Module 2

Mechanics of Machining

Lesson
8
Machining forces and Merchant's Circle Diagram (MCD)

Instructional Objectives

At the end of this lesson, the student would be able to

(i) Ascertain the benefits and state the purposes of determining cutting forces
(ii) Identify the cutting force components and conceive their significance and role
(iii) Develop Merchant's Circle Diagram and show the forces and their relations
(iv) Illustrate advantageous use of Merchant's Circle Diagram

(i) Benefit of knowing and purpose of determining cutting forces.

The aspects of the cutting forces concerned :

- Magnitude of the cutting forces and their components
- Directions and locations of action of those forces
- Pattern of the forces : static and / or dynamic.

Knowing or determination of the cutting forces facilitate or are required for :

- Estimation of cutting power consumption, which also enables selection of the power source(s) during design of the machine tools
- Structural design of the machine – fixture – tool system
- Evaluation of role of the various machining parameters (process – V_C, s_o, t, tool – material and geometry, environment – cutting fluid) on cutting forces
- Study of behaviour and machinability characterisation of the work materials
- Condition monitoring of the cutting tools and machine tools.

(ii) Cutting force components and their significances

The single point cutting tools being used for turning, shaping, planing, slotting, boring etc. are characterised by having only one cutting force during machining. But that force is resolved into two or three components for ease of analysis and exploitation. Fig. 8.1 visualises how the single cutting force in turning is resolved into three components along the three orthogonal directions; X, Y and Z.

The resolution of the force components in turning can be more conveniently understood from their display in 2-D as shown in Fig. 8.2.

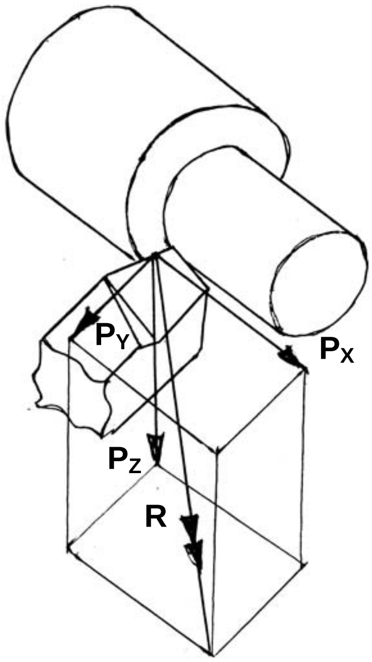


Fig. 8.1 *Cutting force R resolved into P_X, P_Y and P_Z*

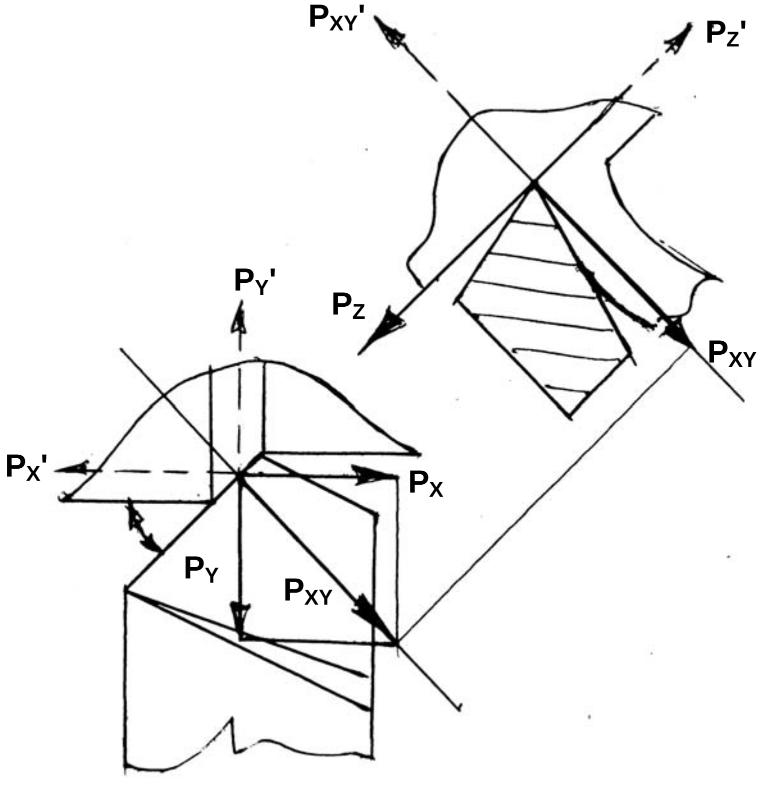


Fig. 8.2 *Turning force resolved into P_Z, P_X and P_Y*

The resultant cutting force, R is resolved as,

$$\overline{R} = \overline{P_Z} + \overline{P}_{XY} \quad (8.1)$$

and $$\overline{P}_{XY} = \overline{P}_X + \overline{P}_Y \quad (8.2)$$

where, $P_X = P_{XY}\sin\phi$ and $P_Y = P_{XY}\cos\phi$ (8.3)

