Sub Module 2.3

Resistance Thermometry

Resistance thermometry depends on the unique relation that exists between resistance of an element and the temperature. The resistance thermometer is usually in the form of a wire and its resistance is a function of its temperature. Material of the wire is usually high purity Platinum. Other materials also may be used. The resistance variation of different materials is indicated by Table 10.

Table 10 Resistance variation of different wire materials

Material	Temperature Range °C	Element Resistance in Ohms at 0°C	Element Resistance in Ohms at 100°C
Nickel	-60 to 180	100	152
Copper	-30 to 220	100	139
Platinum	-200 to 850	100	136

Platinum resistance thermometer is also referred to as (Platinum Resistance Thermometer) PRT (or PT) or Resistance Temperature Detector (RTD). Usually the resistance of the detector at the ice point is clubbed with it and the thermometer is referred to as, for example, PT100, if it has a resistance of 100 Ω at the ice point. The resistance of standard high purity Platinum varies systematically with temperature and it is given by the International standard calibration curve for wire wound Platinum elements:

$$R_t = R_0\left(1 + K_1 t + K_2 t^2 + K_3\{t - 100\}t^3\right), \; -200°C < t < 0°C$$
$$R_t = R_0\left(1 + K_1 t + K_2 t^2\right), \; 0°C < t < 250°C \quad (9)$$

Where

$$K_1 = 3.90802 \times 10^{-3} /°C;\; K_2 = -5.802 \times 10^{-7} /°C^2;\; K_3 = -1.2735 \times 10^{-12} /°C^4$$

The ratio $\frac{R_{100} - R_0}{100 R_0}$ is denoted by and is given as 0.00385/°C. It is seen that the resistance temperature relationship is non linear. The response of a Platinum resistance thermometer is usually plotted in the form of ratio of resistance at temperature t to that at the ice point as a function of temperature as shown in Figure 24. The sensor in the Figure 25 is shown with a three wire is shown with three wire arrangement. The resistance sensor is also available with four wire arrangement. These two aspects will be discussed later.

