Module
2
Mechanics of Machining

Lesson
4
Conversion of tool angles from one system to another

Instructional objectives

At the end of this lesson the students should be able to

(i) State the purposes of conversion of tool angles
(ii) Identify the four different methods of conversion of tool angles
(iii) Employ the graphical method for conversion of
- Rake angles
- clearance angles
- Cutting angles

From ASA to ORS and ORS to ASA systems
(iv) Convert rake angle and clearance angle from ORS to NRS
(v) Demonstrate tool angle's relationship in some critical conditions.

(i) Purposes of conversion of tool angles from one system to another

- To understand the actual tool geometry in any system of choice or convenience from the geometry of a tool expressed in any other systems
- To derive the benefits of the various systems of tool designation as and when required
- Communication of the same tool geometry between people following different tool designation systems.

(ii) Methods of conversion of tool angles from one system to another

- Analytical (geometrical) method: simple but tedious
- Graphical method – Master line principle: simple, quick and popular
- Transformation matrix method: suitable for complex tool geometry
- Vector method: very easy and quick but needs concept of vectors

(iii) Conversion of tool angles by Graphical method – Master Line principle.

This convenient and popular method of conversion of tool angles from ASA to ORS and vice-versa is based on use of Master lines (ML) for the rake surface and the clearance surfaces.

- **Conversion of rake angles**

The concept and construction of ML for the tool rake surface is shown in Fig. 4.1.

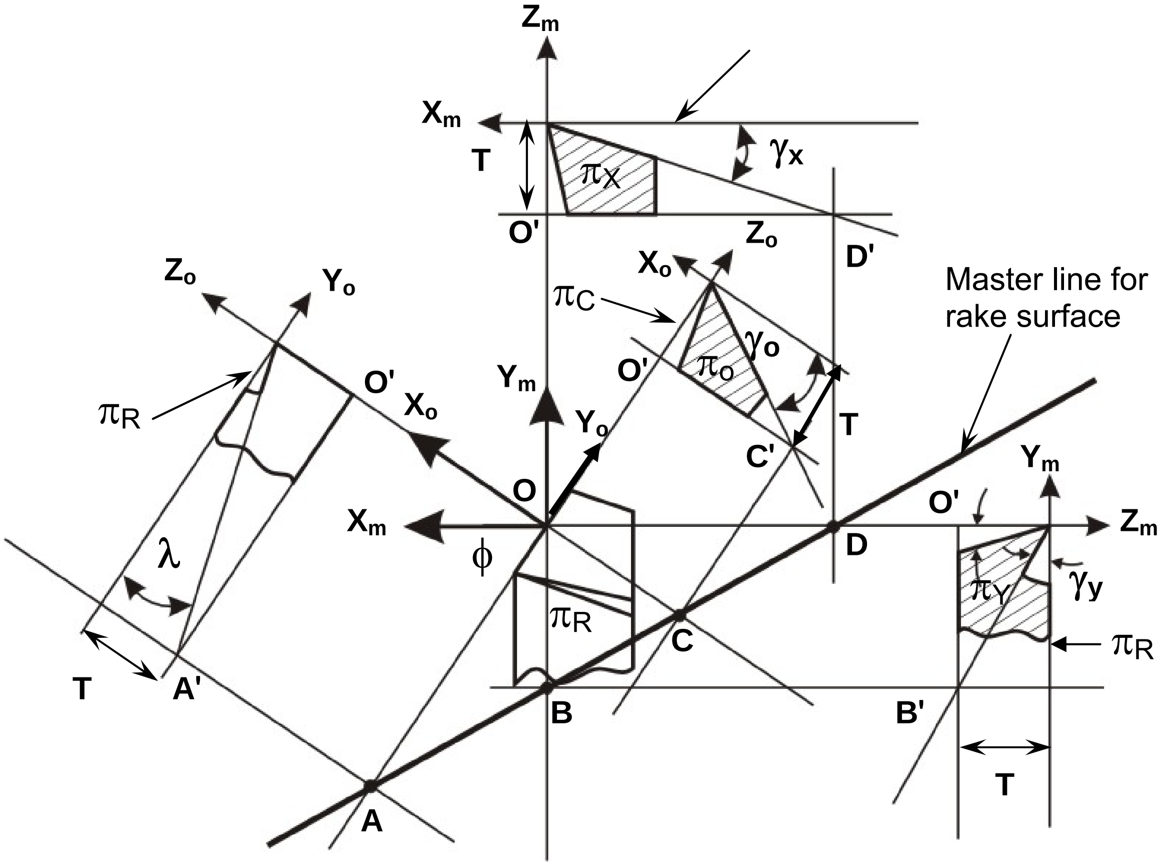


Fig. 4.1 *Master line for rake surface (with all rake angles: positive)*

In Fig. 4.1, the rake surface, when extended along π_X plane, meets the tool's bottom surface (which is parallel to π_R) at point D' i.e. D in the plan view. Similarly when the same tool rake surface is extended along π_Y, it meets the tool's bottom surface at point B' i.e., at B in plan view. Therefore, the straight line obtained by joining B and D is nothing but the line of intersection of the rake surface with the tool's bottom surface which is also parallel to π_R. Hence, if the rake surface is extended in any direction, its meeting point with the tool's bottom plane must be situated on the line of intersection, i.e., BD. Thus the points C and A (in Fig. 4.1) obtained by extending the rake surface along π_o and π_C respectively upto the tool's bottom surface, will be situated on that line of intersection, BD.
This line of intersection, BD between the rake surface and a plane parallel to π_R is called the **"Master line of the rake surface"**.

