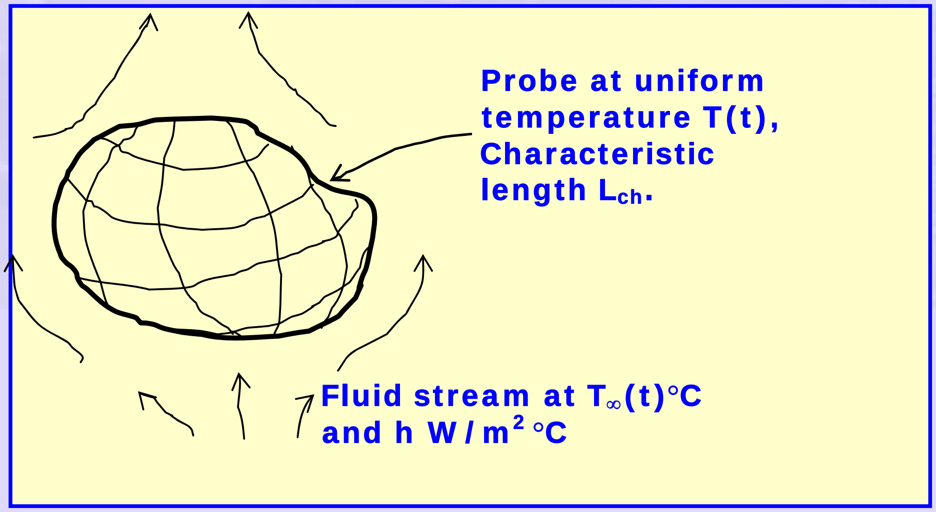


Figure 44 Schematic of a temperature probe placed in a flowing medium

The model assumes that the probe is at a uniform temperature within it at any time t. This means that the probe is considered to be thermally lumped. The medium that flows over the probe is at a temperature that may vary with respect to time. Initially the probe is assumed to be at temperature T_0. Let us assume that the probe is characterized by the following physical parameters:

Density of the probe material = ρ kg/m^3, Volume of the probe = V m^3, Surface area of the probe that is exposed to the flowing fluid = S m^2, Specific heat of the probe material = C J/kg°C, Heat transfer coefficient for heat transfer between the probe and the surrounding medium = h W/m^2°C. By conservation of energy, we have

$$\begin{bmatrix} \text{Rate of change of} \\ \text{internal energy of probe} \end{bmatrix} = \begin{bmatrix} \text{rate of heat transfer between} \\ \text{the probe and the fluid} \end{bmatrix} \quad \textbf{(30)}$$

If we assume that the probe is at a higher temperature as compared to the fluid heat transfer will be from the probe to the fluid and the internal energy of the probe will reduce with time. Using the properties of the probe introduced above, the left hand side of Equation 30 is given by $-\rho VC\frac{dT}{dt}$. The right hand side of Equation 30 is given by $hS(T-T_\infty)$. With these, after some rearrangement, Equation 30 takes the form

$$\frac{dT}{dt} + \frac{hS}{\rho VC}T = \frac{hS}{\rho VC}T_\infty \quad \textbf{(31)}$$

Note that this equation holds even when the probe temperature is *lower* than the fluid temperature. The quantity $\frac{\rho VC}{hS}$ has the unit of time and is referred to as the time constant τ of the first order system (first order since the governing differential equation is a first order ordinary differential equation). The first order time constant involves thermal and geometric properties. The volume to surface area ratio is a characteristic length dimension and is indicated as L_{ch} in Figure 44. Noting that the product of density and volume is the mass M of the probe, the first order time constant may also be written as $\tau = \frac{MC}{hS}$. The time constant may be interpreted in a different way also, using electrical analogy. The quantity MC represents the thermal capacity and the quantity $\frac{1}{hS}$ represents the thermal resistance. Based on this interpretation an electric analog may be made as shown in Figure 45.