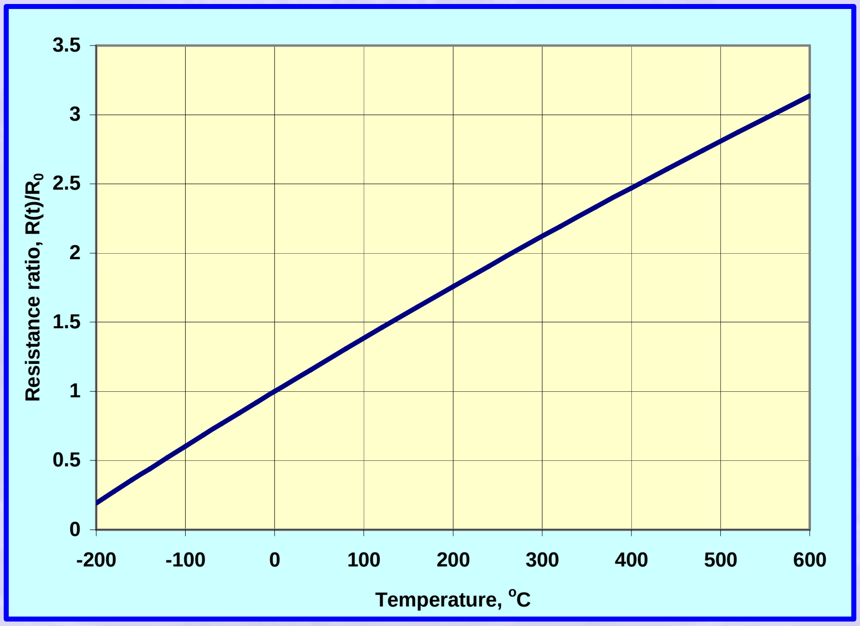


Figure 24 Characteristics of a Platinum resistance thermometer

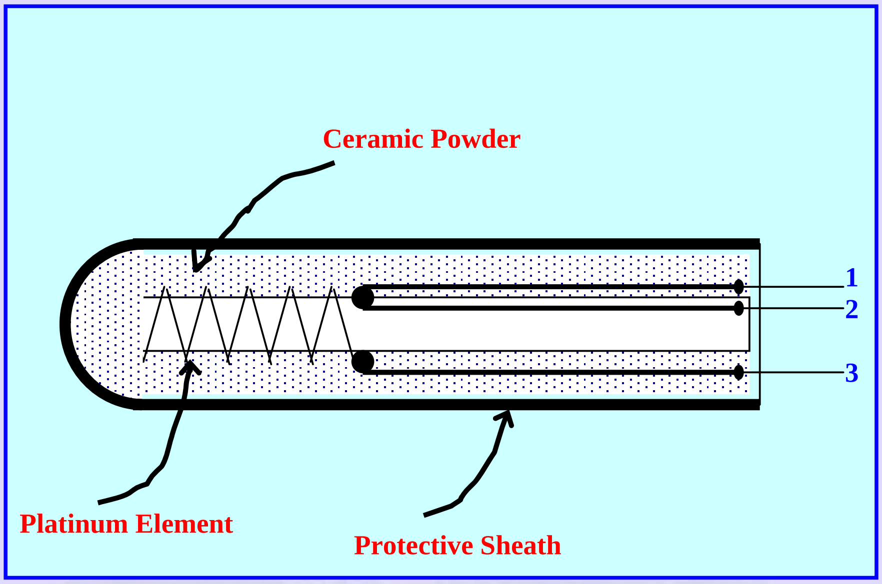


Figure 25 Typical PRT sensor schematic with three wire arrangement

Platinum resistance thermometer and the Callendar equation

As seen from Equation 9 the Platinum resistance thermometer has essentially a non-linear response with respect to temperature. We define a temperature scale defined as the Platinum resistance temperature that is basically given by a linear scale defined through the relation

$$t_{Pt} = \frac{R_t - R_0}{R_{100} - R_0} \times 100 \quad \textbf{(10)}$$

The quantities appearing in the above are:

R_{Pt} = Platinum resistance temperature, R_t = Resistance of sensor at temperature t, R_0 = Resistance of the sensor at the ice point and R_{100} = Resistance of the sensor at the ice point. Obviously the non-linearity will have to be taken into account to get the correct temperature from the linear value obtained by Equation 10. This is done by applying a correction to the Platinum resistance temperature as suggested by Callendar.

From Equation 9 we have

$$R_{100} = R_0\left(1 + 100\,K_1 + 100^2 K_2\right).$$

Hence

$$R_{100} - R_0 = 100K_1 + 100^2 K_2 .$$

We then have

$$\frac{R_t - R_0}{R_{100} - R_0} \times 100 = 100 \times \frac{K_1 t + K_2 t^2}{100K_1 + 100^2 K_2}$$
$$= \frac{K_1 t + K_2 t^2}{K_1 + 100K_2} = \frac{K_1 t + 100K_2 t - 100K_2 t + K_2 t^2}{K_1 + 100K_2}$$
$$= t - \frac{100K_2 t - K_2 t^2}{K_1 + 100K_2} \approx t - \frac{100K_2 t - K_2 t^2}{K_1} = t + \left(\frac{K_2}{K_1} \times 100^2\right) \frac{t}{100}\left[\frac{t}{100} - 1\right]$$

With the K's given earlier, we have $\frac{K_2}{K_1} \times 100^2 = \frac{-5.802 \times 10^{-7}}{3.90802 \times 10^{-3}} \times 10^4 = -1.485 = -\delta$. Thus we have

$$\frac{R_t - R_0}{R_{100} - R_0} \times 100 = t - \delta\left(\frac{t}{100}\right)\left(\frac{t}{100} - 1\right)$$

This may be rephrased as (using the definition given in Equation 10)

$$t \approx \frac{R_t - R_0}{R_{100} - R_0} \times 100 + \delta\left(\frac{t}{100}\right)\left(\frac{t}{100} - 1\right) = t_{Pt} + \underbrace{\delta\left(\frac{t_{Pt}}{100}\right)\left(\frac{t_{Pt}}{100} - 1\right)}_{\text{Correction } c} \qquad \textbf{(11)}$$

This is referred to as the Callendar equation and the second term is the Callendar correction, represented as c. The Callendar correction is evidently zero at both the ice and steam points. The correction is non-zero at all other temperatures. Figure 26 shows the Callendar correction as a function of the Platinum resistance temperature over a useful range of the sensor.

