Sub Module 2.11

Pressure transducers

Here we discuss several pressure transducers that are grouped together, for convenience. These are:

1. Pressure tube with bonded strain gage
2. Diaphragm/Bellows type transducer
3. Capacitance pressure transducer

The common feature of all these transducers is that the pressure to be measured introduces a strain or movement in a mechanical element that is measured by different techniques. The strain is measured by a) strain gage or b) linear voltage displacement transducer (LVDT) or c) the change in capacitance. Detailed discussion of each one of these follows.

1) Pressure tube with bonded strain gage

The principle of a pressure tube with bonded strain gage may be understood by referring to Figure 72.

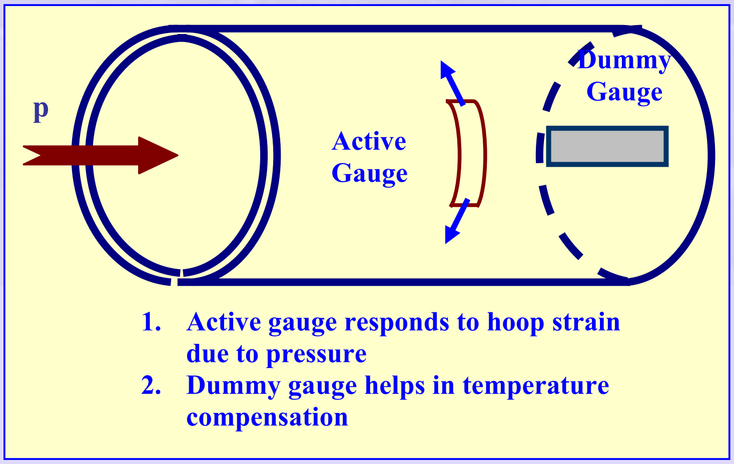


Figure 72 Pressure tube with bonded strain gage

The pressure to be measured is communicated to the inside of a tube of suitable wall thickness one end of which is closed with a thick plate. Because the end plate is very thick it undergoes hardly any strain. The tube, however, experiences a hoop strain due to internal pressure (indicated by the blue arrows). A strain gage mounted on the tube wall experiences the hoop strain. The way the strain gage works is dealt with below.

Strain gage theory:

Strain gage consists of a resistance element whose resistance is a function of its size.

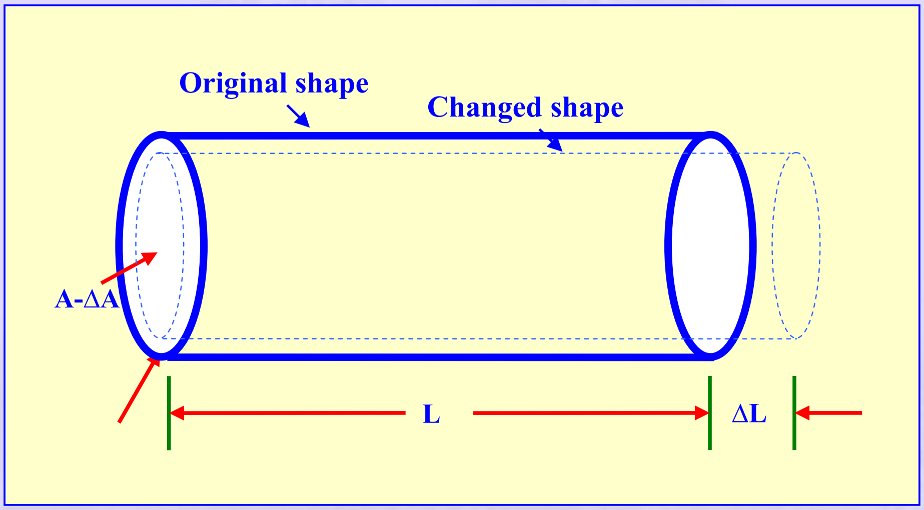


Figure 73 Deformation of a wire element

Electrical resistance of a wire of Length L, area of cross section A and made of a material of specific resistance ρ is given by

$$R = \frac{\rho L}{A} \quad (72)$$

Logarithmic differentiation yields

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad (73)$$

Equation 73 represents the fractional change in the wire resistance due to fractional changes in the specific resistance, the length and the area of cross section due to an imposed deformation of the wire. The specific resistance is a strong function of temperature but a weak function of pressure. If we assume that the element is at a constant temperature the first term on the right hand side of Equation 73 may be dropped to get the relation

$$\frac{dR}{R} = \underbrace{\frac{dL}{L}}_{\substack{\text{Longitudinal}\\\text{Strain}}} - \underbrace{\frac{dA}{A}}_{\substack{\text{Fractional}\\\text{Change in area}}} \quad (75)$$

If the wire is of circular cross section of radius r, we have $A = \pi r^2$ and hence

$$\underbrace{\frac{dA}{A}}_{\substack{\text{Fractional}\\\text{Change in area}}} = 2\underbrace{\frac{dr}{r}}_{\substack{\text{Lateral}\\\text{Strain}}} \quad (76)$$

Recall from mechanics of solids that Poisson ratio ν for the material is defined as the ratio of lateral strain to longitudinal strain. Hence we have

$$\frac{dA}{A} = 2\frac{dr}{r} = -2\nu\frac{dL}{L} \quad (77)$$

Combining these, Equation 75 becomes

$$\frac{dR}{R} = (1+2\nu)\frac{dL}{L} = (1+2\nu)\in \quad (78)$$

In the above the longitudinal strain is represented by $\in$. An incompressible material has Poisson ratio of 0.5. In that case the fractional change in resistance is equal to twice the longitudinal strain. The ratio of fractional resistance change of the wire to the longitudinal strain is called the gage factor (GF) or sensitivity of a strain gage and is thus given by

$$GF = \frac{dR/R}{\in} = (1+2\nu) \quad (79)$$

The gage factor is thus close to 2. Poisson ratio of some useful materials is given in Table 8.

Table 14 Poisson ratio of some useful materials

Poisson's ratio: typical values	
Steel	0.30
Concrete	0.20
Gold	0.42
Glass	0.20-0.25
Elastomers	$\rightarrow$ 0.5

Table 15 presents typical gage factor values with different materials.

Strain gage construction details:

Figure 74 shows the way a strain gage is made. The strain element is a serpentine metal layer obtained by etching, mounted on a backing material that may is bonded on to the surface whose strain needs to be measured. The active direction is as indicated in the figure. When a force is applied normal to this direction the serpentine metal layer opens up like a spring and hence does not undergo any strain in the material (actually the response will be something like 1% of that along the active direction). When the length of the metal foil changes the resistance changes and the change in resistance is the measured quantity. Connecting wires are used to connect the strain gage to the external circuit.