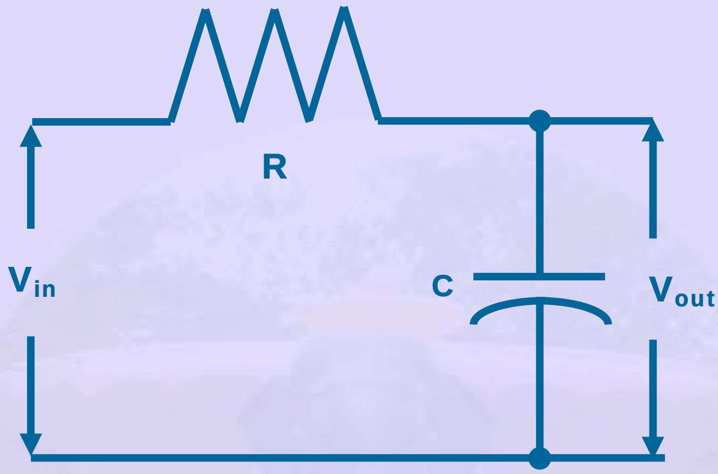


Figure 45 Electrical analog of a first order thermal system

In the electric circuit shown in Figure 45 the input voltage represents the temperature of the fluid, the output voltage represents the temperature of the probe, the resistance R represents the thermal resistance and the capacitance C represents the thermal mass (mass specific heat product) of the probe. Equation 31 may be rewritten as

$$\frac{dT}{dt} + \frac{T}{\tau} = \frac{T_\infty}{\tau} \qquad \textbf{(32)}$$

Note that Equation 32 may be simplified using the integrating factor $e^{\frac{t}{\tau}}$ to write it as

$$\frac{d}{dt}\left(Te^{\frac{t}{\tau}}\right) = T_\infty e^{\frac{t}{\tau}} \qquad \textbf{(33)}$$

This may be integrated to get $Te^{\frac{t}{\tau}} = \int_0^t T_\infty e^{\frac{t}{\tau}} dt + A$ where A is a constant of integration. Using the initial condition $T(t=0) = T_0$ we get, after minor simplification

$$T = T_0 e^{-\frac{t}{\tau}} + e^{-\frac{t}{\tau}} \int_0^t T_\infty e^{\frac{t}{\tau}} dt \qquad \textbf{(34)}$$

This is the general solution to the problem. If the variation of fluid temperature with time is given, we may perform the indicated integration to obtain the response of the probe as a function of time.

Response to step input

If the fluid temperature is constant but different from the initial temperature of the probe, the solution is easily shown to be represented by

$$\frac{T - T_\infty}{T_0 - T_\infty} = \phi = e^{-\frac{t}{\tau}} \quad (35)$$

The temperature difference between the probe and the fluid exponentially decreases with time. The variation is indicated in Figure 46. At the end of one time constant the temperature difference is some 37% of the initial temperature difference. After about 5 time constants the temperature difference is quite negligible.

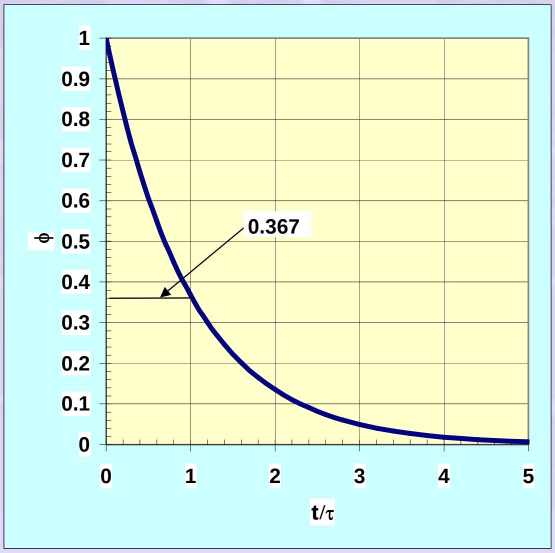


Figure 46 Response of a first order system to a step input

A step input may be experimentally realized by heating the probe to an initial temperature in excess of the fluid temperature and then exposing it quickly to the fluid environment. The probe temperature is recorded as a function of time. If it is plotted in the form $\ln(\phi)$ as a function of t, the slope of the line is negative reciprocal of the time constant. In fact, this is one method of measuring the time constant. Example 15 shows how this is done.