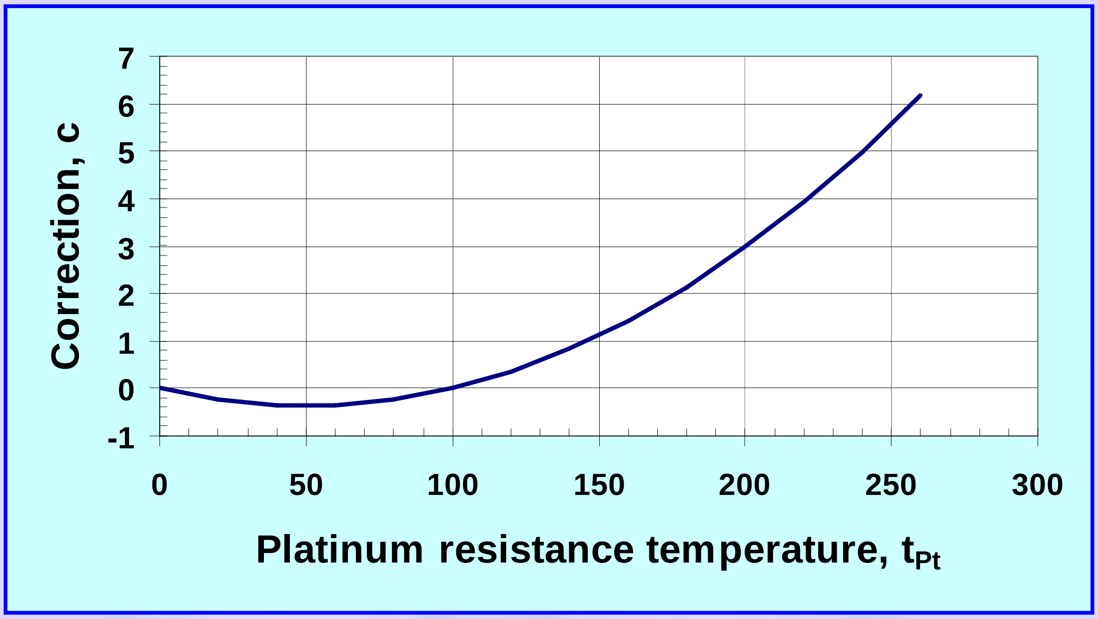


Figure 26 Callendar correction as a function of t_{Pt}

Example 7

- *The resistance of a Platinum resistance sensor of $R_0 = 100\Omega$ was measured to be 119.4Ω. This sensor has α value of 0.00385. What is the corresponding temperature without and with correction?*

  - *We have $R_0 = 100\ \Omega$, $\alpha = 0.00385$ and $\delta = 1.485$.*
  - *Hence* $R_{100} = R_0\left(1+100\alpha\right) = 100\times\left(1+100\times 0.00385\right) = 13.5\Omega$
  - *The measured sensor resistance is given as* $R_t = 119.4\Omega$.
  - *By definition the Platinum resistance temperature is*

$$t_{Pt} = \frac{R_t - R_0}{R_{100} - R_0}\times 100 = \frac{119.4-100}{138.5-100}\times 100 = 50.39^{O}C$$

  - *This is also the uncorrected value of the temperature.*

- *The Callendar correction is calculated as*

$$c = \delta\left(\frac{t_{Pt}}{100} - 1\right)\frac{t_{Pt}}{100} = 1.485 \times \left(\frac{50.39}{100} - 1\right) \times \frac{50.39}{100} = -0.37^{O}C$$

- *The corrected temperature is thus given by*

$$t = t_{Pt} + c = 50.39 - 0.37 = 50.02^{O}C$$

RTD measurement circuits

The resistance of the resistance sensor is determined by the use of a DC bridge circuit. As mentioned earlier there are two variants, viz. the three wire and the four wire systems. These are essentially used to eliminate the effect of the lead wire resistances that may adversely affect the measurement. There are two effects due to the lead wires: 1) they add to the resistance of the Platinum element 2) the resistance of the lead wires may also change with temperature. These two effects are mitigated or eliminated by either the three or four wire arrangements.