Table 15 Typical gage materials and gage factors

Material	Sensitivity (*GF*)
Platinum (Pt 100%)	6.1
Platinum-Iridium (Pt 95%, Ir 5%)	5.1
Platinum-Tungsten (Pt 92%, W 8%)	4.0
Isoelastic (Fe 55.5%, Ni 36% Cr 8%, Mn 0.5%) *	3.6
Constantan / Advance / Copel (Ni 45%, Cu 55%) *	2.1
Nichrome V (Ni 80%, Cr 20%) *	2.1
Karma (Ni 74%, Cr 20%, Al 3%, Fe 3%) *	2.0
Armour D (Fe 70%, Cr 20%, Al 10%) *	2.0
Monel (Ni 67%, Cu 33%) *	1.9
Manganin (Cu 84%, Mn 12%, Ni 4%) *	0.47
Nickel (Ni 100%)	-12.1
* Isoelastic, Constantan, Advance, Copel, Nichrome V, Karma, Armour D, Monel, and Manganin are all trade marks	

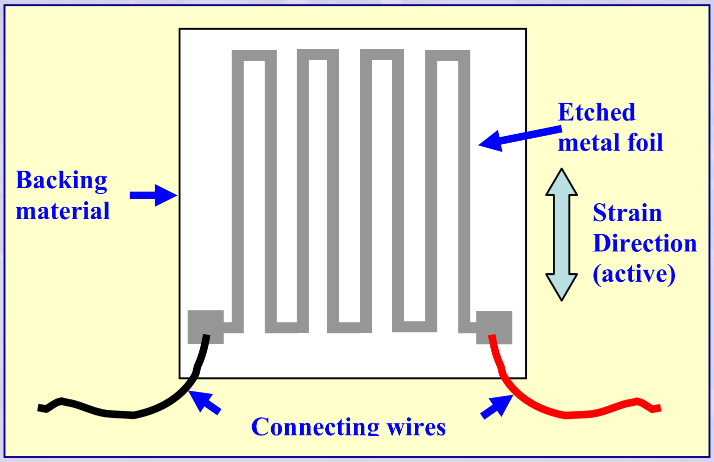


Figure 74 Strain gage construction schematic (not to scale)

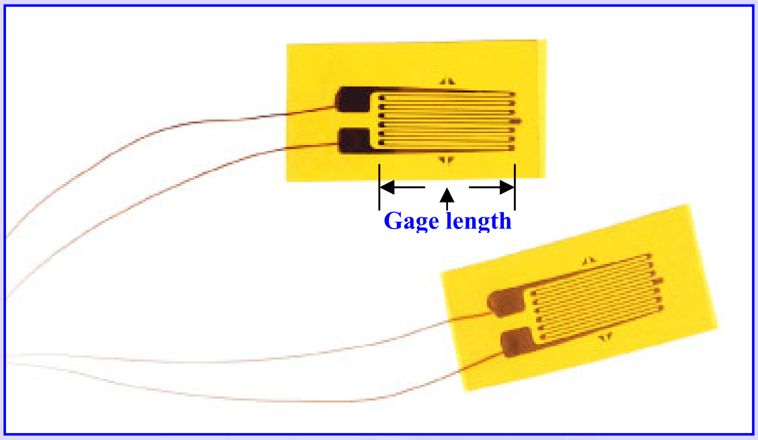


Figure 75 Strain gage elements as they actually appear
(Visit: www.omega.com)

Strain gage elements as supplied by Omega are shown in Figure 75. By making the element in the form of a serpentine foil the actual length of the element is several times the length of the longer side of the element (referred to as gage length in Figure 59). The gage length may vary from 0.2 to 100 mm. For a given strain the change in length will thus be the sum of the changes in length of each leg of the element and hence the resistance change will be proportionately larger. Gages with nominal resistances of 120,350,600 and 700 Ω are available. However 120 Ω gages are very common.

Example 25

- A typical foil strain gage has the following specifications:

  1. Material is Constantan 6 □m thick
  2. Resistance 120 □ ± 0.33%
  3. Gage factor 2 ± 10%

  Such a strain gage is bonded on to a tube made of stainless steel of internal diameter 6 mm and wall thickness 0.3 mm. The tube is subjected to an internal pressure of 0.1 MPa gage. The outside of the tube is exposed to the standard atmosphere. What is the change in resistance of the strain gage if the Young's modulus of stainless steel is 207 GPa?

- Data:

  Applied internal pressure is $p = 0.1\ MPa = 10^5\ Pa$

  Tube internal diameter is $ID = 6\ mm = 0.006\ m$
  Tube wall thickness is $t = 0.3 mm = 3\times10^{-4}\ m$

  Tube is assumed to be thin and the radius r is taken as the mean radius given by

$$r = \frac{(ID+t)}{2} = \frac{(0.006+0.0003)}{2} = 0.00315m$$

  The Young's modulus of stainless steel is given to be $E = 207\ GPa = 2.07\times10^{8}\ Pa$

- The hoop stress due to internal pressure is calculated as

$$\sigma_h = \frac{pr}{t} = \frac{10^5\times0.00315}{0.0003} Pa = 1.05\times10^6\ Pa$$

- The corresponding strain is calculated as

$$\in_h = \frac{\sigma_h}{E} = \frac{1.05\times10^6}{2.07\times10^8} = 0.0051$$

(For Constantan the maximum allowable elongation is 1% and the above value is acceptable)

- With the gage factor of $GF = 2$ the change in resistance as a fraction of the resistance of the strain element is

$$\frac{\Delta R}{R} = GF \in_h = 2 \times 0.0051 = 0.0101$$

- The change in resistance of the strain gage is thus given by

$$\Delta R = R\,GF \in_h = 120 \times 2 \times 0.0051 = 1.02\ \Omega$$

Bridge circuits for use with strain gages:

While discussing resistance thermometers we have already seen how a bridge circuit may be used for making measurements (see Sub Module 2.3). Since a strain gage is similar in its operation to an RTD it is possible to use a similar bridge circuit for measuring the strain and hence the pressure. Lead wire compensation is also possible by using a three wire arrangement. For the sake of completeness some of these will be discussed again here.

The basic DC bridge circuit is shown in Figure 76. The output voltage V_o is related to the input voltage V_s by the relation (derived using Ohm's law)

$$V_o = V_s \frac{(R_1 R_3 - R_2 R_4)}{(R_1 + R_4)(R_2 + R_3)} \quad (80)$$

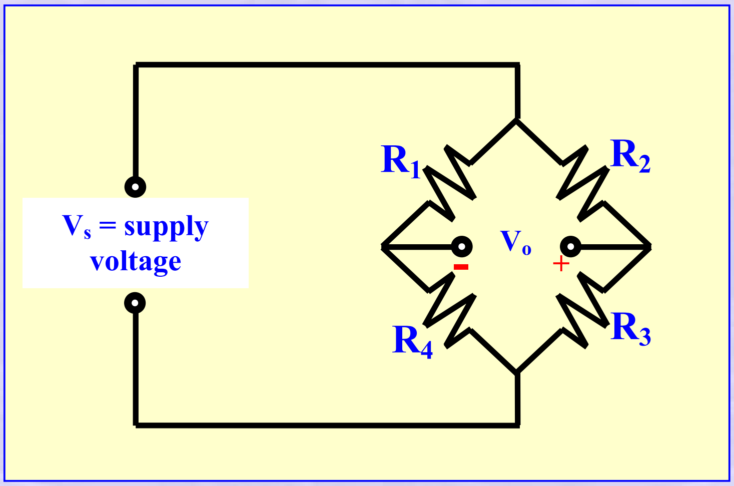


Figure 76 Basic bridge circuit and the nomenclature

If all the resistances are chosen equal (equal to 120 □, for example) when the strain gage is unstrained, the output voltage will be zero. Let us assume that R_3 is the strain gage element and all others are standard resistors. When the strain gage is strained the resistance will change to $R_3 + \Delta R_3$ where $\Delta R_3 \ll R_3$. The numerator of Equation 80 then becomes $R_1 \Delta R_3$. The denominator is approximated by $(R_1 + R_4)(R_2 + R_3 + \Delta R_3) \approx (R_1 + R_4)(R_2 + R_3) = 4R_1^2$ since all resistances are equal when the strain gage is unstrained. We thus see that Equation 80 may be approximated by

$$V_o = V_s \frac{R_1 \Delta R_3}{4R_1^2} = V_s \frac{\Delta R_3}{4R_1} = V_s \frac{\Delta R_3}{4R_3} = V_s \frac{GF \in}{4} \quad (81)$$

