MULTI-LEAF SPRINGS

Multi-leaf springs are widely used for automobile and rail road suspensions. It consists of a series of flat plates, usually of semi- elliptical shape as shown in fig. 4.20. The leaves are held together by means of two U-bolts and a centre clip. Rebound clips are provided to keep the leaves in alignment and prevent lateral shifting of the plates during the operation. The longest leaf, called the master leaf, is bent at both ends to form the spring eye. At the center, the spring is fixed to the axle of the car. Multi- leaf springs are provided with one or two extra full length leaves in addition to the master leaf. These extra full-length leaves are stacked between the master leaf and the graduated-length leaves. The extra full-length are provided to support the transverse shear force.

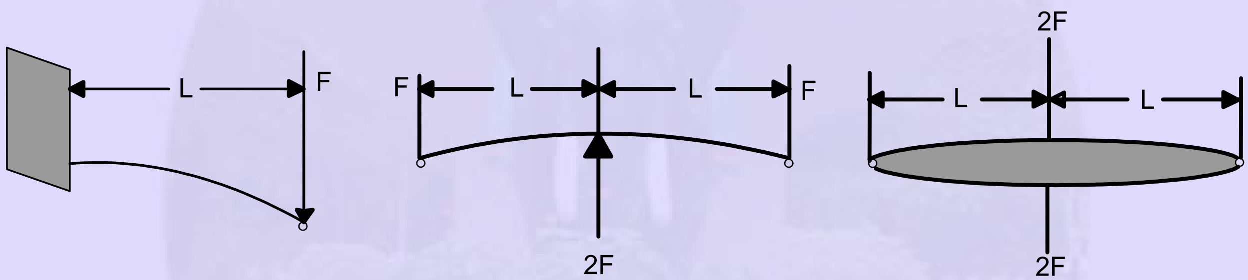


Figure 4.20

For the purpose of analysis, the leaves are divided into two groups namely master leaf along with graduated-length leaves forming one group and extra full-length leaves forming the other. The following notations are used in the analysis:

n_f = number of extra full-length leaves

n_g =number of graduated-length leaves including master leaf

n= total number of leaves

b= width of each leaf (mm)

t= thickness of each leaf (mm)

L=length of the cantilever or half the length of semi- elliptic spring (mm)

F= force applied at the end of the spring (N)

F_f=portion of F taken by the extra full-length leaves (N)

F_g=portion of F taken by the graduated-length leaves (N)

The group of graduated-length leaves along with the master leaf can be treated as a triangular plate, as shown in fig. 4.21. In this case, it is assumed that the individual leaves are separated and the master leaf placed at the centre. The second leaf is cut longitudinally into two halves, each of width (b/2) and placed on each side of the master leaf.

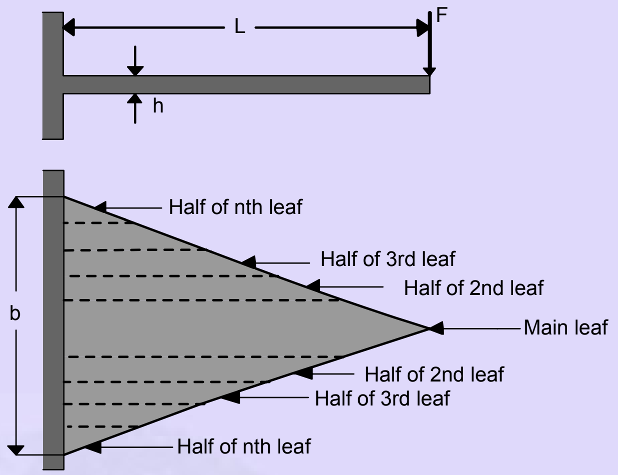


Figure 4.21

A similar procedure is repeated for other leaves

The resultant shape is approximately a triangular plate of thickness t and a maximum width at the support as ($n_g b$). The bending stress in the plate, which is uniform throughout, is given by