where, P_Z = tangential component taken in the direction of Z_m axis
P_X = axial component taken in the direction of longitudinal feed or X_m axis
P_Y = radial or transverse component taken along Y_m axis.

In Fig. 8.1 and Fig. 8.2 the force components are shown to be acting on the tool. A similar set of forces also act on the job at the cutting point but in opposite directions as indicated by P_Z', P_{XY}', P_X' and P_Y' in Fig. 8.2

Significance of P_Z, P_X and P_Y

P_Z : called the main or major component as it is the largest in magnitude. It is also called power component as it being acting along and being multiplied by V_C decides cutting power ($P_Z.V_C$) consumption.
P_y : may not be that large in magnitude but is responsible for causing dimensional inaccuracy and vibration.
P_X : It, even if larger than P_Y, is least harmful and hence least significant.

Cutting forces in drilling

In a drill there are two main cutting edges and a small chisel edge at the centre as shown in Fig. 8.3.
The force components that develop (Fig. 8.3) during drilling operation are :

- a pair of tangential forces, P_{T1} and P_{T2} (equivalent to P_Z in turning) at the main cutting edges
- axial forces P_{X1} and P_{X2} acting in the same direction
- a pair of identical radial force components, P_{Y1} and P_{Y2}
- one additional axial force, P_{Xe} at the chisel edge which also removes material at the centre and under more stringent condition.

P_{T1} and P_{T2} produce the torque, T and causes power consumption P_C as,

$$T = P_T \times ½ (D) \quad (8.3)$$

and $$P_C = 2\pi TN \quad (8.4)$$

where, D = diameter of the drill
and N = speed of the drill in rpm.
The total axial force P_{XT} which is normally very large in drilling, is provided by

$$P_{XT} = P_{X1} + P_{X2} + P_{Xe} \quad (8.5)$$

But there is no radial or transverse force as P_{Y1} and P_{Y2}, being in opposite direction, nullify each other if the tool geometry is perfectly symmetrical.

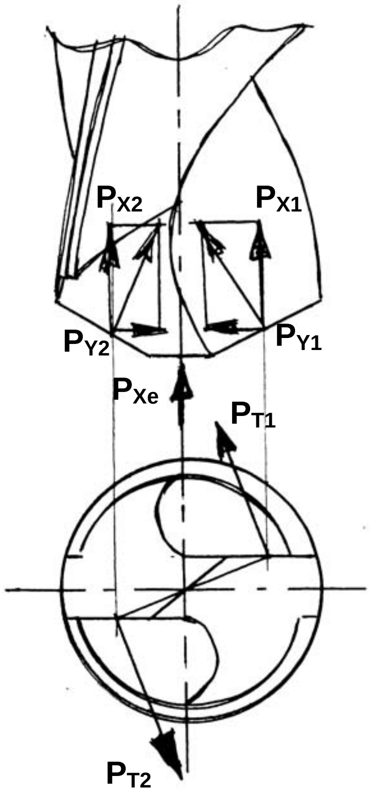


Fig. 8.3 *Cutting forces in drilling.*

Cutting forces in milling

The cutting forces (components) developed in milling with straight fluted slab milling cutter under single tooth engagement are shown in Fig. 8.4.
The forces provided by a single tooth at its angular position, ψ_I are :

- Tangential force P_{Ti} (equivalent to P_Z in turning)
- Radial or transverse force, P_{Ri} (equivalent to P_{XY} in turning)
- R is the resultant of P_T and P_R
- R is again resolved into P_Z and P_Y as indicated in Fig. 8.4 when Z and Y are the major axes of the milling machine.

Those forces have the following significance:

- o P_T governs the torque, T on the cutter or the milling arbour as

$$T = P_T \text{ x } D/2 \quad (8.5)$$

and also the power consumption, P_C as

$$P_C = 2\pi TN \quad (8.6)$$

where, N = rpm of the cutter.