Thus the output voltage is proportional to the strain experienced by the strain gage. For a gage with sensitivity of 2 the ratio of output voltage to input voltage is equal to half the strain. In practice Equation 81 is rearranged to read

$$\in = \frac{4V_o}{V_s} \frac{1}{GF} \quad (82)$$

Since the pressure is proportional to the strain, Equation 82 also gives the pressure within a constant factor. Note that V_o needs to be measured by the use of a suitable voltmeter, likely a milli-voltmeter. In practice one may use an amplifier to amplify the signal by a known factor and measure the amplified voltage by a suitable voltmeter. Another method is to make the resistance R_2 a variable resistor and null the bridge by using a controller as shown in Figure 77.

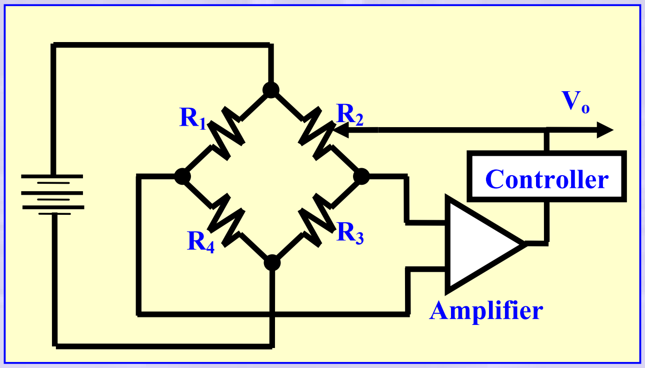


Figure 77 Null balanced bridge

The controller uses the amplified off balance voltage to drive the bridge to balance by varying the resistor R_2. The voltage V_o shown is then a measure of the strain.

Example 26

- The supply voltage in the arrangement depicted in Figure 60 is 5 V. All the resistors are equal when the strain gage is not under load. When the strain gage is loaded along its axis the output voltage registered is 1.13 mV. What is the axial strain of the gage if the sensitivity is 2?
- We use Equation 82 for calculating the axial strain. We have:

$$V_s = 5\ V,\ V_o = 1.13\ mV = 1.13\times10^{-3}\ V\ \ and\ \ GF = 2$$

- The axial strain is then given by

$$\in_a = \frac{4V_o}{V_s}\frac{1}{GF} = \frac{4\times1.13\times10^{-3}}{5\times2} = 4.52\times10^{-4} = 452\ micro-strain$$

Example 27

- Consider the data given in Example 21 again. This gage is connected in a bridge circuit with an input voltage of 9 V. What is the output voltage in this case?

$$\Delta R = R\,GF\,\in_h = 120\times2\times0.0101 = 1.02\ \Omega$$

- From the results in Example 21 the change is resistance has been obtained as $\Delta R_3 = 1.02\ \Omega$. We then have $\frac{\Delta R_3}{R_3} = \frac{1.02}{120} = 0.0085$. With $V_s = 9\ V$ we then have

$$V_o = V_s\frac{\Delta R_3}{4R_3} = 9\times\frac{1.02}{4\times120} = 0.0191\ V = 19.1\ mV$$

Three wire gage for lead wire compensation:

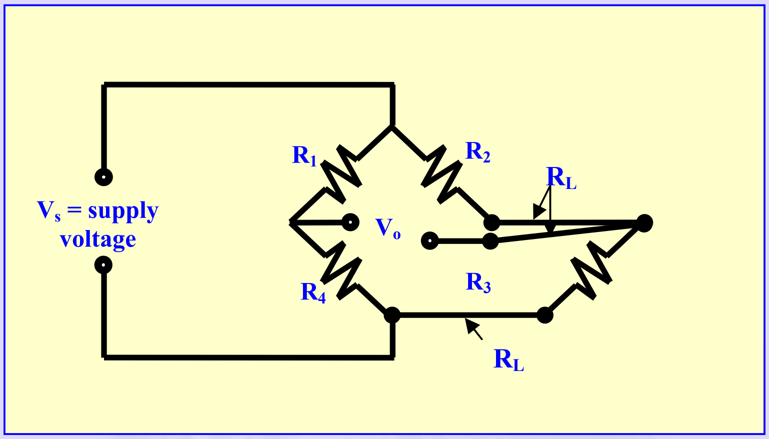


Figure 78 Three wire arrangement for lead wire compensation

The connections are made as shown in Figure 78 while using a strain gage with three leads. Let each lead have a resistance of R_L as shown. Also let $R_2^{'} = R_2 + R_L$ and $R_3^{'} = R_3 + R_L$. Equation 80 is recast as

$$\begin{aligned} V_o &= V_s \frac{\left(R_1 R_3^{'} - R_2^{'} R_4\right)}{\left(R_1 + R_4\right)\left(R_2^{'} + R_3^{'}\right)} \\ &= V_s \frac{R_1(R_3 + R_L) - (R_2 + R_L)R_4}{\left(R_1 + R_4\right)\left(R_2 + R_3 + 2R_L\right)} \\ &= V_s \frac{\left(R_1 R_3 - R_2 R_4\right) + R_L(R_1 - R_4)}{\left(R_1 + R_4\right)\left(R_2 + R_3 + 2R_L\right)} \end{aligned} \qquad \textbf{(83)}$$

If all the resistances are identical under no strain condition the second term in the numerator is zero and hence the balance condition is not affected by the lead resistance. Let $R_3 + \Delta R_3$ be the resistance in the strained condition for the gage. Again we assume that $\Delta R_3 \ll R_3$ and, in addition, we assume that $\Delta R_3 \ll R_L$. With these Equation 83 may be approximated by the relation

$$V_o = V_s \frac{\left(R_1 R_3 - R_2 R_4 + R_1 \Delta R_3\right)}{\left(R_1 + R_4\right)\left(R_2 + R_3 + 2R_L\right)} \approx V_s \frac{\Delta R_3}{4R_1\left(1 + \frac{R_L}{R_1}\right)} \approx V_s \frac{\Delta R_3}{4R_3}\left(1 - \frac{R_L}{R_1}\right) \qquad \textbf{(84)}$$

In writing the above we have additionally assumed that the lead resistance is small compared to the resistance of the gage. Thus Equation 81 is recast as

$$V_o \approx V_s \frac{GF \in}{4}\left(1 - \frac{R_L}{R_1}\right) \qquad \textbf{(85)}$$

Thus the three wire arrangement provides lead wire compensation but with a reduced effective gage factor of $GF\left(1-\frac{R_L}{R_1}\right)$.