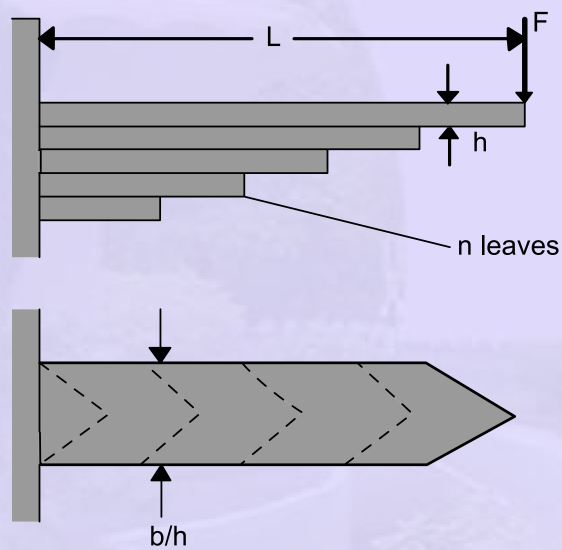


Figure 4.22

$$\left(\sigma_b\right)_g = \frac{M_b y}{I} = \frac{\left(F_g L\right)(t/2)}{\left[\frac{1}{12}\left(n_g b\right)(t^3)\right]}$$ a

$$\left(\sigma_b\right)_g = \frac{6F_g L}{n_g b t^2}$$

It can be proved that the deflection δ_g at the load point of the triangular plate is given by

$$\delta_g = \frac{F_g L^3}{2EI_{max}} = \frac{F_g L^3}{2E\left[\frac{1}{12}\left(n_g b\right)(t^3)\right]} \qquad b$$

$$\delta_g = \frac{6F_g L^3}{En_g bt^3}$$

Similarly, the extra full length leaves can be treated as a rectangular plate of thickness t and uniform width ($n_f b$), as shown in Fig 4.22 The bending stress at the support is given by

$$(\sigma_b)_f = \frac{M_b y}{I} = \frac{(F_f L)(t/2)}{\left[\frac{1}{12}(n_f b)(t^3)\right]} \qquad c$$

$$(\sigma_b)_f = \frac{6F_f L}{n_f bt^2}$$

The deflection at the load point is given by

$$\delta_f = \frac{F_f L^3}{2EI} = \frac{F_f L^3}{2E\left[\frac{1}{12}\left(n_f b\right)(t^3)\right]}$$

$$\delta_g = \frac{4F_f L^3}{En_f bt^3} \qquad d$$

$$\delta_g = \delta_f$$

$$\frac{6F_g L^3}{En_g bt^3} = \frac{4F_f L^3}{En_f bt^3}$$

$$\text{or} \qquad \frac{F_g}{F_f} = \frac{2n_g}{3n_f} \qquad e$$

Also

$$F_g + F_f = F \qquad \text{(f)}$$

From Eqs(e) and (f),

$$F_f = \frac{3n_f F}{\left(3n_f + 2n_g)\right)}$$

$$F_f = \frac{2n_g F}{\left(3n_f + 2n_g)\right)}$$

Substituting these valued in Eqs(a) and (c),

$$\left(\sigma_b\right)_g = \frac{12FL}{\left(3n_f + 2n_g\right)bt^2}$$

$$\left(\sigma_b\right)_f = \frac{18FL}{\left(3n_f + 2n_g\right)bt^2} \qquad \text{h}$$

It is seen from the above equations that bending stresses in full-length leaves are 50% more than those in graduated length leaves. The deflection at the end of the spring is determined from Eqs(b) and (h). It is given by

$$\delta = \frac{12FL^3}{\left(3n_f + 2n_g\right)Ebt^3}$$

Multi-leaf springs are designed using load stress and load deflection equations. The standard dimensions for the width and thickness of the leaf section are as follows:

Nominal thickness (mm): 3.2, 4.5, 5, 6, 6.5, 7, 7.5 8,9, 10,11,12,14, and 16.

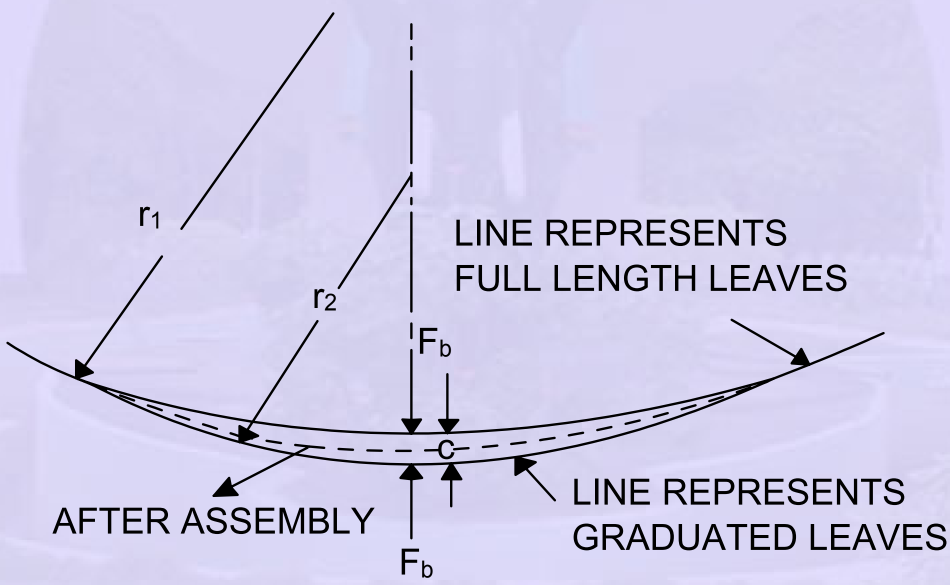
Nominal width (mm) 32, 40, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100 and 125.