From the diagram in Fig. 4.1,

$$OD = T\cot\gamma_X$$

$$OB = T\cot\gamma_Y$$

$$OC = T\cot\gamma_o$$

$$OA = T\cot\lambda$$

Where, T = thickness of the tool shank.
The diagram in Fig. 4.1 is redrawn in simpler form in Fig. 4.2 for conversion of tool angles.

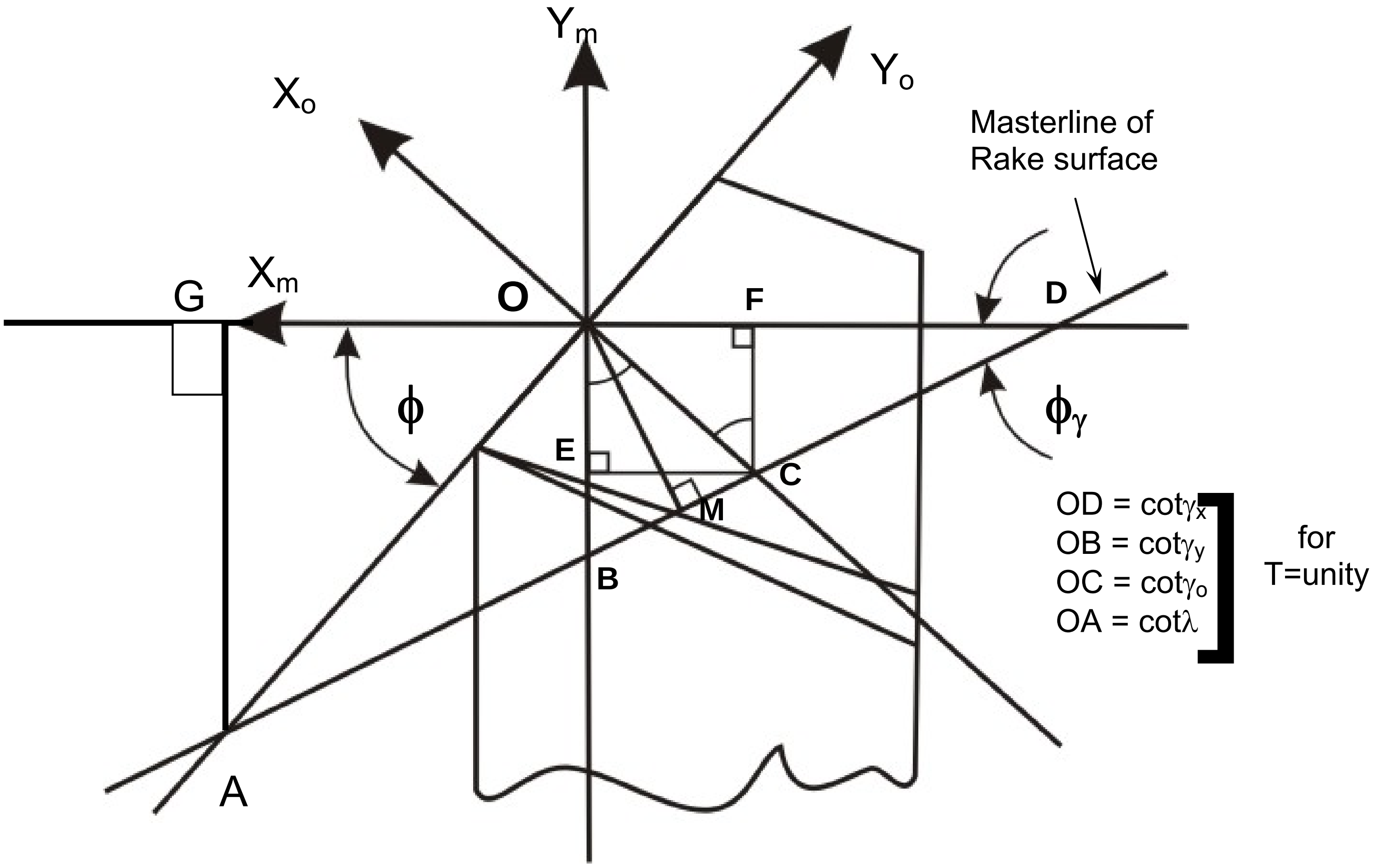


Fig. 4.2 *Use of Master line for conversion of rake angles.*

- **Conversion of tool rake angles from ASA to ORS**

γ_o and λ (in ORS) = f (γ_x and γ_y of ASA system)

$$\tan\gamma_o = \tan\gamma_x \sin\varphi + \tan\gamma_y \cos\varphi \quad (4.1)$$

and
$$\tan\lambda = -\tan\gamma_x \cos\varphi + \tan\gamma_y \sin\varphi \quad (4.2)$$

Proof of Equation 4.1:

With respect to Fig. 4.2,
Consider, ΔOBD = Δ OBC + Δ OCD
Or, ½ OB.OD = ½ OB.CE + ½ OD.CF
Or, ½ OB.OD = ½ OB.OCsinϕ + ½ OD.OCcosϕ
Dividing both sides by ½ OB.OD.OC,

$$\frac{1}{OC} = \frac{1}{OD}\sin\varphi + \frac{1}{OB}\cos\varphi$$

i.e. $\tan\gamma_o = \tan\gamma_x \sin\varphi + \tan\gamma_y \cos\varphi$ — Proved.