The other forces, P_R, P_Z, P_Y etc are useful for design of the Machine – Fixture – Tool system.

In case of multitooth engagement;

Total torque will be $D/2.\sum P_{Ti}$ and total force in Z and Y direction will be $\sum P_Z$ and $\sum P_Y$ respectively.

One additional force i.e. axial force will also develop while milling by helical fluted cutter

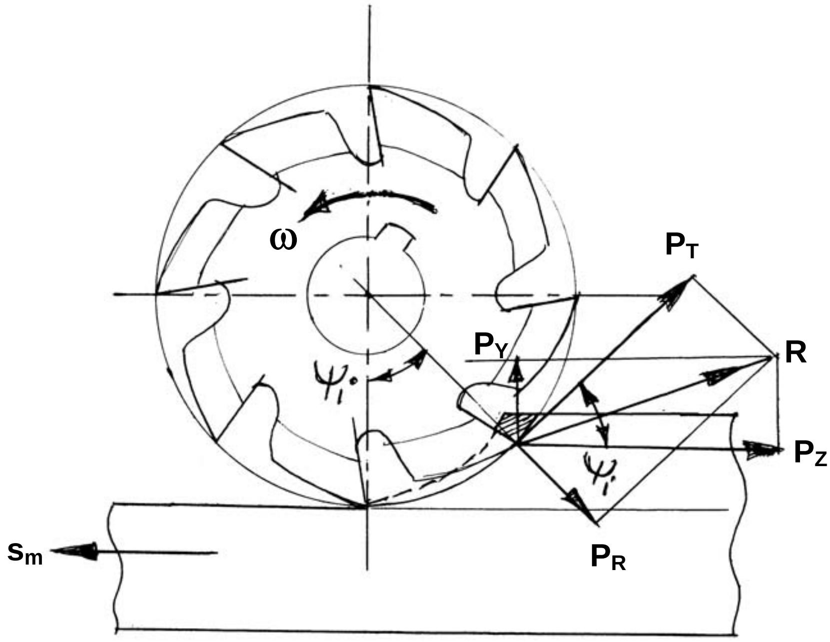


Fig. 8.4 *Cutting forces developed in plain milling (with single tooth engagement)*

(iii) Merchant's Circle Diagram and its use

In orthogonal cutting when the chip flows along the orthogonal plane, π_O, the cutting force (resultant) and its components P_Z and P_{XY} remain in the orthogonal plane. Fig. 8.5 is schematically showing the forces acting on a piece of continuous chip coming out from the shear zone at a constant speed. That chip is apparently in a state of equilibrium.

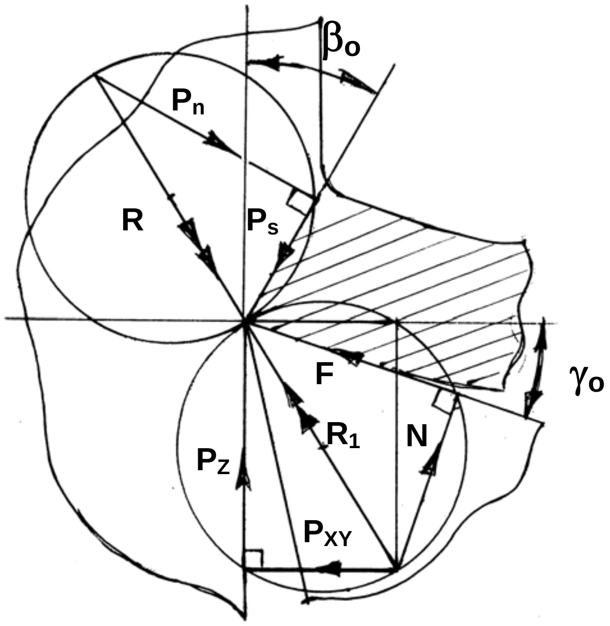


Fig. 8.5 *Development of Merchants Circle Diagram.*

The forces in the chip segment are :

- From job-side :
  - P_S – shear force and
  - P_n – force normal to the shear force

  where, $\overline{P_s} + \overline{P_n} = \overline{R}$ (resultant)
- From tool side :
  - $\overline{R_1} = \overline{R}$ (in state of equilibrium)
  - where $\overline{R_1} = \overline{F} + \overline{N}$
  - N = force normal to rake face
  - F = friction force at chip tool interface.