The lead wires are usually of higher diameter than the diameter of the sensor wire to reduce the lead wire resistance. In both the three and four wire arrangements, the wires run close to each other and pass through regions experiencing similar temperature fields (refer Figure 25). Hence the change in the resistance due to temperature affects all the lead wires by similar amounts. The resistances of the lead wires are compensated by a procedure that is described below.

Bridge circuit for resistance thermometry:

Three wire arrangement for lead wire compensation

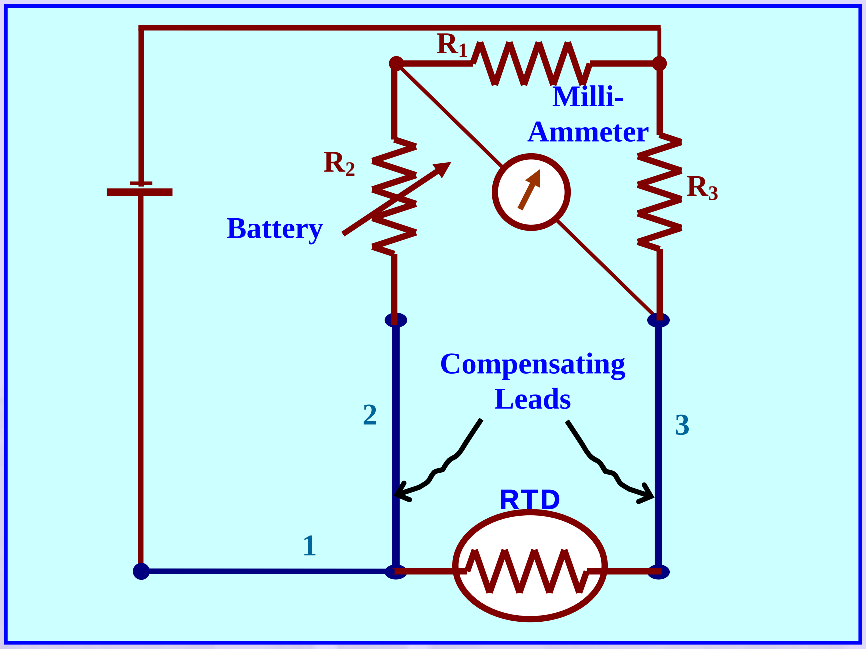


Figure 27 Bridge circuit with lead wire compensation (three wire arrangement)

Figure 27 shows the bridge circuit that is used with three lead wires. The resistances R_1 and R_3 are chosen to be equal and the same as R_0 of the RTD. Two lead wires (labeled 2 and 3) are connected as indicated adding equal resistances to the two arms of the bridge. The third lead wire (labeled 1) is used to connect to the battery. Thus the bridge will indicate null (milli-ammeter will indicate zero) when $R_2 = R_0$ when the RTD is maintained at the ice point. During use, when the RTD is at temperature t, the resistance R_2 is adjusted to restore balance. If the lead wires have resistances equal to R_{s2} and R_{s3}, we have

$$R_t + R_{s3} = R_2 + R_{s2} \text{ or } R_t = R_2 + (R_{s2} - R_{s3}) \quad (12)$$

If the two lead wires are of the same size the bracketed terms should essentially be zero and hence the lead wire resistances have been compensated for.

Four wire arrangement for lead wire compensation

The four wire arrangement is a superior arrangement, with reference to lead wire compensation, as will be shown below. Figure 28 is the bridge arrangements that are used for this purpose.