Similarly Equation 4.2 can be proved considering;

Δ OAD = Δ OAB + Δ OBD

i.e., ½ OD.AG = ½ OB.OG + ½ OB.OD

where, AG = OAsinϕ

and OG = OAcosϕ

Now dividing both sides by ½ OA.OB.OD,

$$\frac{1}{OB}\sin\varphi = \frac{1}{OD}\cos\varphi + \frac{1}{OA}$$

$$\tan\lambda = -\tan\gamma_x\cos\varphi + \tan\gamma_y\sin\varphi \quad \text{— Proved.}$$

The conversion equations 4.1 and 4.2 can be combined in a matrix form,

$$\begin{bmatrix}\tan\gamma_o \\ \tan\lambda\end{bmatrix} = \begin{bmatrix}\sin\phi & \cos\phi \\ -\cos\phi & \sin\phi\end{bmatrix}\begin{bmatrix}\tan\gamma_x \\ \tan\gamma_y\end{bmatrix} \tag{4.3}$$

(ORS) (ASA)

where, $\begin{bmatrix}\sin\phi & \cos\phi \\ -\cos\phi & \sin\phi\end{bmatrix}$ is the transformation matrix.

- Conversion of rake angles from ORS to ASA system

γ_x and γ_y (in ASA) = f(γ_o and $\lambda$of ORS)

$$\tan\gamma_x = \tan\gamma_o\sin\varphi - \tan\lambda\cos\varphi \tag{4.4}$$

$$\text{and} \quad \tan\gamma_y = \tan\gamma_o\cos\varphi + \tan\lambda\sin\varphi \tag{4.5}$$

The relations (4.4) and (4.5) can be arrived at indirectly using Equation 4.3.

By inversion, Equation 4.3 becomes,

$$\begin{bmatrix}\tan\gamma_x \\ \tan\gamma_y\end{bmatrix} = \begin{bmatrix}\sin\phi & -\cos\phi \\ \cos\phi & \sin\phi\end{bmatrix}\begin{bmatrix}\tan\gamma_o \\ \tan\lambda\end{bmatrix} \tag{4.6}$$

from which equation 4.4 and 4.5 are obtained.

The conversion equations 4.4 and 4.5 can also be proved directly from the diagram in Fig. 4.2

Hints

To prove equation 4.4, proceed by taking (from Fig. 4.2)

Δ OAD = Δ OAC + Δ OCD,

[involving the concerned angles γ_o, λ and γ_x i.e., OC, OA and OD]

And to prove Equation 4.5, proceed by taking

Δ OAC = Δ OAB + Δ OBC

[involving the concerned angles γ_o, λ and γ_y i.e., OC, OA and OB]

- Maximum rake angle (γ_{max} or γ_m)

The magnitude of maximum rake angle (γ_m) and the direction of the maximum slope of the rake surface of any single point tool can be easily derived from its geometry specified in both ASA or ORS by using the diagram of Fig. 4.2. The smallest intercept OM normal to the Master line (Fig. 4.2) represents γ_{max} or γ_m as

$OM = \cot \gamma_m$

Single point cutting tools like HSS tools after their wearing out are often resharpened by grinding their rake surface and the two flank surfaces.
The rake face can be easily and correctly ground by using the values of γ_m and the orientation angle, ϕ_γ (visualized in Fig. 4.2) of the Master line.

- Determination of γ_m and ϕ_γ from tool geometry specified in ASA system.

In Fig. 4.2,

Δ OBD = ½ OB.OD = ½ BD.OM

or, ½ OB.OD = ½ $\sqrt{OB^2 + OD^2}.OM$

Dividing both sides by ½ OB.OD.OM

$$\frac{1}{OM} = \sqrt{\frac{1}{OD^2} + \frac{1}{OB^2}}$$

or $$\tan\gamma_m = \sqrt{\tan^2\gamma_x + \tan^2\gamma_y} \qquad (4.7)$$

Again from Δ OBD

$$\tan\varphi_\gamma = \frac{OB}{OD}$$

or $$\phi_\gamma = \tan^{-}\left(\frac{\tan\gamma_x}{\tan\gamma_y}\right) \qquad (4.8)$$

- γ_m and ϕ_γ from tool geometry specified in ORS

Similarly from the diagram in Fig. 4.2, and taking Δ OAC, one can prove

$$\tan\gamma_m = \sqrt{\tan^2\gamma_o + \tan^2\lambda} \qquad (4.9)$$

$$\phi_\gamma = \phi - \tan^{-}\left(\frac{\tan\lambda}{\tan\gamma_o}\right) \qquad (4.10)$$

- **Conversion of clearance angles from ASA system to ORS and vice versa by Graphical method.**

Like rake angles, the conversion of clearance angles also make use of corresponding Master lines. The Master lines of the two flank surfaces are nothing but the dotted lines that appear in the plan view of the tool (Fig. 4.3).
The dotted line are the lines of intersection of the flank surfaces concerned with the tool's bottom surface which is parallel to the Reference plane π_R. Thus according to the definition those two lines represent the Master lines of the flank surfaces.
Fig. 4.4 shows the geometrical features of the Master line of the principal flank of a single point cutting tool.
From Fig. 4.4,

$OD = T\tan\alpha_x$

$OB = T\tan\alpha_y$

$OC = T\tan\alpha_o$

OA = Tcotλ where, T = thickness of the tool shank.