The resulting cutting force R or R_1 can be resolved further as

$$\overline{R_1} = \overline{P_Z} + \overline{P_{XY}}$$

where, P_Z = force along the velocity vector
and P_{XY} = force along orthogonal plane.

The circle(s) drawn taking R or R_1 as diameter is called Merchant's circle which contains all the force components concerned as intercepts. The two circles with their forces are combined into one circle having all the forces contained in that as shown by the diagram called Merchant's Circle Diagram (MCD) in Fig. 8.6

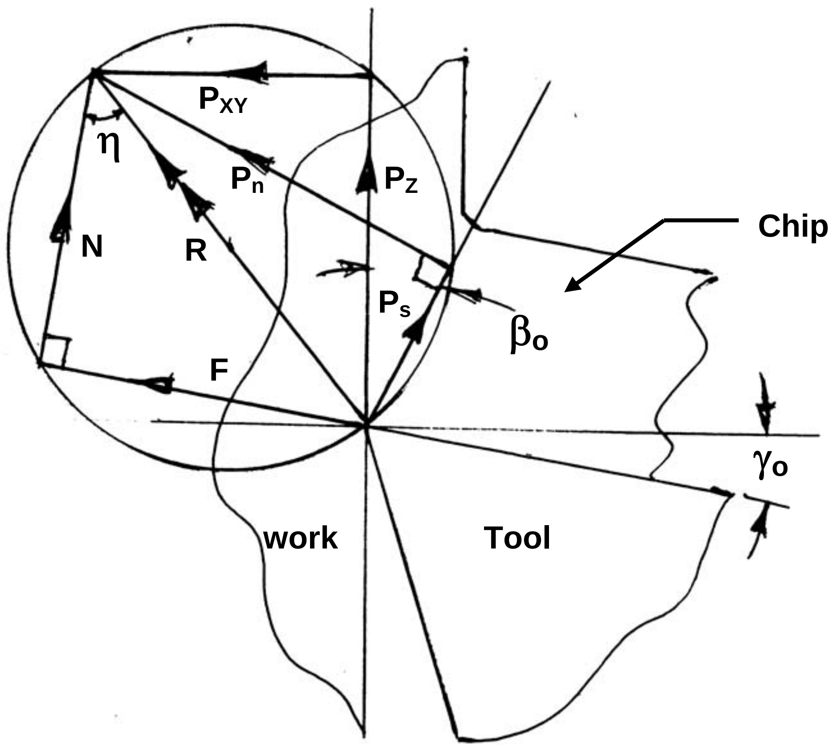


Fig. 8.6 *Merchant' s Circle Diagram with cutting forces.*

The significance of the forces displayed in the Merchant's Circle Diagram are :

P_S – the shear force essentially required to produce or separate the chip from the parent body by shear

P_n – inherently exists along with P_S

F – friction force at the chip tool interface

N – force acting normal to the rake surface

P_Z – main force or power component acting in the direction of cutting velocity

P_{XY} – $\overline{P}_X + \overline{P}_Y$

The magnitude of P_S provides the yield shear strength of the work material under the cutting condition.

The values of F and the ratio of F and N indicate the nature and degree of interaction like friction at the chip-tool interface. The force components P_X, P_Y, P_Z are generally obtained by direct measurement. Again P_Z helps in determining cutting power and specific energy requirement. The force components are also required to design the cutting tool and the machine tool.

(iv) Advantageous use of Merchant's Circle Diagram (MCD)

Proper use of MCD enables the followings :

- Easy, quick and reasonably accurate determination of several other forces from a few known forces involved in machining
- Friction at chip-tool interface and dynamic yield shear strength can be easily determined
- Equations relating the different forces are easily developed.

Some limitations of use of MCD

- Merchant's Circle Diagram(MCD) is valid only for orthogonal cutting
- by the ratio, F/N, the MCD gives apparent (not actual) coefficient of friction
- It is based on single shear plane theory.