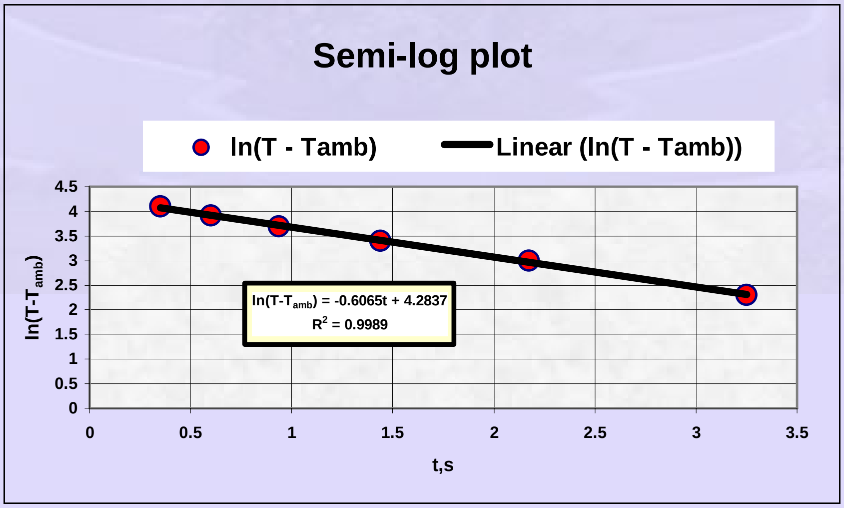
Example 15

- *A temperature probe was heated by immersing it in boiling water and is then quickly transferred in to a fluid medium at a temperature of T_{amb} = 25°C. The temperature difference between the probe and the medium in which it is immersed is recorded as given below:*

t (s)	*0.35*	*0.6*	*0.937*	*1.438*	*2.175*	*3.25*
$T-T_{amb}$	*60*	*50*	*40*	*30*	*20*	*10*

- *What is the time constant of the probe in this situation?*

o *The data is plotted on a semi-log graph as shown here. It is seen that it is well represented by a straight line whose equation is given as an inset in the plot. EXCEL was used to obtain the best fit.*



o *The slope of the line is -0.6065 and hence the time constant is*

$$\tau = -\frac{1}{\text{slope}} = -\frac{1}{-0.6065} = 1.6487 \approx 1.65\ \text{s}$$

- *The correlation coefficient of the linear fit is -0.9994. This shows that the data has been collected carefully.*

A note on time constant

It is clear from our discussion above that the time constant of a system (in this case the temperature probe) is not s property of the system. It depends on parameters that relate to the system as well as the parameters that define the interaction between the system and the surrounding medium (whose temperature we are trying to measure, as it changes with time). The time constant is the ratio of thermal mass of the system to the conductance (reciprocal of the thermal resistance) between the system and the medium. It is also clear now how we can manipulate the time constant. Thermal mass reduction is one possibility. The other possibility is the reduction of the thermal resistance. This may be achieved by increasing the interface area between the system and the medium. In general this means a reduction in the characteristic dimension L_{ch} of the system. A thermocouple attached to a thin foil will accomplish this. The characteristic dimension is equal to half the foil thickness, if heat transfer takes place from both sides of the foil. Another way of accomplishing this is to use very thin thermocouple wires so that the bead at the junction has very small volume and hence the thermal mass. Indeed these are the methods used in practice and thin film sensors are commercially available.

Response to a ramp input

In applications involving material characterization heating rate is controlled to follow a predetermined program heating. The measurement of the corresponding temperature is to be made so that the temperature sensor follows the temperature very closely. Consider the case of linear heating and possibly linear temperature rise of a medium. Imagine an oven being turned on with a constant amount of electrical heat input. We would like to measure the temperature of the oven given by

$$T_\infty(t) = T_0 + R\,t \qquad \textbf{(36)}$$

The general solution to the problem is given by (using Equation 34)

$$T\,e^{t/\tau} = A + \frac{T_0}{\tau}\int_0^t e^{t/\tau}dt + \frac{R}{\tau}\int_0^t te^{t/\tau}dt \qquad \textbf{(37)}$$

where A is a constant of integration. The first integral on the right hand side is easily obtained as $\tau\left(e^{t/\tau}-1\right)$. Second integral on the right hand side is obtained by integration by parts, as follows.

$$\int_0^t t\,e^{t/\tau}dt = t\tau\,e^{t/\tau}\Big|_0^t - \tau\int_0^t e^{t/\tau}dt = t\tau\,e^{t/\tau} - \tau^2\left(e^{t/\tau}-1\right) \qquad \textbf{(38)}$$

If the initial temperature of the first order system is T_i, then $A = T_i$, since both the integrals vanish for t = 0 (the lower and upper limit will be the same). On rearrangement, the solution is

$$T(t) = \left(T_i - T_0 + R\,\tau\right)e^{-t/\tau} + \left(T_0 + R\,t - R\,\tau\right) \qquad \textbf{(39)}$$

We notice that as $t \to \infty$ the transient part tends to zero (transient part is the exponential decaying part) and the steady part (this part survives for t >>) yields

$$T_o + Rt - T(t) = R\tau\,. \qquad \textbf{(40)}$$

The steady state response has a lag equal to R with respect to the input.

