Sub Module 1.2

2. Errors in measurements

Errors accompany any measurement, however well it is conducted. The error may be inherent in the measurement process or it may be induced due to variations in the way the experiment is conducted. The errors may be classified as:

1. Systematic errors (Bias):

Systematic errors due to faulty or improperly calibrated instruments. These may be reduced or eliminated by careful choice and calibration of instruments. Sometimes bias may be linked to a specific cause and estimated by analysis. In such a case a correction may be applied to eliminate or reduce bias. Bias is an indication of the accuracy of the measurement. Smaller the bias more accurate the data

2. Random errors:

Random errors are due to non-specific causes like natural disturbances that may occur during the measurement process. These cannot be eliminated. The magnitude of the spread in the data due to the presence of random errors is a measure of the precision of the data. Smaller the random error more precise is the data. Random errors are statistical in nature. These may be characterized by statistical analysis.

We shall explain these through the familiar example shown in Figure 3. Three different individuals with different skill levels are allowed to complete a round of target practice. The outcome of the event is shown in the figure.

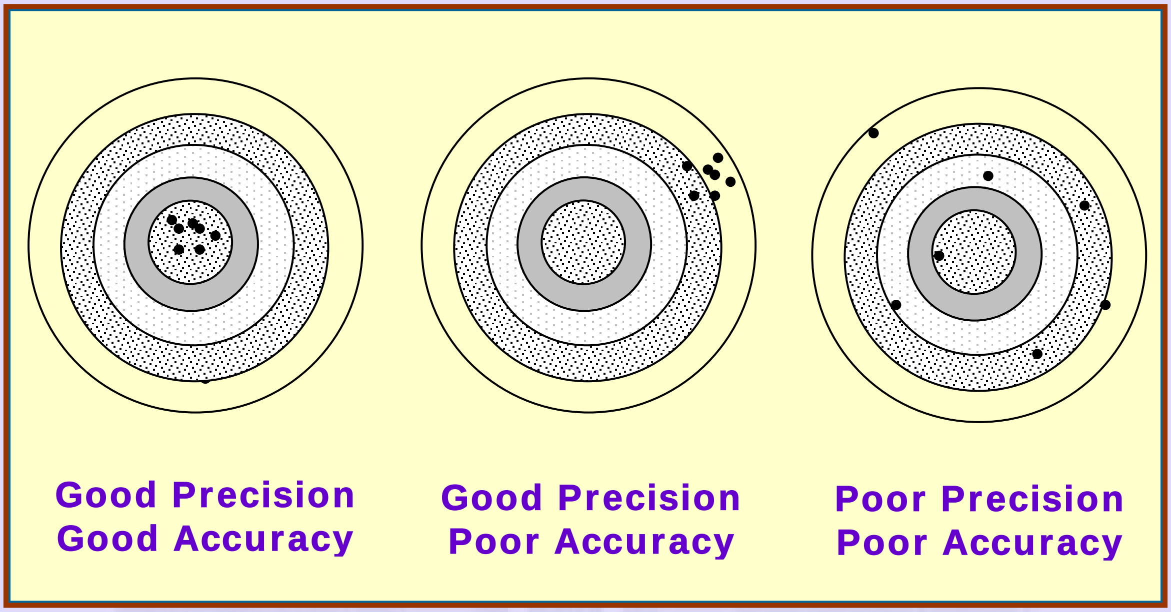

Figure 3 Precision and accuracy explained through a familiar example

It is evident that the target at the left belongs to a highly skilled shooter. This is characterized by all the shots in the inner most circle. The result indicates good accuracy as well as good precision. A measurement made well must be like this case! The individual in the middle is precise but not accurate. Maybe it is due to a faulty bore of the gun. The individual at the right is an unskilled person who is behind on both counts. Most beginners will fall into this category. The analogy is quite realistic since most students performing a measurement in the laboratory may be put into one of the three categories. A good experimentalist has to work hard to excel in it!