The advantages of constructing and using MCD has been illustrated as by an example as follows ;

Suppose, in a simple straight turning under orthogonal cutting condition with given speed, feed, depth of cut and tool geometry, the only two force components P_Z and P_X are known by experiment i.e., direct measurement, then how can one determine the other relevant forces and machining characteristics easily and quickly without going into much equations and calculations but simply constructing a circle-diagram. This can be done by taking the following sequential steps :

- Determine P_{XY} from $P_X = P_{XY}\sin\phi$, where P_X and ϕ are known.
- Draw the tool and the chip in orthogonal plane with the given value of γ_o as shown in Fig. 8.4
- Choose a suitable scale (e.g. 100 N = 1 cm) for presenting P_Z and P_{XY} in cm
- Draw P_Z and P_{XY} along and normal to $\overline{V_C}$ as indicated in Fig. 8.6
- Draw the cutting force R as the resultant of P_Z and P_{XY}
- Draw the circle (Merchant's circle) taking R as diameter

- Get F and N as intercepts in the circle by extending the tool rake surface and joining tips of F and R
- Divide the intercepts of F and N by the scale and get the values of F and N
- For determining P_s (and P_n) the value of the shear angle β_o has to be evaluated from

  $$\tan\beta_o = \frac{\cos\gamma_o}{\zeta - \sin\gamma_o}$$

  where γ_o is known and ζ has to be obtained from

  $$\zeta = \frac{a_2}{a_1} \text{ where } a_1 = s_o\sin\phi$$

  s_o and ϕ are known and a_2 is either known, if not, it has to be measured by micrometer or slide calliper
- Draw the shear plane with the value of β_o and then P_s and P_n as intercepts shown in Fig. 8.6.
- Get the values of P_s and P_n by dividing their corresponding lengths by the scale
- Get the value of apparent coefficient of friction, μ_a at the chip tool interface simply from the ratio, $\mu_a = \frac{F}{N}$
- Get the friction angle, η, if desired, either from $\tan\eta = \mu_a$ or directly from the MCD drawn as indicated in Fig. 8.6.
- Determine dynamic yield shear strength (τ_s) of the work material under the cutting condition using the simple expression

  $$\tau_s = \frac{P_s}{A_s}$$

  where, A_s = shear area as indicated in Fig. 8.7

  $$= \frac{a_1 b_1}{\sin\beta_o} = \frac{ts_o}{\sin\beta_o}$$

  t = depth of cut (known)

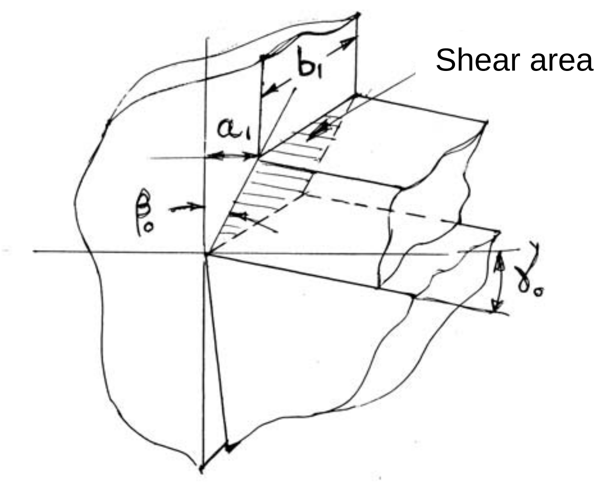


Fig. 8.7 *Shear area in orthogonal turning*

Evaluation of cutting power consumption and specific energy requirement

Cutting power consumption is a quite important issue and it should always be tried to be reduced but without sacrificing MRR.

Cutting power consumption, P_C can be determined from,

$$P_C = P_Z.V_C + P_X.V_f \tag{8.4}$$

where, V_f = feed velocity

$= Ns_o/1000$ m/min [N=rpm]

Since both P_X and V_f, specially V_f are very small, $P_X.V_f$ can be neglected and then $P_C \cong P_Z.V_C$

Specific energy requirement, which means amount of energy required to remove unit volume of material, is an important machinability characteristics of the work material. Specific energy requirement, U_s, which should be tried to be reduced as far as possible, depends not only on the work material but also the process of the machining, such as turning, drilling, grinding etc. and the machining condition, i.e., V_C, s_o, tool material and geometry and cutting fluid application.

Compared to turning, drilling requires higher specific energy for the same work-tool materials and grinding requires very large amount of specific energy for adverse cutting edge geometry (large negative rake).