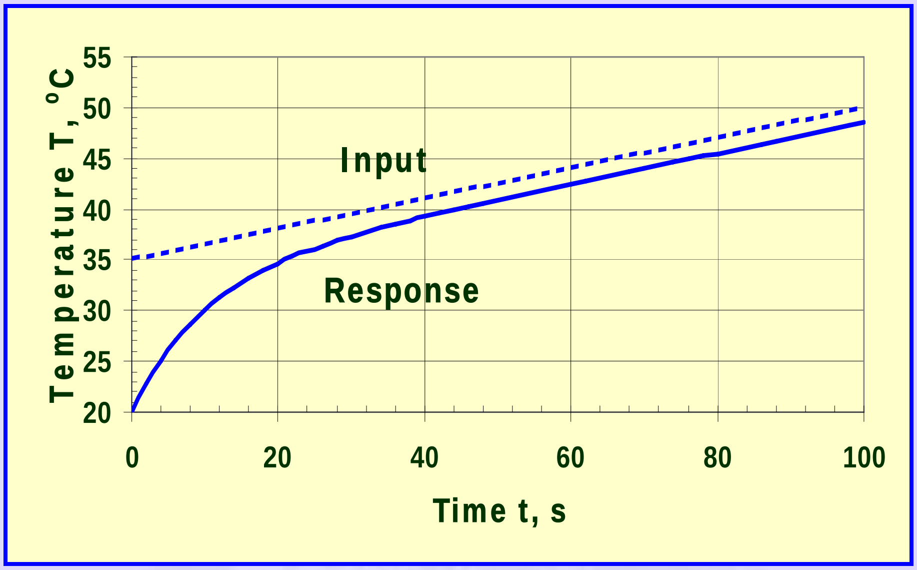


Figure 47 Typical response of a first order system to ramp input

Figure 47 shows the response of first order system to a ramp input. The case shown corresponds to Ti= 20°C, T_0 = 35°C, R = 0.15°C/s and τ = 10 s. For t $\to\infty$ (t > 5τ = 50 s) the probe follows the linear temperature rise with a lag of Rτ = 0.15×10 = 1.5°C. In this case it is advisable to treat this as a systematic error and add it to the indicated temperature to get the correct oven temperature.

Response to a periodic input

There are many applications that involve periodic variations in temperature. For example, the walls of an internal combustion engine cylinder are exposed to periodic heating and hence will show periodic temperature variation. Of course, the waveform representing the periodic temperature variation may be of a complex shape (non sinusoidal). In that case the waveform may be split up in to its Fourier components. The response of the probe can also be studied as that due to a typical Fourier component and combine such responses to get the actual response. Hence we look at a periodic sinusoidal input given by

$$T_\infty = T_a \cos(\omega t) \qquad \textbf{(41)}$$

In the above expression T_a is the amplitude of the input wave and is the circular frequency. We may use the general solution given by Equation 34 and perform the indicated integration to get the response of the probe. The steps are left as exercise to the student. Finally the response is given by

$$T = \underbrace{T_o e^{-t/\tau} - \frac{T_a\, e^{-\frac{t}{\tau}}}{\left(1+\omega^2\tau^2\right)}}_{\text{Transient response}} + \underbrace{\frac{T_a \cos\left(\omega t - \tan^{-1}(\omega\tau)\right)}{\sqrt{1+\omega^2\tau^2}}}_{\text{Steady state response}} \qquad (42)$$

Again for large t, the transient terms drop off and the steady sate response survives. There is a reduction in the amplitude of the response and also a time lag with respect to the input wave. Amplitude reduction and the time lag (or phase lag) depend on the product of the circular frequency and the time constant. The variations are as shown in Figure 48.