Example 28

- A three wire strain gage with a lead resistance of $R_L = 4\ \Omega$ and a gage resistance under no strain of $R = 120\ \Omega$ is used to measure a strain of 0.001. What will be the output voltage for an input voltage of 5 V if a) the lead resistance effect is not included b) if the lead wire resistance is included? The gage factor GF has been specified to be 1.95.

Case a

- The effective gage factor in this case is the same as GF = 1.9. The output voltage with a strain of $\in$ = 0.001 is given by

$$V_o \approx V_s \frac{GF \in}{4} = 5 \times \frac{1.9 \times 10^{-3}}{4} = 0.00238\ V = 23.8\ mV$$

Case b

- The lead resistance is given to be 4 □. The resistance of the gage is R = 120 □. Hence the effective gage factor is

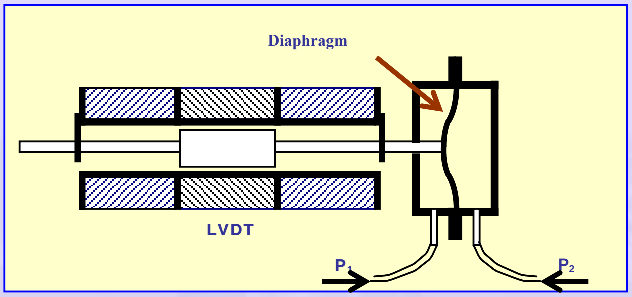
$$GF_{eff} = GF\left(1-\frac{R_L}{R}\right) = 1.9 \times \left(1-\frac{4}{120}\right) = 1.837$$

- The corresponding output is given by

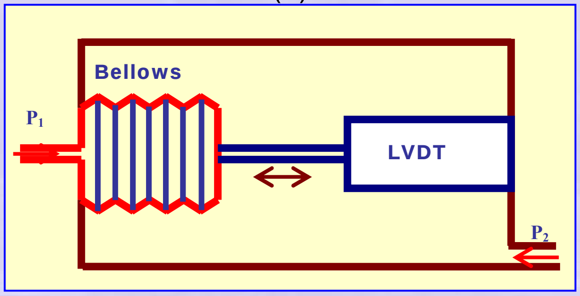
$$V_o \approx V_s \frac{GF_{eff} \in}{4} = 5 \times \frac{1.837 \times 10^{-3}}{4} = 0.0023\ V = 23.0\ mV$$

2) Diaphragm/Bellows type transducer:

Pressure signal is **converted** to a **displacement** in the case of diaphragm/bellows type pressure gage. The diaphragm or bellows acts as a **spring element** that undergoes a displacement under the action of the pressure. Schematic of diaphragm and bellows elements are shown respectively in Figure 79 (a) and (b).



(a)



(b)

Figure 79 Schematic of diaphragm and bellows type pressure gages that use LVDT for displacement measurement

In the above figure a Linear Variable Differential Transformer (**LVDT**) is used as a displacement transducer. The working principle of LVDT will be discussed later on. In the case of the diaphragm type gage one may also use a strain gage for measuring the strain in the diaphragm by fixing it at a suitable position on the diaphragm (Figure 80).

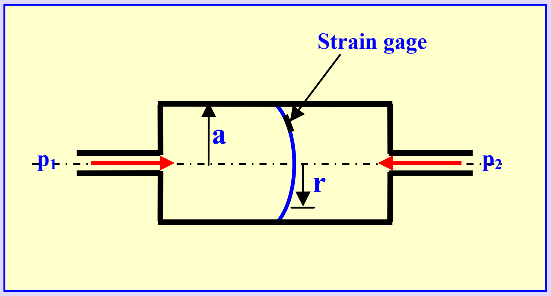


Figure 80 Diaphragm gage with strain gage for Displacement measurement

In the case of the bellows transducer the relationship between the pressure difference and the displacement is given by

$$\delta = \frac{(p_1 - p_2)A}{K} \quad \textbf{(86)}$$

Here δ is the displacement of the bellows element, A is the inside area of the bellows and K is the spring constant of the bellows element. In the case of the diaphragm element the following formula from strength of materials is useful:

$$\delta(r) = \frac{3(p_1 - p_2)(a^2 - r^2)^2}{16Et^3}(1-\nu^2) \quad \textbf{(87)}$$

In the above formula δ is the deflection at radius r, 'a' is the radius of the diaphragm, t is its thickness, E and ν are respectively the Young's modulus and Poisson ratio of the material of the diaphragm. This formula assumes that the displacement of the diaphragm is **small** and the deformation is much smaller the elastic limit of the material. The perimeter of the diaphragm is assumed to be rigidly fixed. The force is assumed to be uniformly distributed over the surface of the diaphragm. It is seen that the maximum deflection takes place at the center of the diaphragm

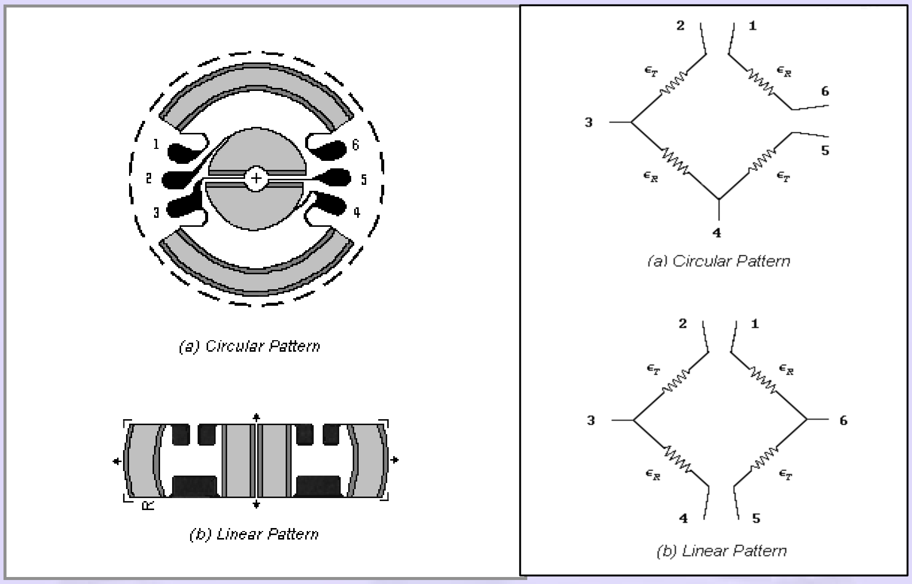


Figure 81 Strain gages for mounting on a diaphragm gage

(Figures from Vishay Intertechnology, Inc., Manufacturers of Diaphragm pressure gages) Terminals are provided for external connections and for compensating resistors. Different strain gage elements of the bridge respond to radial ($\in_R$) and tangential ($\in_T$) strains

Since the strain gage occupies a large area over the diaphragm it is necessary to integrate the strain of the element over its extent. The displacement given by Equation 87 is normal to the plane of the diaphragm. Because of this displacement the diaphragm undergoes both radial as well as circumferential strains. As indicated in Figure 81 strain gage elements respond to both the radial as well as tangential (circumferential) strains. The numbers represent the leads and the strain gages are connected in the form of a full bridge. The ratio of output to input is then given by the **approximate formula** (from Vishay Intertechnology, Inc., Manufacturers of Diaphragm pressure gages)

$$\frac{V_o}{V_s} = 0.75\frac{(p_1 - p_2)(1-\nu^2)}{E}\left(\frac{a}{t}\right)^2 \times 10^3 \; mV/V \qquad (88)$$

The above formula assumes that the diaphragm is perfectly rigidly held over its perimeter. This is achieved by making the diaphragm an integral part of the body of the pressure transducer. It is also assumed that the sensitivity of the strain gage is 2. While measuring transient pressures it is necessary that the transients do not have significant components above about 20% of the natural frequency (f_n) of the diaphragm given by the following expression.

$$f_n = \frac{0.469t}{a^2}\sqrt{\frac{E}{\rho\left(1-\nu^2\right)}} \tag{89}$$

In the above expression ρ is the density of the material of the diaphragm in kg/m^3. The diaphragm is again assumed to be rigidly clamped along its periphery.