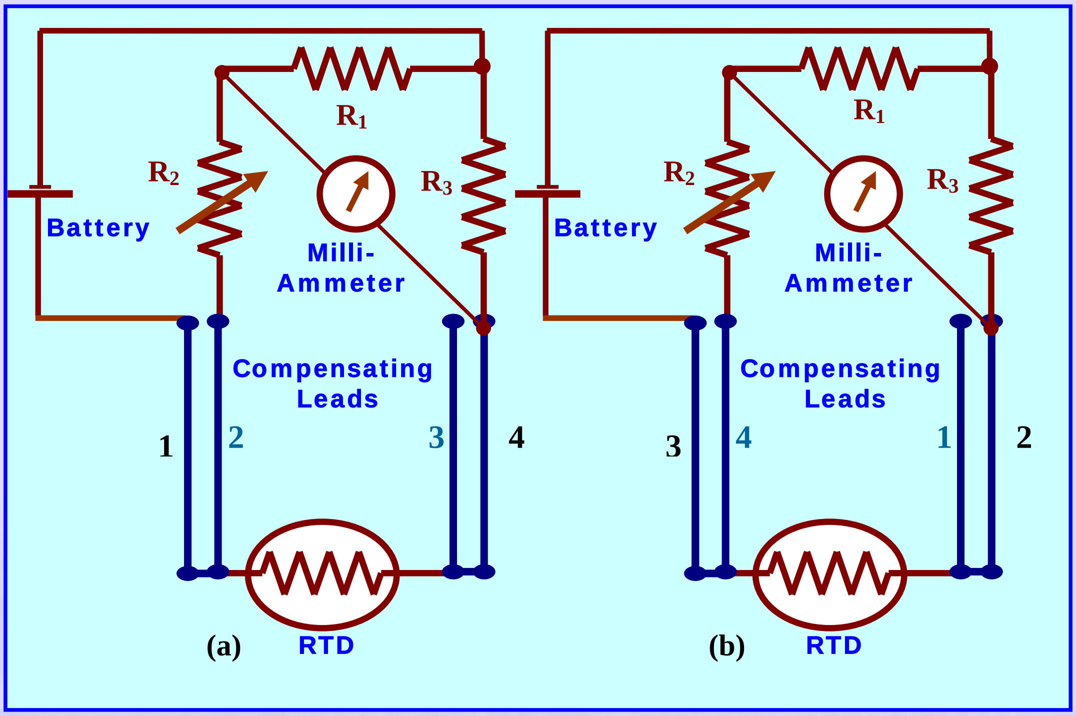


Figure 28 Bridge circuit with lead wire compensation (four wire arrangement)

The choice of the resistances is made as given for the three wire arrangement. If the lead wires have resistances equal to R_{s1} - R_{s4}, we have the following.

Condition for bridge balance in arrangement shown in Figure 28(a):

$$R_t + R_{s4} = \underbrace{R_{2(a)}}_{\substack{\text{For balanc} \\ \text{arrangement a}}} + R_{s2} \qquad (12)$$

Condition for bridge balance in arrangement shown in Figure 28(b):

$$R_t + R_{s2} = \underbrace{R_{2(b)}}_{\substack{\text{For balanc} \\ \text{arrangement b}}} + R_{s4} \qquad (13)$$

We see that by addition of Equations 5 and 6, we get

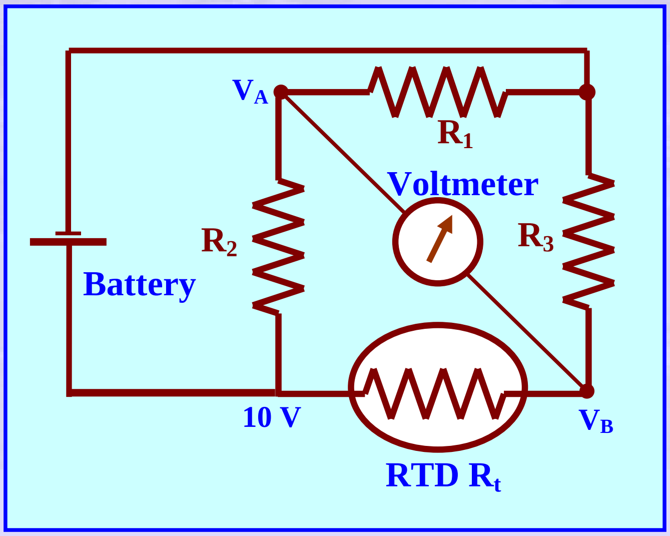
$$R_t = \frac{R_{2(a)} + R_{2(b)}}{2} \quad (14)$$