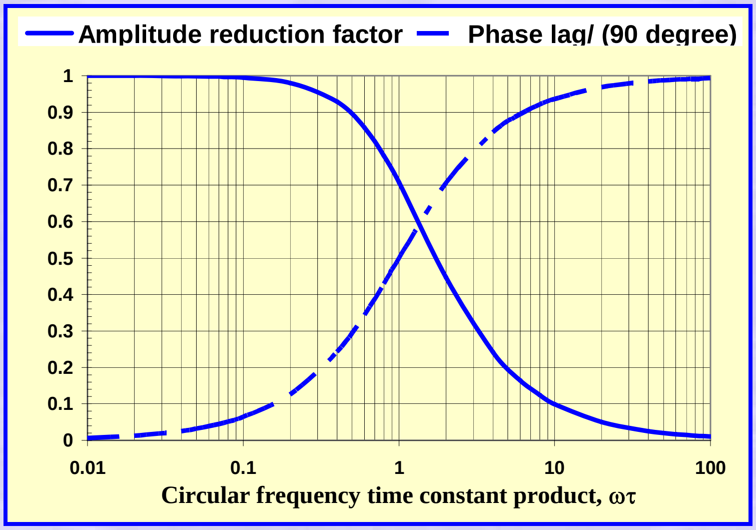


Figure 48 Response of a first order system to periodic input

In order to bring out the features of the response of the probe, we make a plot (Figure 49) that shows both the input and output responses, for a typical case.

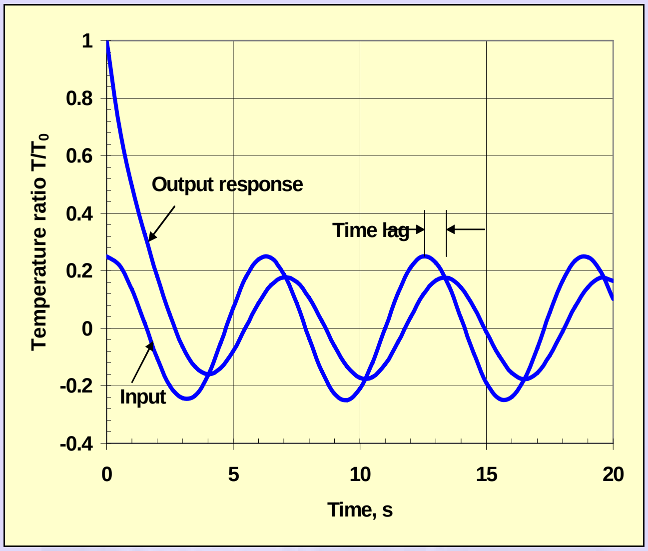


Figure 49 Response of a first order system to periodic input

The case shown in Figure 33 corresponds to $\frac{T_a}{T_o} = 0.25;\ \omega = 1\,\text{rad/s}\ \text{and}\ \tau = 1\ \text{s}$. The output response has an initial transient that adjusts the initial mismatch between the probe temperature and the imposed temperature. By about 4 to 5 time constants (4 to 5 seconds since the time constant has been taken as 1 second) the probe response has settled down to a response that follows the input but with a time lag and an amplitude reduction as is clear from Figure 33.

Example 16

The time constant of a first order thermal system is given as 0.55 min. The uncertainty in the value of the time constant is given to be $\pm$0.01 min. The initial temperature excess of the system over and above the ambient temperature is 45^{o}C. It is desired to determine the system temperature excess and its uncertainty at the end of 50 s from the start.

Hint: It is known that the temperature excess follows the formula

$\frac{T(t)}{T(0)} = e^{-\frac{t}{\tau}}$ *where T(t) is the temperature excess at any time t, T(0) is the temperature excess at t = 0 and τ is the time constant.*

- *We shall convert all times given to s so that things are consistent. The time constant is* $\tau = 0.55 \text{ min} = 0.55 \times 60 \text{ s} = 33 \text{ s}$
- *We need the temperature excess at* $t = 50 \text{ s}$ *from the start. Hence*

$$T(50) = T(0)e^{-\frac{t}{\tau}} = 45e^{-\frac{50}{33}} = 9.89°\text{C}$$

- *We would like to calculate the uncertainty in this value. We shall assume that this is due to the error in the time constant alone.*

$$\Delta\tau = \pm 0.1 \text{ min} = \pm 0.1 \times 60 = \pm 0.6 \text{ s}$$

- *The influence coefficient I_τ is given by*

$$I_\tau = \left.\frac{\partial T}{\partial \tau}\right|_{t=50} = 45 \times \left(\frac{50}{33^2}\right)e^{-\frac{50}{33}} = 0.454°\text{C/s}$$