Specific energy, U_s is determined from

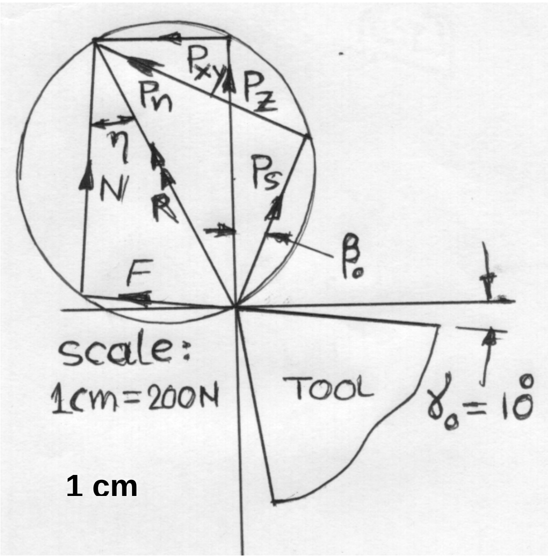
$$U_s = \frac{P_Z.V_C}{MRR} = \frac{P_Z}{ts_o}$$

Exercise - 8
Solution of some Problems

Problem 1

During turning a ductile alloy by a tool of $\gamma_o = 10^o$, it was found P_Z= 1000 N, P_X= 400 N, P_Y= 300 N and ζ = 2.5. Evaluate, using MCD, the values of F, N and μ as well as P_S and P_n for the above machining.

Solution :



- force, $P_{XY} = \sqrt{P_X^2 + P_Y^2}$ = $\sqrt{(400)^2 + (300)^2}$ = 500 N
- Select a scale: 1 cm=200N
- Draw the tool tip with γ_o =10^o
  In scale, P_Z=1000/200= 5 cm and P_{XY}=500/200=2.5cm

- Draw P_Z and P_{XY} in the diagram
- Draw R and then the MCD
- Extend the rake surface and have F and N as shown
- Determine shear angle, β_o

  tan β_o= cosγ_o / (ζ - sinγ_o)

  = cos10^o/(2.5 – sin10^o) = 0.42

  β_o= tan^{-1}(0.42) = 23^o
- Draw P_S and P_n in the MCD
- From the MCD, find F = 3x200 = 600 N; N = 4.6x200 = 920 N;

  μ = F/N = 600/920 = 0.67

  P_S = 3.4x200 = 680; P_n= 4.3x200 = 860 N

Problem 2

During turning a steel rod of diameter 160 mm at speed 560 rpm, feed 0.32 mm/rev. and depth of cut 4.0 mm by a ceramic insert of geometry

0^o, —10^o, 6^o, 6^o, 15^o, 75^o, 0 (mm)

The followings were observed :

P_Z =1600 N, P_X=800 N and chip thickness=1mm. Determine with the help of MCD the possible values of F, N, m_a, P_S, P_n, cutting power and specific energy consumption.

Solution

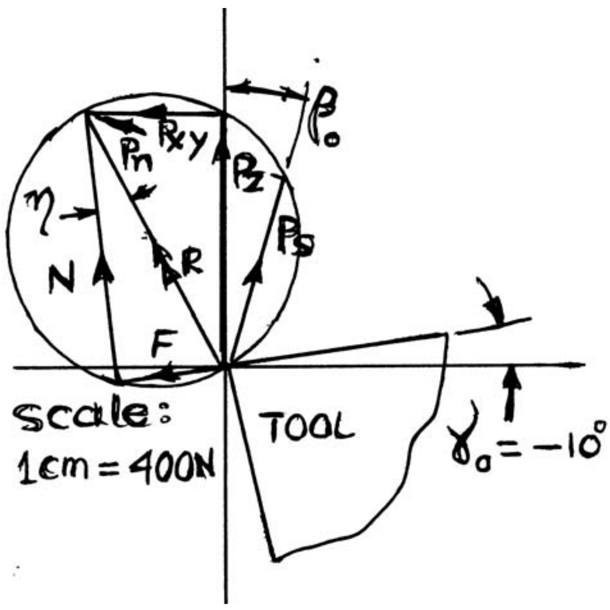
- P_{XY}=P_X/sinϕ = 800/sin75^o = 828 N
- Select a scale: 1cm = 400N
- Draw the tool tip with γ_o = -10^o
- Draw P_Z and P_{XY} in scale as shown
- Draw resultant and MCD

  shear angle, β_o

  tan β_o = cosγ_o / (ζ - sin γ_o)