The lead wire resistances thus drop off and the correct resistance is nothing but the mean of the two measurements. Since the lead wire resistances actually drop off, the four wire scheme is superior to the three wire scheme.

Example 8

- *An RTD has* $\alpha_{20} = 0.004/°C$. *If* $R_{20} = 106\ \Omega$ *(resistance at 20°C), determine the resistance at 25°C. The above RTD is used in a bridge circuit with* $R_1 = R_2 = R_3 = 100\ \Omega$ *and the supply voltage is 10 V. Calculate the voltage the detector must be able to resolve in order to measure a 1°C change in temperature around 20°C.*

- *Note the definition of* α *viz.* $\alpha_t = \left.\frac{1}{R}\frac{dR}{dt}\right|_t = \frac{s_t}{R_t}$ *at any temperature. Symbol s stands for the slope of the resistance versus temperature curve for the sensor. (The earlier definition assumes that* α *is constant and is evaluated using the resistance values at the ice and steam points.)*
- *The circuit used for measurement is shown in the following figure.*



- *With the given data of* $\alpha_{20} = 0.004/°C$ *the slope may be determined as*

$$s_{20} = \alpha_{20} R_{20} = 0.004 \times 106 = 0.424\ \Omega/°C$$

- *Assuming the response of the sensor to be linear over small changes in temperature, the resistance of the sensor at 25°C may be determined as*

$$R_{25} = R_{20} + (25-20)\times s_{20} = 106 + 5\times 0.424 = 108.12\ \Omega$$

- *Infer all voltages with reference to the negative terminal of the battery taken as zero (ground). The voltmeter reads the potential difference between A and B. If there is change of temperature of 1°C the temperature of the RTD may either be 21°C or 19°C.*
- *Case (a): t = 21°C. The potentials are given by the following:*

$$V_A = 10 - \frac{10}{R_1 + R_2} R_2 = 10 - \frac{10}{100+100}\times 100 = 5\ \text{V}$$

- *If there is a change of 1.0°C in temperature the resistance changes by 0.424 Ω as given by the slope. The resistance of the RTD will be 106.424 Ω in this case.*
- *The potential V_B is then given by*

$$V_B = 10 - \frac{10}{R_{21} + R_3} R_{21} = 10 - \frac{10}{106.424+100}\times 106.424 = 4.844\ \text{V}$$

- *The voltmeter should read*

$$V_A - V_B = 5 - 4.844 = 0.156\ \text{V or } 156\ \text{mV}$$

- *Case (b): t = 19°C. The potentials are given by the following:*

$$V_A = 10 - \frac{10}{R_1 + R_2} R_2 = 10 - \frac{10}{100+100}\times 100 = 5\ \text{V}$$

- *If there is a change of -1.0°C in temperature the resistance changes by -0.424 Ω as given by the slope. The resistance of the RTD will be 105.576 Ω in this case.*
- *The potential V_B is then given by*

$$V_B = 10 - \frac{10}{R_{19} + R_3} R_{21} = 10 - \frac{10}{105.576+100}\times 105.576 = 4.864\ \text{V}$$

- *The voltmeter should read*

$$V_A - V_B = 5 - 4.864 = 0.136 \text{ V or } 136 \text{ mV}$$

- *The smaller of these or 0.136 V or 136 mV is the resolution of the voltmeter required for 1^{o}C resolution. Practically speaking we may choose a voltmeter with 100 mV resolution for this purpose.*

Effect of self heating

The bridge arrangement for measuring the sensor resistance involves the passage of a current through the sensor. Heat is generated by this current passing through the RTD. The heat has to be dissipated by an increase in the sensor temperature compared to the medium surrounding the sensor. Thus the self heating leads to a systematic error. Assume that the conductance (dissipation constant) for heat transfer from the RTD to the surrounding medium is P_D W/K. The temperature excess of the RTD is given by $\Delta t = \left[\frac{I^2 R_t}{P_D}\right]$ °C. Example 9 demonstrates this.