Another example:

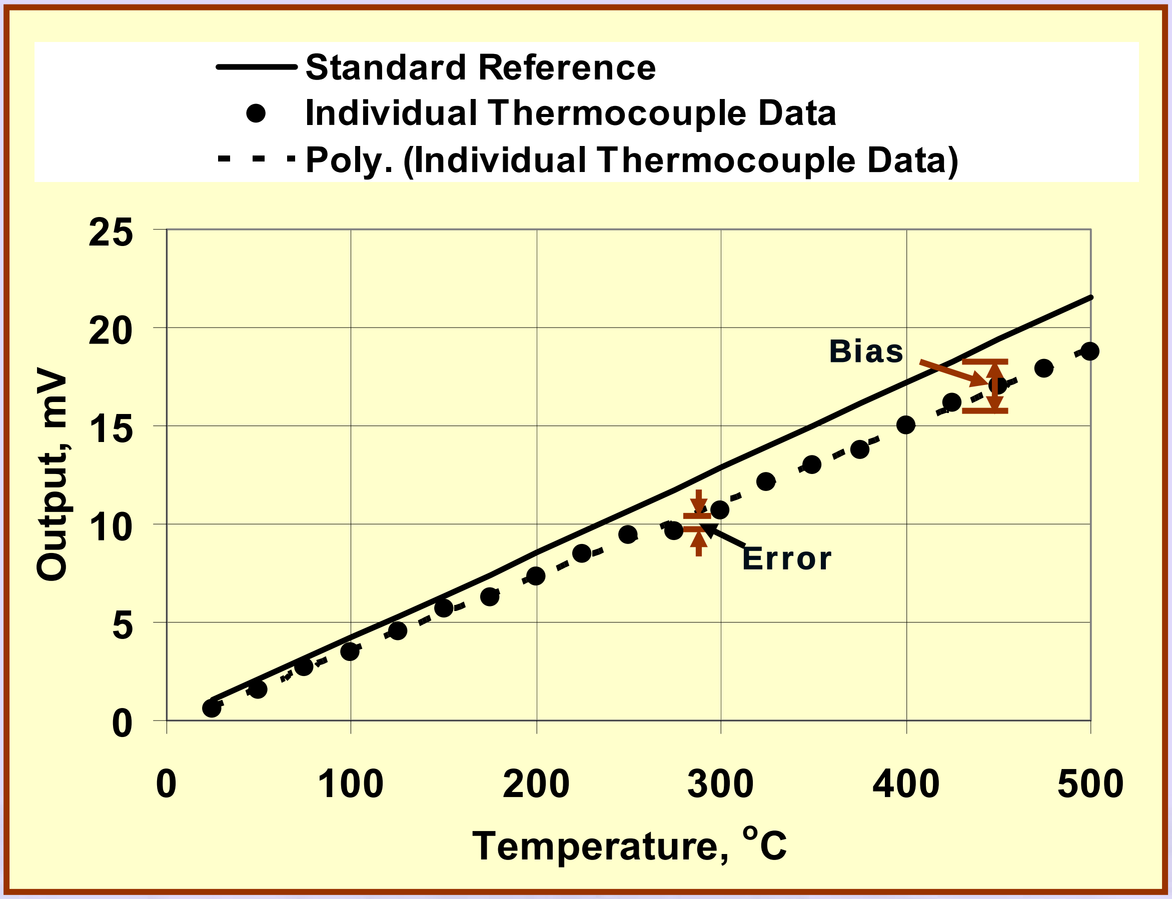

Figure 4 Example showing the presence of systematic and random errors in data.

The results shown in Figure 4 compare the response of a particular thermocouple (that measures temperature) and a standard thermocouple. The measurements are reported between room temperature (close to 20°C) and 500°C. That there is a systematic variation between the two is clear from the figure that shows the trend of the measured temperatures indicated by the particular thermocouple. The systematic error appears to vary with the temperature. The data points indicated by the full symbols appear also to hug the trend line. However the data points do not lie on it. This is due to random errors that are always present in any measurement. Actually the standard thermocouple would also have the random errors that are not indicated in the figure. We have deliberately shown only the trend line for the standard thermocouple.

Sub Module 1.3

3. Statistical analysis of experimental data

Statistical analysis and best estimate from replicate data:

- Let a certain quantity X be measured repeatedly to get

$$X_i,\ i=1,n \tag{1}$$

- Because of random errors these are all *different*.
- How do we find the best estimate X_b for the true value of X?
- It is reasonable to assume that the best value be such that the measurements are as precise as they can be!
- In other words, the experimenter is confident that he has conducted the measurements with the best care and he is like the skilled shooter in the target practice example presented earlier!
- Thus, we minimize the variance with respect to the best estimate X_b of X.
- Thus we minimize:

$$S = \sum_{i=1}^{n} \left[X_i - X_b\right]^2 \tag{2}$$