  where, ζ = a_2/a_1 = $a_2/(s_o \sin\phi)$ = 3.2

  β_o = tan^{-1}(cos(-10^o))/{(3.2 - sin(-10^o))}= 16.27^o



- Draw P_S and P_n as shown
- Using the scale and intercepts determine
  F = 1.75xscale = 700 N
  N = 4.40xscale = 1760 N
  μ_a = F/N = 700/1760 = 0.43
  P_S = 3.0 x scale = 1200 N
  P_n = 3.3 x scale = 1320 N
- Cutting Power, P_C $P_C = P_Z.V_C$ where
  $V_C = \pi DN/1000 = \pi$ x160x560/1000 = 281.5 m/min
  So, P_C = 8 KW.
- Specific energy = $P_Z/(ts_o)$ = 1600/(4 x 0.32)= 1250 N-mm/mm^3

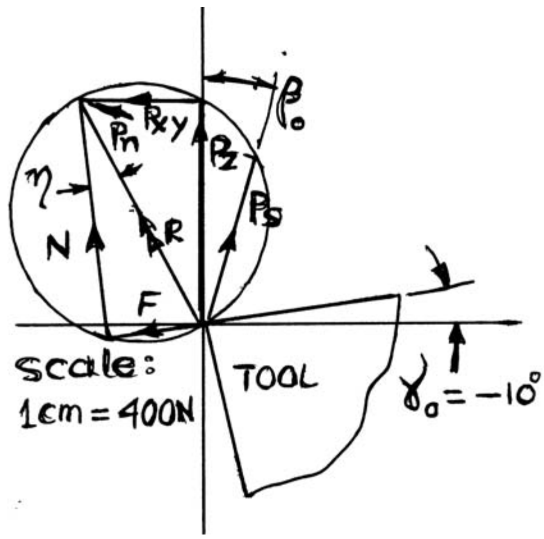
Problem 3

For turning a given steel rod by a tool of given geometry if shear force P_S, frictional force F and shear angle γ_o could be estimated to be 400N and 300N respectively, then what would be the possible values of P_X P_Y and P_Z?
[use MCD]

Solution:

- tool geometry is known. Let rake angle be γ_o and principal cutting edge angle be ϕ.
- Draw the tool tip with the given value of γ_o as shown.
- Draw shear plane using the essential value of β_o
- using a scale (let 1cm=400N) draw shear force P_S and friction force F in the respective directions.
- Draw normals on P_S and F at their tips as shown and let the normals meet at a point.
- Join that meeting point with tool tip to get the resultant force

- Based on resultant force R draw the MCD and get intercepts for P_Z and P_{xy}
- Determine P_Z and P_{xy} from the MCD
- P_Z= __ x scale = ____
- P_{xy} = __ x scale = ___
- $P_Y = P_{XY} \cos\phi$
- $P_X = P_{XY} \sin\phi$



Problem - 4

During shaping like single point machining/turning) a steel plate at feed, 0.20 mm/stroke and depth 4 mm by a tool of $\lambda = \gamma = 0^o$ and $\phi = 90^o$ P_Z and P_X were found (measured by dynamometer) to be 800 N and 400 N respectively, chip thickness, a_2 is 0.4 mm. From the aforesaid conditions and using Merchant's Circle Diagram determine the yield shear strength of the work material in the machining condition?

Solution

- It is orthogonal ($\lambda = 0^o$) cutting \ MCD is valid
- draw tool with $\gamma_o = 0^o$ as shown
- $P_{XY} = P_X/\sin\phi = 400/\sin 90^o = 400$ N
- Select a scale : 1 cm= 200N
- Draw P_Z and P_{XY} using that scale

  $P_Z = 800/200 = 4$ cm,

  $P_{XY} = 400/200 = 2$ cm
- Get R and draw the MCD
- Determine shear angle, β_o from

  $\tan\beta_o = \cos\gamma_o/(\zeta - \sin\gamma_o)$, $\gamma_o = 0^o$ and

  $\zeta = a_2/a_1$ $a_1 = (s_o \sin\phi) = 0.2 \times \sin 90^o = 0.2$

  $\beta_o = \tan^{-1}(0.2/0.4) = 26^o$
- Draw P_S along the shear plane and find $P_S = 2.5 \times 200 = 500$ N
- Now, $\tau_S = P_S/A_S$;

$A_S = (ts_O)/\sin\beta_o = 4 \times 0.2/\sin 26^0 = 1.82 \text{ mm}^2$

or, $\tau_S = 500/1.82$

$= 274.7 \text{ N/mm}^2$