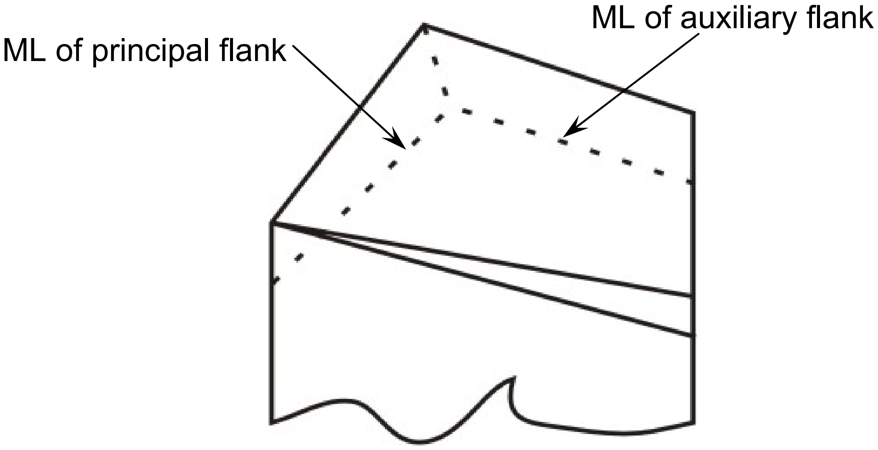


Fig. 4.3 *Master lines (ML) of flank surfaces.*

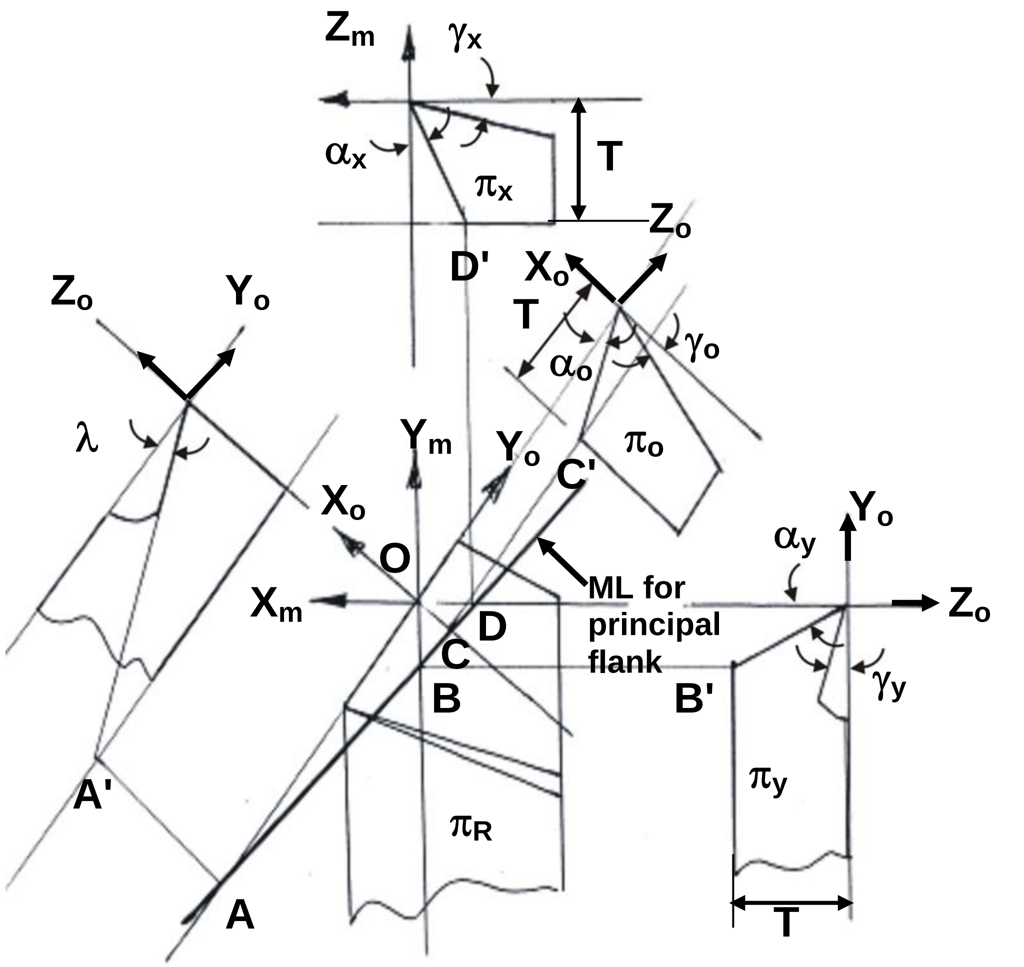


Fig. 4.4 *Master line of principal flank.*

The diagram in Fig. 4.4 is redrawn in simpler form in Fig. 4.5 for conversion of clearance angles.
The inclination angle, λ basically represents slope of the rake surface along the principal cutting edge and hence is considered as a rake angle. But λ appears in the analysis of clearance angles also because the principal cutting edge belong to both the rake surface and the principal flank.