Example 29

- Steel diaphragm of 15 mm diameter is used in a pressure transducer to measure a maximum possible pressure of 10 MPa. The transducer is required to give an output of $3\ mV/V$ at the maximum pressure. Use the following data to determine the thickness of the diaphragm. Also determine the maximum out of plane deflection of the diaphragm and the its natural frequency.

  Young's modulus $E = 207\ GPa$
  Poisson ratio $\nu = 0.285$
  The gage is constructed as shown in Figure 81.

- From the given data we have:

$$\frac{V_o}{V_s} = 3\ mV/V\,;\, a = \frac{15}{2} mm = 0.075\ m\,;\left(p_1 - p_2\right) = 10\ MPa = 10^7\ Pa$$

- We use Equation 88 to obtain the diaphragm thickness as

$$t = a\sqrt{\frac{0.75\left(p_1 - p_2\right)\left(1-\nu^2\right)\times 10^3}{E}}\frac{1}{\sqrt{\frac{V_o}{V_s}}}$$

$$= 0.075\times\sqrt{\frac{0.75\times 10^7\times\left(1-0.285^2\right)\times 10^3}{2.07\times 10^{11}}}\times\frac{1}{\sqrt{3}}$$

$$= 0.000593\ m = 0.593\ mm$$

- The maximum out of plane deflection is obtained by putting $r = 0$ in Equation 87.

- $$\delta\left(r=0\right) = \frac{3\times 10^7\times 0.0075^4\times\left(1-0.285^2\right)}{16\times 2.07\times 10^{11}\times 0.000593^3} = 0.0001263\ m = 0.126\ mm$$

- The ratio of the maximum deflection to the diaphragm thickness is

$$\frac{\delta(0)}{t} = \frac{0.000126}{0.000593} = 0.213$$

- This ratio is less than 0.25 and hence is satisfactory.

- The natural frequency is given by

$$f_n = \frac{0.469 \times 0.000593}{0.0075^2}\sqrt{\frac{207 \times 10^9}{7830 \times \left(1 - 0.285^2\right)}} = 26522\ Hz$$

- The sensor is useful below about 5 kHz

Diaphragm/Bellows type gage with LVDT:

As indicated earlier a diaphragm/bellows element may be used to measure pressure by directly measuring the small displacement very accurately. This may be done by using a Linear Variable Differential Transformer (LVDT) as shown in Figure 63. A schematic of LVDT is shown in Figure 82.

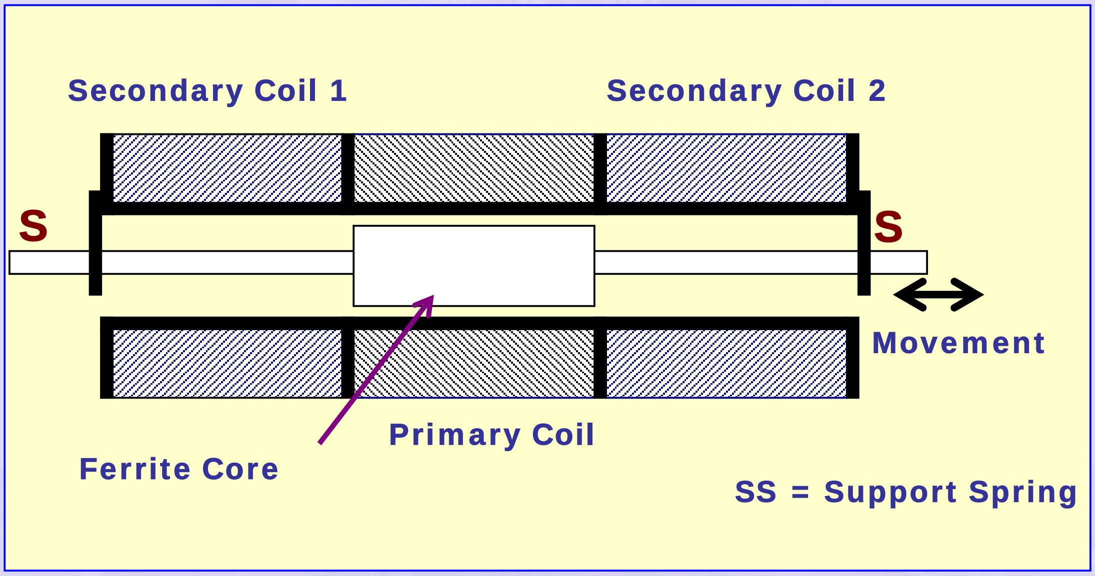


Figure 82 Schematic of an LVDT

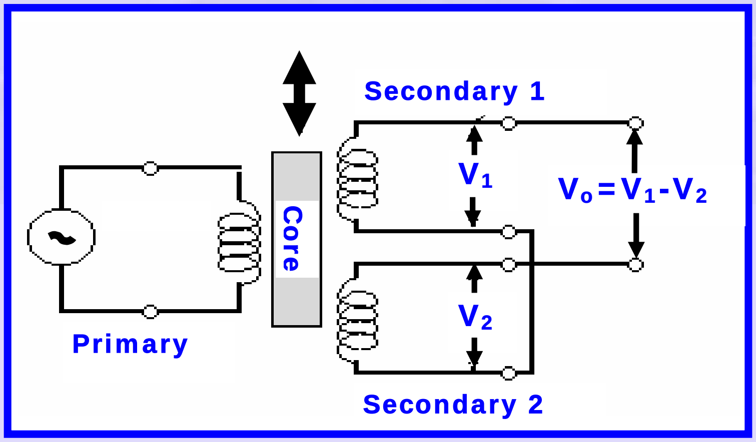


Figure 83 Operation of the LVDT

LVDT consists of three coaxial coils with the primary coil injected with an **ac current**. The coils are mounted on a **ferrite core** carried by a stainless steel rod. The two secondary coils are connected in opposition as shown in Figure 83.

The stainless steel rod is restrained by springs as indicated. One end of the rod is connected to the diaphragm/bellows element as indicated in Figure 79. When the ferrite core is symmetrical with respect to the two secondary coils the ac voltages induced by **magnetic coupling** across the two secondary coils are equal. When the core moves to the right or the left as indicated the magnetic coupling changes and the two coils generate different voltages and the voltage difference is proportional to the displacement (see Figure 84).