Example 9

- *An RTD has* $\alpha_{20} = 0.005\ /°C$, $R_{20} = 500\ \Omega$ *and a dissipation constant of* $P_D = 30\ mW/°C$ *at 20°C. The RTD is used in a bridge circuit with* $R_1 = R_3 = 500\Omega$ *and* R_2 *is a variable resistor used to null the bridge. If the supply voltage is 10 V and the RTD is placed in a bath at* $0°C$*, find the value of* R_3 *to null the bridge. Take the effect of self heating into account.*
    - *Note: Figure in Example 2 is appropriate for this case also.*

- *Given data:* $\alpha_{20} = 0.005/°C,\ R_{20} = 500\ \Omega,\ P_D = 30\times10^{-3} W/°C,\ V_s = 10\ V$
- *Since* $R_1 = R_3 = 500\Omega$*, at null* $R_{RTD} = R_2$*. Thus the current through the RTD is* $\frac{V_s}{(R_2+500)}$ *and hence the dissipation in the RTD is* $\left[\frac{V_s}{(R_2+500)}\right]^2 R_2$*. The self heating leads to a temperature change of* $\left[\frac{V_s}{(R_2+500)}\right]^2 \frac{R_2}{P_D}\ °C$*. The temperature of the RTD is thus* $\left[\frac{V_s}{(R_2+500)}\right]^2 \frac{R_2}{P_D}\ °C$ *instead of* 0^oC *as it should have been.*
- *The resistance of the RTD is thus given by (assuming linear variation of resistance with temperature)*

$$R_2 = R_{20}[1-\alpha_{20}(20-\left[\frac{V_s}{(R_2+500)}\right]^2 \frac{R_2}{P_D}]$$

- *This has to be solved for* R_2 *to get the variable resistance which will null the bridge.*
- *The solution may be obtained by iteration. The iteration starts with the trial value*

$$R_2^0 = R_{20}(1-20\alpha_{20}) = 500\times(1-0.005\times20) = 450\ \Omega$$

- o *Substitute this in the right hand side of the previous expression to get*

$$R_2^1 = R_{20}[1-\alpha_{20}(20-\left[\frac{V_s}{\left(R_2^0+500\right)}\right]^2\frac{R_2^0}{P_D}]$$

$$= 500\times\left[1-0.005\times\left\{20-\left[\frac{10}{450+500}\right]^2\times\frac{450}{30\times10^{-3}}\right\}\right] = 454.16\ \Omega$$

- o *It so happens that we may stop after just one iteration! Thus the required resistance to null the bridge is 454.16 Ω.*

Example 10

- *Use the values of RTD resistance versus temperature shown in the table to find the equation for the linear approximation of resistance between 100 and 130°C. Assume* $T_0 = 115°C$.

t °C	100	105	110	115	120	125	130
R Ω	573.40	578.77	584.13	589.48	594.84	600.18	605.52

- *For the linear fit we calculate the* α *value by using the mean slope near the middle of the table. We use the values shown in blue to get*

$$\alpha_0 = \frac{594.84 - 584.13}{120 - 110} = 0.00182$$

- *The linear fit is:*

$$R_{ft} = R_{115}\left[1 + \alpha_0\left(t - 115\right)\right]$$

- *We make a table to compare the linear fit with the data.*

t	R_{ft}	R(data)	Difference
100	573.39	573.40	0.01
105	578.75	578.77	0.02
110	584.12	584.13	0.01
115	589.48	589.48	0.00
120	594.84	594.84	0.00
125	600.21	600.18	-0.03
130	605.57	605.52	-0.05

Linear fit appears to be very good