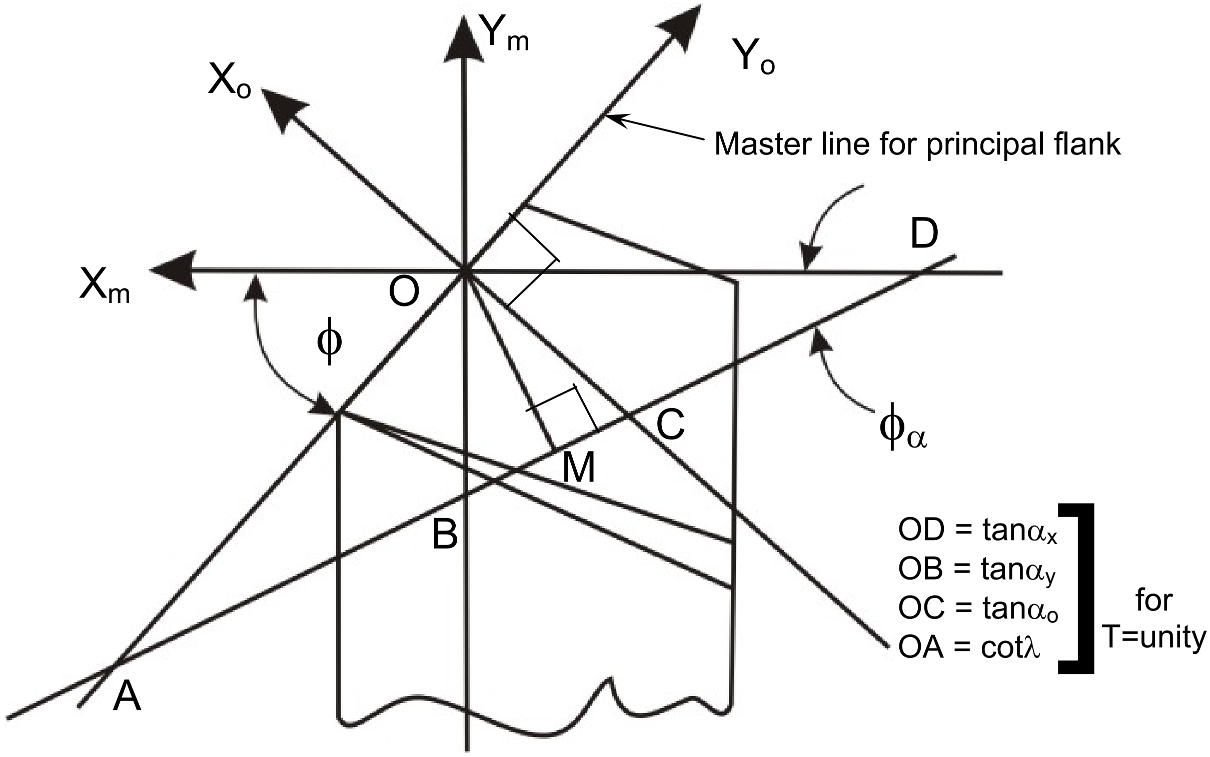


Fig. 4.5 *Use of Master line for conversion of clearance angles.*

- Conversion of clearance angles from ASA to ORS

Angles, α_o and λ in ORS = f(α_x and α_y in ASA system)
Following the same way used for converting the rake angles taking suitable triangles (in Fig. 4.2), the following expressions can be arrived at using Fig. 4.5:

$$\cot\alpha_o = \cot\alpha_x \sin\varphi + \cot\alpha_y \cos\varphi \qquad (4.11)$$

and $$\tan\lambda = -\cot\alpha_x \cos\varphi + \cot\alpha_y \sin\varphi \qquad (4.12)$$

Combining Equation 4.11 and 4.12 in matrix form

$$\begin{bmatrix} \cot\alpha_o \\ \tan\lambda \end{bmatrix} = \begin{bmatrix} \sin\phi & \cos\phi \\ -\cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} \cot\alpha_x \\ \cot\alpha_y \end{bmatrix} \tag{4.13}$$

- Conversion of clearance angles from ORS to ASA system

α_x and α_y (in ASA) = f(α_o and λ in ORS)
Proceeding in the same way using Fig. 4.5, the following expressions are derived

$$\cot\alpha_x = \cot\alpha_o \sin\varphi - \tan\lambda\cos\varphi \tag{4.14}$$

and $$\cot\alpha_y = \cot\alpha_o \cos\varphi + \tan\lambda\sin\varphi \tag{4.15}$$

The relations (4.14) and (4.15) are also possible to be attained from inversions of Equation 4.13 as indicated in case of rake angles.

- Minimum clearance, α_{min} or α_m

The magnitude and direction of minimum clearance of a single point tool may be evaluated from the line segment OM taken normal to the Master line (Fig. 4.5) as OM = tanα_m
The values of α_m and the orientation angle, ϕ_α (Fig. 4.5) of the principal flank are useful for conveniently grinding the principal flank surface to sharpen the principal cutting edge.
Proceeding in the same way and using Fig. 4.5, the following expressions could be developed to evaluate the values of α_m and ϕ_α

- o From tool geometry specified in ASA system

$$\cot\alpha_m = \sqrt{\cot^2\alpha_x + \cot^2\alpha_y} \tag{4.16}$$

and $$\phi_\gamma = \tan^{-1}\left(\frac{\cot\alpha_x}{\cot\alpha_y}\right) \tag{4.17}$$

- o From tool geometry specified in ORS

$$\cot\alpha_x = \sqrt{\cot^2\alpha_0 + \tan^2\lambda} \tag{4.18}$$

and $$\phi_\alpha = \phi - \tan^{-1}\left(\frac{\tan\lambda}{\cot\alpha_o}\right) \tag{4.19}$$

Similarly the clearance angles and the grinding angles of the auxiliary flank surface can also be derived and evaluated.