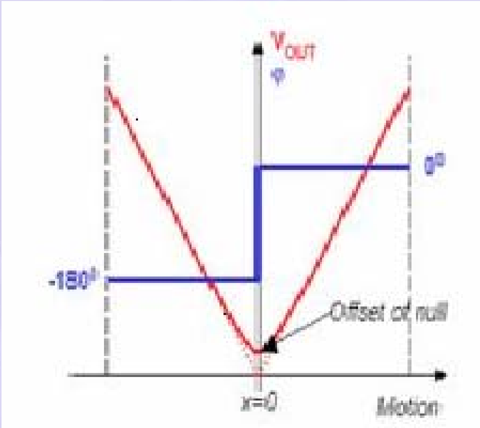


Figure 84 Output of an LVDT excited by alternating current supply. The phase information is with respect to the phase of the supply.

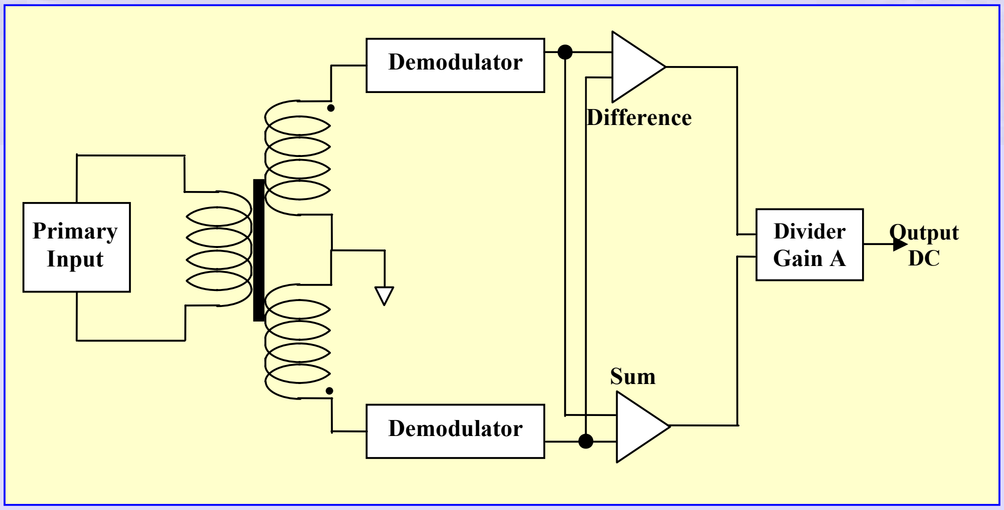


Figure 85 Schematic of a typical LVDT signal conditioner

We describe the typical LVDT circuit shown in Figure 85. The primary input voltage is 1 – 3 V rms. Full scale **secondary** voltage is between 0.1 – 3 V rms. Gain is set between 2 – 10 to increase the output voltage to ± 10 V. The two secondary coils are connected in opposition and sum and difference signals are obtained from the two coils, after demodulating the two voltages. The ratio of these is taken and is output with a gain A as indicated in the figure. The output is independent of the gain settings of the sum and difference amplifiers as long as they have the same gain.

Table 16 Common specifications for commercially available LVDT

Input:	Power input is a 3 to 15 V (rms) sine wave with a frequency between 60 to 20,000 Hz (the two most common signals are 3 V, 2.5 kHz and 6.3 V, 60 Hz).
Stroke:	Full-range stroke ranges from ±125 μm to ±75 mm
Sensitivity:	Sensitivity usually ranges from 0.6 to 30 mV per 25 μm under normal excitation of 3 to 6 V. Generally, the higher the frequency the higher the sensitivity.
Nonlinearity:	Inherent nonlinearity of standard units is on the order of 0.5% of full scale.

In Example 29 the diaphragm undergoes a maximum deflection of 0.126 mm or 136 μm. As Table 16 shows we can certainly pick a suitable LVDT for this purpose.

3) Capacitance diaphragm gage:

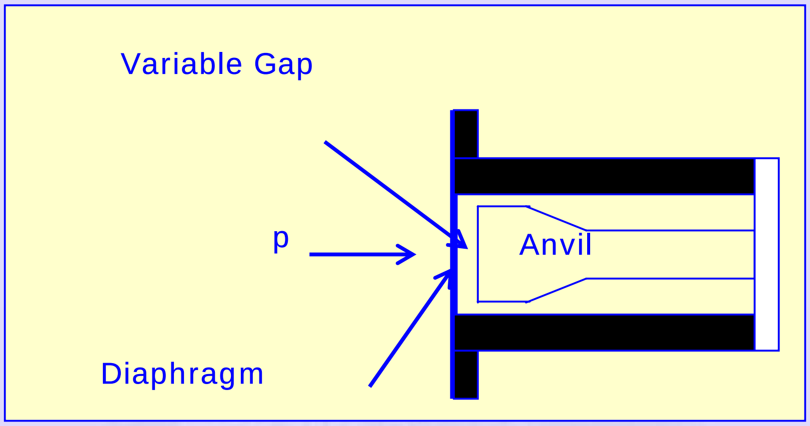


Figure 86 Capacitance pressure transducer

Figure 86 shows the schematic of a capacitance type pressure transducer. The **gap** between the stretched diaphragm and the anvil (remains fixed because of its mass) varies with applied pressure due to the small displacement of the diaphragm. This changes the capacitance of the gap. The theoretical basis for this is derived below.

Consider a parallel plate capacitor of plate area A cm^2 and gap between plates of x cm. Let the gap contain a medium of dielectric constant κ. The capacitance of the parallel plate capacitor is then given in pico-Farads (1 pF = 10^{-12} Farad) by

$$C = 0.0885\kappa \frac{A}{x} pF \tag{90}$$

The dielectric constants of some common materials are given in Table 17.

Table 17 Dielectric constants of some common materials

Material	κ	Material	κ
Vacuum	1	Plexiglass®	3.4
Air (1 atmosphere)	1.00059	Polyethylene	2.25
Air (10 atmospheres)	1.0548	PVC	3.18
Glasses	5 – 10	Teflon	2.1
Mica	3 – 6	Germanium	16
Mylar	3.1	Water	80.4
Neoprene	6.7	Glycerin	42.5

Logarithmic differentiation of Equation 90 shows that

$$\Delta C = -\frac{C\Delta x}{x} = -0.0885\kappa\frac{A}{x^2} \quad \textbf{(91)}$$

As an example consider a capacitance type gage with air as the medium, $A = 1\ cm^2$, $x = 0.3\ mm = 0.03\ cm$. We then have $C = \frac{0.0885 \times 1}{0.03} = 2.95\ pF$. The sensitivity defined by the partial derivative of C with respect to x is $S = \frac{\partial C}{\partial x} = -\frac{C}{x} = -\frac{2.95}{0.03} = -98.3\ pF/cm$.

Bridge circuit for capacitance pressure gage:

Schematic of a bridge circuit for a capacitance pressure gage is shown in Figure 87. The circuit is driven by alternating current input at high frequency and coupled using a transformer, as shown in the figure.

