Energy considerations

Kinetic energy is absorbed during slippage of a clutch and this energy appears as heat.

The clutch or brake operation is completed at the instance in which the two angular velocities θ1 and θ2 become equal. Let the time required for the entire operation be t1, then,

$$t_1 = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T(I_1 + I_2)}$$

This is derived by writing the equations of motion involving inertia

i.e

$$I_1 \ddot{\theta}_1 = -T \qquad \dot{\theta}_1 = -\frac{T}{I_1} t + \omega_1$$

$$I_2 \ddot{\theta}_2 = -T \qquad \dot{\theta}_2 = -\frac{T}{I_2} t + \omega_2$$

$$\dot{\theta} = \dot{\theta}_1 - \dot{\theta}_2 = -\frac{T}{I_1} t + \omega_1 - \left(\frac{T}{I_2} t + \omega_2\right)$$

$$= \omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right) t$$

from which $t = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T(I_1 + I_2)}$ as at the instance of completion of clutching operation ω_1-ω_2 = 0

Assuming the torque to be constant, the rate of energy dissipation during the operation is then,

$$U = T\dot{\theta} = T\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right).t\right]$$

The total energy dissipated during the clutching operation or braking cycle is obtained by integrating the above equation from t=0 to t = t1 . The result can be summed up as,

$$E = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

$$U = T\dot{\theta} = T\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right).t\right]$$

$$E = \int_0^{t_1} u\,dt = T\int_0^{t_1}\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right)t\right]dt$$

$$E = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

Thus the energy absorbed during clutch slip is a function of the magnitude of the inertia and the angular velocities only. This energy compared to the brake energy may be negligible. Heat dissipation and temperature rise are governed by the same equations presented during brakes. To contain the temperature rise when very frequent clutching operations, wet clutches rather than dry clutches are often use

WORKED OUT EXAMPLE 1

Design an automotive plate clutch to transmit a torque of 550 N-m. The coefficient of friction is 0.25 and the permissible intensity of pressure is 0.5 N/mm2. Due to space limitations, the outer diameter of the friction disc is fixed as 250 mm.

Using the uniform wear theory, calculate:

The inner diameter of the friction disc

the spring force required to keep the clutch in engaged position

Solution: As noted the friction disc of the automotive clutch is fixed between the flywheel on one side and the pressure plate on the other. The friction lining is provided on both sides of the friction disc.

Therefore two pairs of contacting surfaces-one between the fly wheel and the friction disc and the other between the friction disc and the pressure plate. Therefore, the torque transmitted by one pair of contacting surfaces is (550/2) or 275 N-m

$$(M_t)_f = \pi\mu p_a r_i \left(r_o^2 - r_i^2\right)$$
$$\left(275\times10^3\right) = \pi(0.25)(0.5)r_i\left(125^2 - r_i^2\right)$$

From the Eqr $8r_i\left(125^2 - r_i^2\right) = 5602254$

Rearranging the terms, we have

The above equation is solved by the trial and error method. It is a cubic equation, with following three roots:

(i) ri = 87.08 mm

(ii) ri = 56.145 mm

(iii) ri =-143.23 mm

Mathematically, all the three answer are correct. The inner radius cannot be negative. As a design engineer, one should select the inner radius as 87.08 mm, which results in a minimum area of friction lining compared with 56.145. For minimum cost of friction lining.

r_i=87 mm

Actuating force needed can be determined using the equation

$$F_a = 2\pi p_a r_i\left(r_o - r_i\right) = 2\pi(0.5)(87)\left(125-87\right) = 10390.28\text{N}$$

WORKED OUT EXAMPLE 2

A multiple-disc wet clutch is to be designed for transmitting a torque of 85 N.m. Space restriction limit the outside disk diameter to 100 mm. Design values for the molded friction material and steel disks to be used are f=0.06(wet) and pmax =1400 kPa. Determine appropriate values for the disc inside diameter, the total number of discs, and the clamping force.

Solution

Known: A multiple – disc with outside disc diameter, $d_0 \leq$ 100 mm,

dynamic friction coefficient, f=0.06(wet)

and maximum disc allowable pressure, p_{max} =1400 kPa,

To transmits a torque, T= 85 N.m

Find: Determine the disc inside diameter d_i, the total number of disks N, and the clamping force F_a.

Decisions and Assumptions

Use the largest allowable outside disc diameter, d_o=100 mm (r_o =50 mm).

Select r_i =29 mm (based on the optimum d/D ratio of 0.577)

The coefficient of friction f is a constant.

The wear rate is uniform at the interface.

The torque load is shared equally among the disc.

Design Analysis:

Using design equation for torque under constant wear gives

$$N = T / \left[\pi p_{max} r_i f \left(r_o^2 - r_i^2 \right) \right] = 6.69$$

Since N must be an even integer, use N= 8. It is evident that this requires a total of 4+5, or nine discs, remembering that the outer disks have friction surfaces on one side only. 3. With no other changes, this will give a clutch that is over designed by a factor of 8/6.69= 1.19. Possible alternatives include (a) accepting the 19 percent over design, (b) increasing ri, (c) decreasing ro, and (d) leaving both radii unchanged and reducing both pmax and F by a factor of 1.19

4. With the choice of alternative d, the clamping force is computed to be just sufficient to produce the desired torque:

$$T = Ff\left(\frac{r_o + r_i}{2}\right)N = 85\ \text{N.m} = F(0.06)\left(\frac{0.050 + 0.029}{2}\,\text{m}\right)8,$$

$$F = 4483\ \text{N}$$

Rounding up the calculated value of F, we

Find that the final proposed answers are (a) inside diameter= 58 mm, (b) clamping force= 4500 N and (C) a total of nine discs.