- o Interrelationship amongst the cutting angles used in ASA and ORS

The relations are very simple as follows:

ϕ (in ORS) = 90^o - ϕ_s (in ASA) (4.20)

and ϕ_1(in ORS) = ϕ_e (in ASA) (4.21)

(iv) Conversion of tool angles from ORS to NRS

The geometry of any single point tool is designated in ORS and NRS respectively as,

λ, γ_o, α_o, α_o', ϕ_1, ϕ, r (mm) – ORS
λ, γ_n, α_n, α_n', ϕ_1, ϕ, r (mm) – NRS

The two methods are almost same, the only difference lies in the fact that γ_o, α_o and α_o' of ORS are replaced by γ_n, α_n and α_n' in NRS.
The corresponding rake and clearance angles of ORS and NRS are related as ,

$$\tan\gamma_n = \tan\gamma_o\cos\lambda \qquad (4.22)$$

$$\cot\alpha_n = \cot\alpha_o\cos\lambda \qquad (4.23)$$

and $$\cot\alpha_n' = \cot\alpha_o'\cos\lambda' \qquad (4.24)$$

The equation 4.22 can be easily proved with the help of Fig. 4.6.

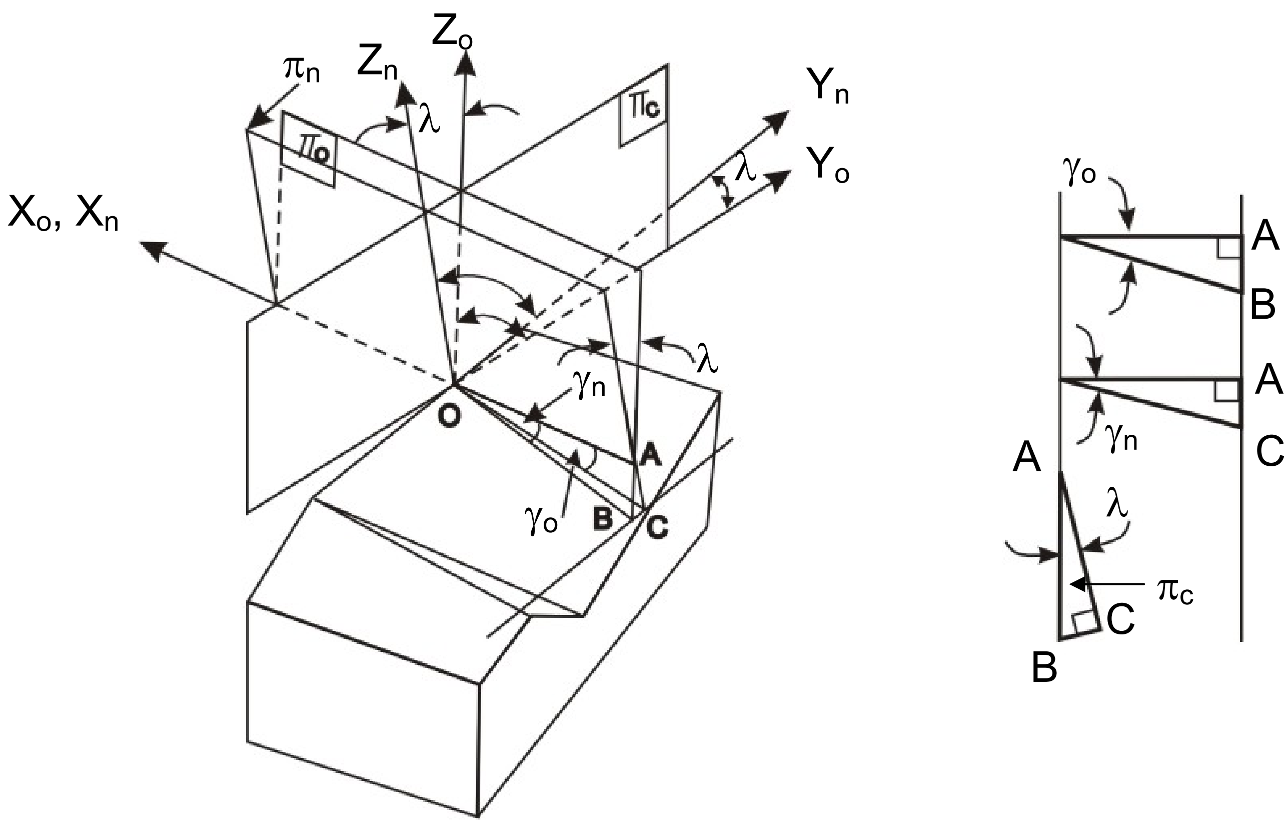


***Fig. 4.6** Relation between normal rake (γ_n) and orthogonal rake (γ_o)*

The planes π_o and π_n are normal to Y_o and Y_n (principal cutting edge) respectively and their included angle is λ when π_o and π_n are extended below OA (i.e. π_R) they intersect the rake surface along OB and OC respectively.
Therefore,

$\angle AOB = \gamma_o$
$\angle AOC = \gamma_n$
where, $\angle BAC = \lambda$

Now $\quad AC = AB\cos\lambda$

Or, $\quad OA\tan\gamma_n = (OA\tan\gamma_o)\cos\lambda$

So, $\quad \mathbf{\tan\gamma_n = \tan\gamma_o\cos\lambda}$ $\qquad$ proved

The equation (4.23) relating α_n and α_o can be easily established with the help of Fig. 4.7.