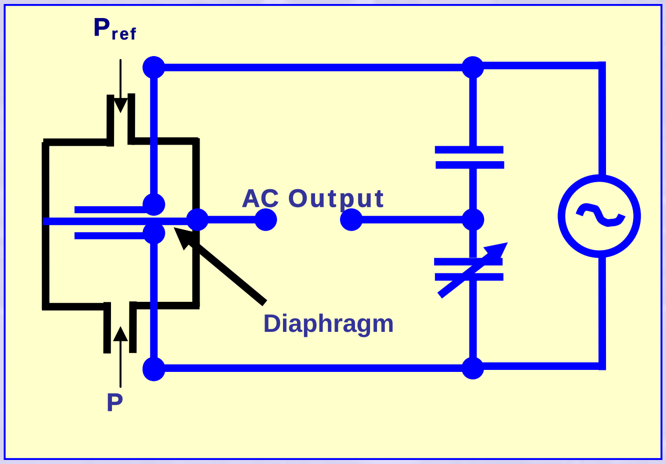


Figure 87 Bride circuit for capacitance pressure gage

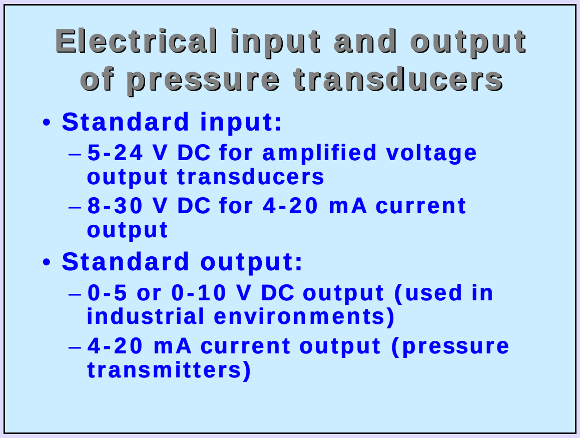


Figure 88 Summary of typical electrical input and output values

Either a balanced or unbalanced mode of operation of the bridge circuit is possible. In the balanced mode of operation the variable capacitor is adjusted to bring the bridge back to balance. The position of a dial attached to the variable capacitor may be marked in pressure units. Alternately the output may be related to the pressure by direct calibration. Figure 88 indicates the typical electrical input output pairs for pressure transducers. In the current output type of instrument 4 mA corresponds to 0 pressure and 20 mA corresponds to full scale (dependent on the range of the pressure transducer). Current transmitters of this type will be discussed later. The full scale may be a few mm of water column up to pressures in the MPa range. The scale is linear over the entire range of the transducer. Note that this type of gage may also be used for measurement of pressures lower than the atmospheric pressure.

Measurement of transient pressures:

In continuation of what we have said about transient pressures, we note the following points:

- **Transient behavior of pressure measuring instruments is needed to understand how they will respond to transient pressures.**

- **Measurement of transient or time varying pressure requires a proper choice of the instrument and the coupling element between the pressure signal and the transducer.**
- **Usually process pressure signal is communicated between the measurement point and the transducer via a tube.**
- **The tube element imposes a "resistance" due to viscous friction.**
- **Viscous effects are normally modeled using laminar fully developed flow assumption (discussed earlier while considering the transient response of a U tube manometer).**

Before we proceed with the analysis we shall look at a few general things. These are in the nature of **comparisons** with what has been discussed earlier while dealing with transients in thermal systems and in the case of the U tube manometer. We shall keep electrical analogy in mind in doing this.

Thermal system:

Thermal Capacity C (J/K) = Product of mass (M, kg) and specific heat (c, J/kg K). With temperature T representing the driving potential, h_e representing the enthalpy the specific heat capacity is given by $c = \frac{\partial h_e}{\partial T}$. Then, we have, $C = Mc = \rho V \frac{\partial h_e}{\partial T}$. Thermal resistance is given by $R = \frac{1}{hS}$ where h is the heat transfer coefficient and S is the surface area. The time constant is given by $\tau = RC = \frac{Mc}{hS}$ (using analogy with an electrical circuit).

Pressure measurement in a liquid system:

Driving potential is the head of liquid h (m). Volume of liquid V (m^3) replaces enthalpy in a thermal system. Thus the **capacitance** is $C = \frac{dV}{dh}$ and has units of m^2. We have already seen that the resistance to liquid flow is defined through the relation $R=-\frac{dp}{d\dot{m}}$ with p standing for pressure and $\dot{m}$ standing for the mass flow rate. Again the time constant is $\tau = RC = -\frac{dV}{dh}\frac{dp}{d\dot{m}}$.

Gas system:

Potential for bringing about change is the pressure p. The mass of the gas m replaces the enthalpy in the thermal system. The **capacitance** is given by $C=\frac{dm}{dp}$. With m being given by the product of density and volume, we have $C=\frac{dm}{dp}=\frac{d(\rho V)}{dp}$. The capacitance depends on the process that the gas undergoes while the pressure transducer responds to the transient. If we assume the process to be polytropic, then $\frac{p}{\rho^n}=\text{Constant}$, where n has an appropriate value ≥ 1. If, for example, the volume of the gas is held constant, we have $C=V\frac{d\rho}{dp}$. For the polytrope, logarithmic differentiation gives $\frac{dp}{p}-n\frac{d\rho}{\rho}=0$ or $\frac{d\rho}{dp}=\frac{\rho}{np}$. With this the capacitance becomes $C=V\frac{\rho}{np}$. Assuming the gas to be an ideal gas $p=\rho R_g T$ and hence the capacitance is $C=\frac{V}{nR_gT}$. The resistance to flow is again given by the **Hagen-Poiseuelle law** as in the case of a liquid. The time constant is then given by $\tau = R\frac{V}{nR_gT}$.

Transient response of a bellows type pressure transducer:

The bellows element may be considered as equivalent to a piston of area A (in m^2) spring arrangement as shown in Figure 89. Let the bellows spring constant be K (in N/m). Let the displacement of the bellows element be b (in m). Let the bellows gage be connected to a reservoir of liquid of density ρ (kg/m^3) at pressure P (in Pa). At any instant of time let p (in Pa) be the pressure inside the bellows element. The displacement of the spring is then given by the relation

$$b = \frac{\text{Force}}{\text{Spring constant}} = \frac{pA}{K} \quad (92)$$

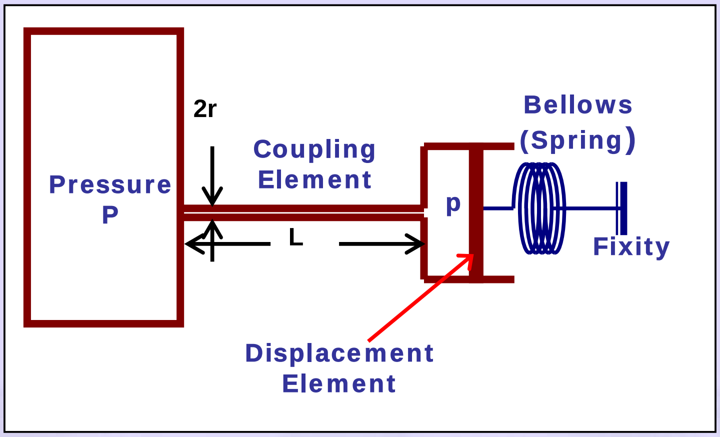


Figure 89 Transient in a bellows type gage

Let the connecting tube between the reservoir and the gage be of length L (in m) and radius r (in .m). If the pressure P is greater than the pressure p, the liquid will flow through the intervening tube at a mass flow rate given by $\dot{m} = \rho A \frac{db}{dt}$. We know from the definition of connecting tube resistance that this should equal $\dot{m} = \frac{P-p}{R}$. All we have to do is to equate these two expressions to get the equation governing the transient.