Hence the uncertainty in the estimated temperature excess is:

$$\Delta T(50) = \pm I_\tau \Delta\tau = \pm 0.454 \times 0.6 = 0.272°\text{C}$$

Example 17

- *A certain first order system has the following specifications:*
  - *Material: copper shell of wall thickness 1 mm, outer radius 6 mm*
  - *Fluid: Air at 30 $^{\circ}$C*
  - *Initial temperature of shell: 50 $^{\circ}$C*
- *How long should one wait for the temperature of the shell to reach 40 $^{\circ}$C? Assume that heat transfer is by free convection. Use suitable correlation (from a heat transfer text) to solve the problem.*

- *Heat transfer coefficient calculation:*

Heat transfer between the shell and the air is by natural convection. The appropriate correlation for the Nusselt number is given by $\mathrm{Nu} = 2 + 0.43\,\mathrm{Ra}^{1/4}$ *where Ra is the Rayleigh number. The characteristic length scale is the sphere diameter. The air properties are calculated at the mean temperature at t = 0.*

From the given data, we have

$$D = 12 \text{ mm} = 0.012 \text{ m},\ T_m = (50+30)/2 = 40°C$$

The air properties required are read off a table of properties:

$$\nu = 16.96\times10^{-6}\,\mathrm{m^2/s},\ \mathrm{Pr} = 0.71,\ k = 0.027\ \mathrm{W/m°C}$$

The isobaric compressibility of air is calculated based on ideal gas assumption. Thus $\beta = \frac{1}{T_{amb}} = \frac{1}{30+273} = 3.3\times10^{-3}\ \mathrm{K^{-1}}$

The temperature difference for calculating the Rayleigh number is taken as the mean shell temperature during the cooling process minus the ambient temperature. We are interested in determining the time to cool from 50 to 40 $^{\circ}$C. Hence the mean shell temperature is $\overline{T}_{Shell} = \frac{50+40}{2} = 45°C$. *The temperature difference is* $\Delta T = \overline{T}_{Shell} - T_{amb} = 45 - 30 = 15°C$. *The value of the* *acceleration due to gravity is taken as* $g = 9.8 \text{ m/s}^2$. *The Rayleigh number is then calculated as*

$$Ra = \frac{g\beta \Delta T D^3}{\nu^2} Pr = \frac{9.8 \times 3.3 \times 10^{-3} \times 15 \times 0.012^3}{\left(16.96 \times 10^{-6}\right)^2} \times 0.71 = 2069$$

- *The Nusselt number is then calculated as*

  $$Nu = 2 + 0.43 Ra^{1/4} = 2 + 0.43 \times 2069^{1/4} = 4.9$$

  The heat transfer coefficient is then calculated as

  $$h = \frac{Nu\,k}{D} = \frac{4.9 \times 0.027}{0.012} = 11.07 \text{ W/m}^2\text{K}$$

- *Time constant calculation:*

  Copper shell properties are

  $$\rho = 8954 \text{ kg/m}^3,\ C = 383.1 \text{ J/kg°C}$$

  Copper shell thickness is $\delta = 0.001 \text{ m}$

  Mass of the copper shell is calculated as

  $$M = \rho \pi D^2 \delta = 8954 \times \pi \times 0.012^2 \times 0.001 = 4.051 \times 10^{-3} \text{ kg}$$

  Surface area of shell exposed to the fluid is $S = \pi D^2 = \pi \times 0.012^2 = 4.524 \times 10^{-4} \text{ m}^2$

  The time constant is then estimated as

  $$\tau = \frac{MC}{hS} = \frac{4.051 \times 10^{-3} \times 383.1}{11.07 \times 4.524 \times 10^{-4}} = 310 \text{ s}$$

- *Cooling follows an exponential process. Hence we have, the time* t_{40} *at which the shell temperature is 40°C,*

$$t_{40} = -310 \ln\left(\frac{40-30}{50-30}\right) = 214.9 \text{ s}$$