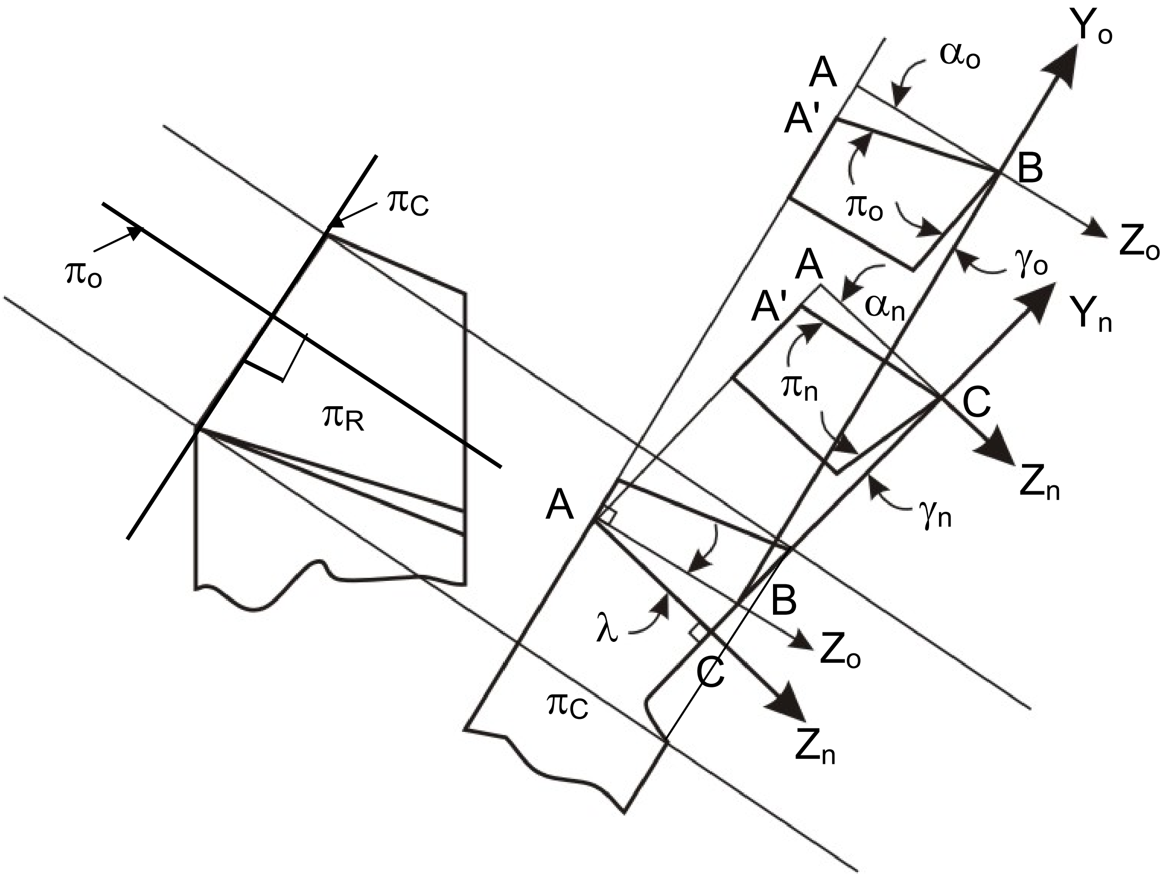


Fig. 4.7 Relation between normal clearance, α_n and orthogonal clearance, α_o

From Fig. 4.7,

$$AC = AB\cos\lambda$$

Or $AA'\cot\alpha_n = AA'\cot\alpha_o\cos\lambda$

$\therefore \quad \mathbf{\cot\alpha_n = \cot\alpha_o\cos\lambda}$ $\qquad$ proved

Similarly it can be proved,

$$\cot\alpha_n' = \cot\alpha_o'\cos\lambda'$$

where λ' is the inclination angle of the auxiliary cutting edge.

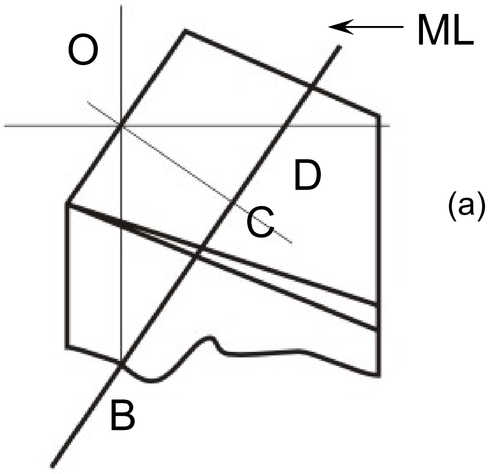
(v) Tool geometry under some critical conditions

- Configuration of Master lines (in graphical method of tool angle conversion) for different tool geometrical conditions.

The locations of the points 'A', 'B', 'C', 'D' and 'M' along the ML will be as shown in Fig. 4.2 when all the corresponding tool angles have some positive values. When any rake angle will be negative, the location of the corresponding point will be on the other side of the tool.

Some typical configurations of the Master line for rake surface and the corresponding geometrical significance are indicated in Fig. 4.8.