Figure 4.3.4

The leaves are usually made of steels, 55Si2Mn9-, 50Cr1 or 50Cr1V23. The plates are hardened and tempered. The factor of safety based on the yield strength is between 2 to 2.5 for the automobiles suspension.

Nipping Of Leaf Springs

As discussed in the previous section, the stresses in extra full length leaves are 50% more than the stresses in graduated –length leaves. One of the methods of equalizing the stresses in different leaves is to pre-stress the spring. The pre-stressing is achieved by bending the leaves to different radii of curvature, before they are assembled with the centre clip. As shown in Figure the full-length leaf is given a greater radius of curvature than the adjacent leaf.



The radius of curvature decreases with shorter leaves. The initial gap C between the extra full-length leaf and the graduated-length leaf before the assembly is called a nip. Such pre-stressing, achieved by a difference in radii of curvature, is known as nipping. Nipping is common in automobile suspension springs.

Rewriting Eqs(a) and (c) of the previous section,

$$\left(\sigma_b\right)_g = \frac{6F_g L}{n_g bt^2}$$

$$\left(\sigma_b\right)_f = \frac{6F_f L}{n_f bt^2}$$

Assuming that pre-stressing results in stress- equalization,

$$\left(\sigma_b\right)_g = \left(\sigma_b\right)_f$$

From(a) and (c),

$$\frac{F_g}{F_f} = \frac{n_g}{n_f} \qquad \text{i}$$

Also, $$P_g + P_f = P \qquad \text{ii}$$

Solving Eqs(i) and (ii),

$$F_g = \frac{n_g F}{n}$$

$$F_f = \frac{n_f F}{n} \qquad \text{iii}$$

$$n = n_g + n_f \qquad \text{iv}$$

Where

Rewriting Eqs (b) and (d) of the previous section,

$$\delta_g = \frac{6F_g L^3}{E n_g bt^3}$$

$$\delta_f = \frac{4F_f L^3}{E n_f bt^3}$$

Under the maximum force P, the deflection of graduated-length leaves will exceed the deflection of extra full length leaves by an amount equal to the initial nip C.

$$C = \frac{6F_g L^3}{En_g bt^3} = \frac{4F_f L^3}{En_f bt^3}$$

Substituting (iii) and (iv) in the above equation,

$$C = \frac{2FL^3}{Enbt^3}$$

The initial pre-load P_i required to close the gap C between the extra full-length leaves an graduated-length leaves is determined by considering the initial deflection of leaves.

Under the action of pre-load P_i

$$C = \left(\delta_g\right)_i + \left(\delta_f\right)_i$$

$$\frac{2FL^3}{Enbt^3} = \frac{6\left(F_i/2\right)L^3}{En_g bt^3} = \frac{4\left(F/2_i\right)L^3}{En_f bt^3} \qquad \text{iv}$$

$$F_i = \frac{2n_g n_f F}{n\left(3n_f + 2n_g\right)}$$

Or,

The resultant stress in the extra full-length leaves is obtained by superimposing the stresses due to initial pre-load P_i and the external force P. From Eq.(c).

$$\left(\sigma_b\right)_f = \frac{6\left(P_f - 0.5P_i\right)L}{n_g bt^2}$$

Substituting Eq (g) of the previous section and Eq. in the above expression, we get

$$\left(\sigma_b\right)_f = \frac{6FL}{nbt^2}$$

Since the stresses are equal in all leaves, the above expression is written as

$$\sigma_b = \frac{6FL}{nbt^2}$$

The deflection of the multi-leaf spring due to the external force P is the same as the given by above equation.