$$\rho A \frac{db}{dt} = \frac{P-p}{R} = \frac{P - \frac{K}{A}b}{R} \quad (93)$$

The latter part of the equality follows from Equation 92. The above equation may be rearranged in the form

$$\left(\frac{A^2 \rho R}{K}\right)\frac{db}{dt} + b = \frac{A}{K}P \quad (94)$$

`We see that the transient of a bellows type gage is governed by a **first order** ordinary differential equation with a **time constant** $\tau = \frac{A^2 \rho R}{K}$. Hence the bellows type gage is a first order system just like the first order behavior exhibited by a first order thermal system. We infer from the above that the capacitance of a bellows type gage is given by $C = \dfrac{A^2\rho}{K}$.

Example 30

- A bellows pressure gage of area 1 cm^2, spring constant K = 4.4 N/cm is connected by a 2.5 mm ID tubing that is 15 m long in a process application. Measurement is of pressure of water at 30°C. Determine the time constant of this gage.

- The given data for the gage:

$$A = 1\ cm^2 = 10^{-4}\ m^2,\ K = 4.4\,N/cm = 440\,N/m$$

- The given data for the connecting tube:

$$r = \frac{ID}{2} = \frac{2.5}{2}mm = 1.25\,mm = 0.00125\,m, L = 15\,m$$

- Properties of water required are taken from tables. These are

$$\rho = 995.7\ kg/m^3,\ \nu = 0.801\times10^{-6}\,m^2/s$$

- The tube resistance is calculated as

$$R = \frac{8\nu L}{\pi r^4} = \frac{8\times0.801\times10^{-6}\times15}{\pi\times0.00125^4} = 1.2532\times10^7\left(m\ s\right)^{-1}$$

- The capacitance is given by

$$C = \frac{A^2\rho}{K} = \frac{\left(10^{-4}\right)^2\times995.7}{440} = 2.26\times10^{-8}\,s^2m$$

- The time constant is then given by

$$\tau = RC = 1.2532\times10^7\times2.26\times10^{-8} = 0.284\ s$$

Force balancing element for measuring pressure:

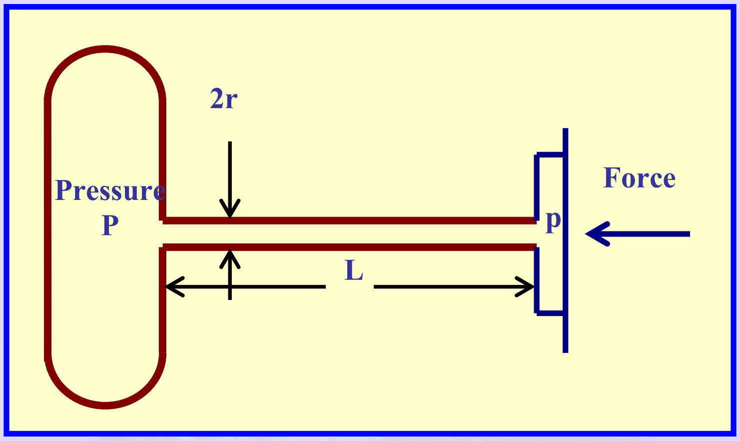


Figure 90 Transient in a force balance transducer

Figure 90 shows the arrangement in a **force balance** element. It consists of a diaphragm gage that is maintained in its **zero displacement condition** by applying a force that balances the pressure being measured. The required force itself is a measure of the pressure. Let us assume that the pressure being measured is that of a gas like air. If the diaphragm displacement is always in the null the volume of the gas within the gage is constant. If the pressure P is greater than pressure p mass flow takes place from the pressure source in to the gage to increase the density of the gas inside the gage. The time constant for this system is just the one discussed earlier and is given by

$$\tau = R\frac{V}{nR_g T} \quad (95)$$

In case the instrument is not the null type (where the diaphragm displacement is zero at all times), the capacity should include both that due to density change and that due to displacement. Since the displacement is very small, it is appropriate to sum the two together and write

$$C = \frac{A^2\rho}{K} + \frac{V}{nR_g T} \quad (96)$$

The above is valid as long as the transients are small so that the changes in density of the fluid are also small. In most applications this condition is satisfied.

Example 31

- A bellows type pressure gage of effective area A = 0.25 cm^2, spring constant K = 100 N/m is connected by a 1.5 mm ID 10 m long stainless tubing in a process application. The volume of the effective air space in the bellows element is V = 2 cm^3. Determine the time constant if the pressure measurement is a pressure around 5 bars and the temperature is 27°C.

- We assume that the pressure hovers around 5 bars during the transients. The air properties are taken from appropriate tables.

- Air properties:

$$p = 5\ bars = 5\times10^5\ Pa,\ T = 27°C = 300\ K,\ R_g = 287\ J/kg\ K$$

- The density of air is calculated as

$$\rho = \frac{p}{R_g T} = \frac{5\times10^5}{287\times300} = 5.807\ kg/m^3$$

- The dynamic viscosity of air is (from table of properties) $\mu = 1.846\times10^{-5}\ kg/m\ s$ The kinematic viscosity is hence given by

$$\nu = \frac{\mu}{\rho} = \frac{1.846\times10^{-5}}{5.807} = 3.179\times10^{-6}\ m^2/s$$

- The capacity is calculated based on Equation 96

$$C = \frac{A^2\rho}{K} + \frac{V}{nR_g T} = \frac{\left(0.25\times10^{-4}\right)^2\times5.807}{100} + \frac{2\times10^{-6}}{1.4\times287\times300} = 5.289\times10^{-11}\ m\,s^2$$

- In the above we have assumed a polytropic index of 1.4 that corresponds to an adiabatic process.

- The fluid resistance is calculated now as

$$R = \frac{8\nu L}{\pi r^4} = \frac{8\times3.179\times10^{-6}\times10}{\pi\times\left(\frac{0.0015}{2}\right)^4} = 2.559\times10^8(m\ s)^{-1}$$

- The time constant of the transducer is then given by

$$\tau = RC = 2.559\times10^8\times5.289\times10^{-11} = 0.0135\ s$$

Example 32

- Rework Example 28 assuming that the gage operates in the null mode.
- The capacity is given by

$$C = \frac{V}{nR_g T} = \frac{2\times10^{-6}}{1.4\times287\times300} = 1.659\times10^{-11}\, m\, s^2$$

- In the above we have assumed a polytropic index of 1.4 that corresponds to an adiabatic process.
- The fluid resistance is calculated now as

$$R = \frac{8\nu L}{\pi r^4} = \frac{8\times3.179\times10^{-6}\times10}{\pi\times\left(\frac{0.0015}{2}\right)^4} = 2.559\times10^8 (m\ s)^{-1}$$

- The time constant of the transducer is then given by

$$\tau = RC = 2.559\times10^8\times1.659\times10^{-11} = 0.0042\ s$$