Configuration of ML **Tool geometry**

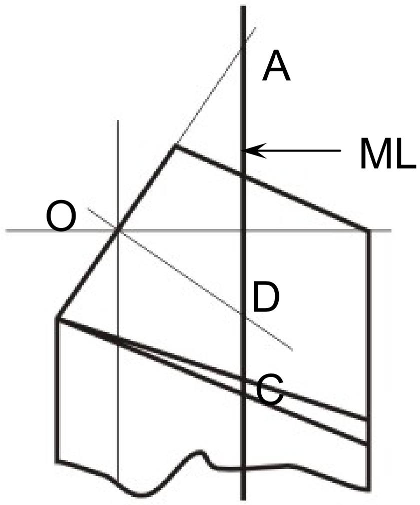


(a)

for ML parallel to π_C

γ_x = positive
γ_y = positive
γ_o = positive
λ = 0
$\gamma_m = \gamma_o$

A at infinity

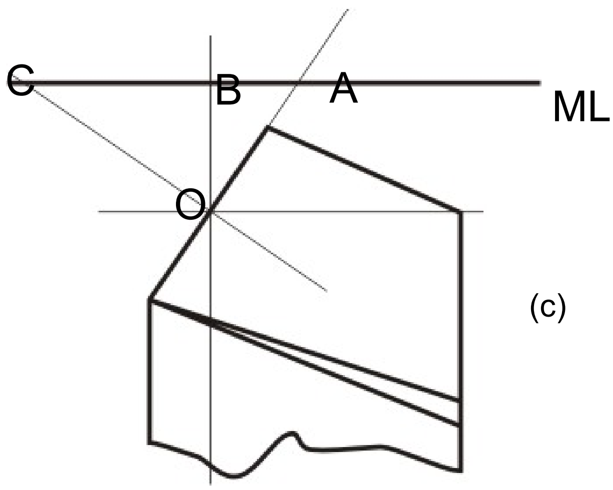


(b)

for ML parallel to π_Y

γ_x = positive
γ_y = 0
γ_o = positive
λ = negative
$\gamma_m = \gamma_x$

B at infinity



(c)

for ML parallel to π_X

γ_x = 0
γ_y = negative
γ_o = negative
λ = negative
$|\gamma_m| = |\gamma_y|$

Fig. 4.8 Tool geometry and Master line (rake face) in some typical conditions.

- Tool angles' relations in some critical conditions

From the equations correlating the cutting tool angles, the following critical observations are made:

- When $\phi = 90^o$; $\gamma_x = \gamma_o$ for $\pi_X = \pi_o$
- When $\lambda = 0$; $\gamma_n = \gamma_o$
  $\alpha_n = \alpha_o$
- When $\lambda=0$ and $\phi = 90^o$; $\gamma_n=\gamma_o=\gamma_x$ pure orthogonal cutting
  $(\pi_N=\pi_o=\pi_X)$

Exercise – 4

A. Quiz test

Select the correct answer from the given four options

1. The master line for the rake surface of the turning tool of geometry : -
   10^o, 0^o, 8^o, 6^o, 15^o, 30^o, 0.1 (inch)
   (a) machine longitudinal plane
   (b) machine transverse plane
   (c) cutting plane
   (d) orthogonal plane

2. If the approach angle of a turning tool be 30^o, the value of its principal cutting edge angle will be
   (a) O deg.
   (b) 30^o deg.
   (c) 60^o deg.
   (d) 90^o deg.

3. The value of side rake of the turning tool of geometry : -
   0^o, 10^o, 8^o, 6^o, 20^o, 60^o, 0 (mm) will be
   (a) 0^o deg.
   (b) 10^o deg.
   (c) 8^o deg.
   (d) 6^o deg.

4. The values of orthogonal clearance and normal clearance of a turning tool will be same if,
   (a) $\phi=0$
   (b) $\alpha_X = \alpha_Y$

(c) $\lambda = 0$
(d) none of the above

5. The angle between orthogonal plane and normal plane of a turning tool is
   (a) γ_o
   (b) ϕ
   (c) γ_n
   (d) λ

B. Problem

1. Determine the values of normal rake of the turning tool whose geometry is designated as : 10^o, -10^o, 8^o, 6^o, 15^o, 30^o, 0 (inch)?

2. Determine the value of side clearance of the turning tool whose geometry is specified as 0^o, -10^o, 8^o, 6^o, 20^o, 60^o, 0 (mm) ?

Solutions of Exercise – 4

A. Quiz test

1 – (a)
2 – (c)
3 – (b)
4 – (c)
5 – (d)

B. Problems

Ans. 1

Tool geometry given :

10^o, -10^o, 8^o, 6^o, 15^o, 30^o, 0 (inch)
γ_y, γ_x, α_y, α_x, ϕ_e, ϕ_s, r – ASA

$\tan\gamma_n = \tan\gamma_o \cos\lambda$
where,

$\tan\gamma_o = \tan\gamma_x \sin\phi + \tan\gamma_y \cos\phi$
$= \tan(-10^o)\sin(90^o - 30^o) + \tan(10^o)\cos(90^o - 30^o)$
$= -0.065$

So, $\gamma_o = -3.7^o$

And $\tan\lambda = -\tan\gamma_x \cos\phi + \tan\gamma_y \sin\phi$
$= -\tan(-10^o)\cos(90^o - 30^o) + \tan(10^o)\sin(90^o - 30^o)$
$= 0.2408$

So, $\lambda = 13.54^o$

$\tan\gamma_n = \tan\gamma_o \cos\lambda = \tan(-3.7)\cos(13.54) = -0.063$

So, $\gamma_n = -3.6^o$ Ans.

Ans. 2

Tool geometry given : 0^o, -10^o, 8^o, 6^o, 20^o, 60^o, 0 (mm)

λ, γ_o, α_o, α_o', ϕ_1, ϕ, r (mm)

$$\cot\alpha_x = \cot\alpha_o \sin\phi - \tan\lambda\cos\phi$$
$$= \cot 8.\sin 60 - \tan 0.\cos 60$$
$$= \cot 8.\sin 60$$
$$= 6.16$$

So, $\alpha_x = 9.217^o$ Ans.