Stressing other than fully reversed loadings.

Quite frequently it is necessary to determine the strength of parts corresponding to stress situations other than complete reversals. Many times in design the stresses fluctuate without passing through zero. Some of the stress time relationships and the components of stresses involved with such situations and the relations among them will be discussed now.

One type is *zero-to-max-to zero*, where a part which is carrying no load is then subjected to a load, and later, the load is removed, so the part goes back to the no-load condition. An example of this type of loading is a chain used to haul logs behind a tractor.

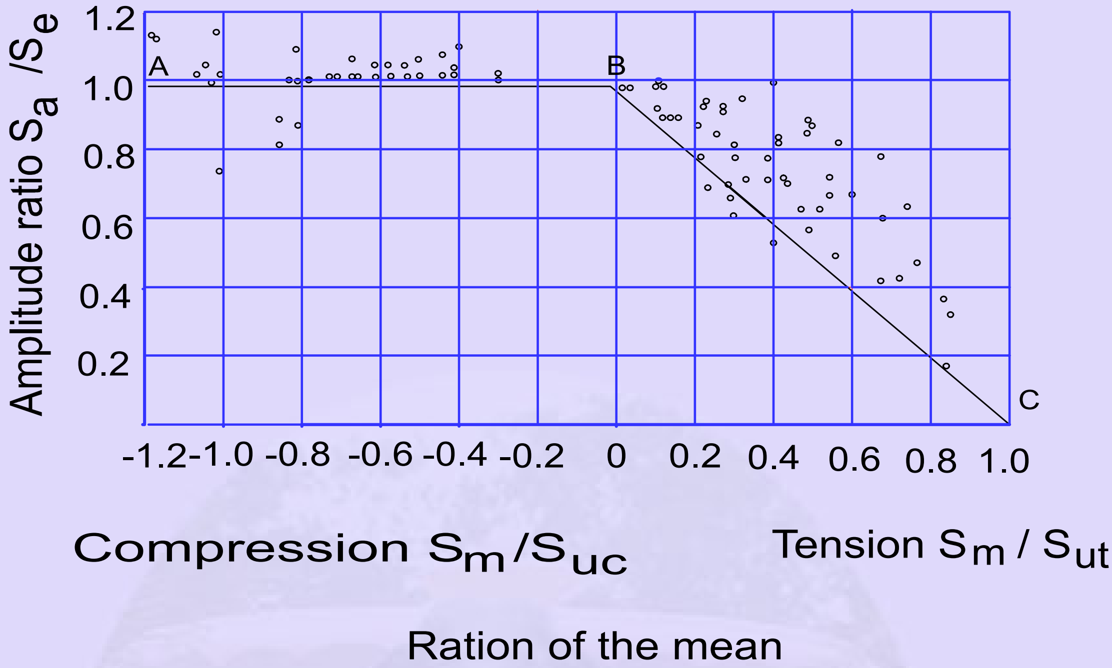
Another type of fatigue loading is a *varying load superimposed on a constant load*. The suspension wires in a railroad bridge are an example of this type. The wires have a constant static tensile load from the weight of the bridge, and an additional tensile load when a train is on the bridge. For such type of stressing how to proceed will be looked now.

Cyclic Stressing

As the name implies, the induced stresses vary in some pattern with time. This can be due to variation in the applied load itself or because of the conditions of use as seen earlier. Let us assume that the pattern of such a variation is sinusoidal. Then the following are the basic terminology associated with variable stresses. The definitions included here are elementary. They are introduced for clarity and convenience.

Maximum stress: σ_{max}

The largest or highest algebraic value of a stress in a stress cycle. Positive for tension



Minimum stress: σ_{max}

The smallest or lowest algebraic value of a stress in a stress cycle. Positive for tension.

Nominal stress: σ_{nom}

As obtained or calculated from simple theory in tension, bending and torsion neglecting geometric discontinuities

$$\sigma_{nom} = F/A \text{ or } M/Z \text{ or } T.r/J$$

$$\text{Hence } \sigma_{max} = F_{max}/ A \text{ or } M_{max}/Z \text{ or } T_{max.}r/J_p$$

$$\sigma_{min} = Fmin/ A \text{ etc}$$

Mean stress (Mid range stress) : σ_m The algebraic mean or average of the maximum and minimum stress in one cycle.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress range: σ_r The algebraic difference between the maximum and minimum stress in one cycle.

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

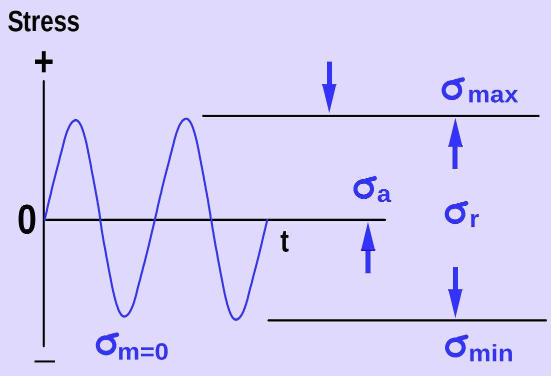
Stress Amplitude: σ_a Half the value of the algebraic difference between the maximum and minimum stress in one cycle or half the value of the stress range.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_r}{2}$$

Types of Variations

(a) (Completely)Reversible stressing:

Stress variation is such that the mean stress is zero; Same magnitude of maximum and minimum stress, one in tension and the other in compression .Now for Completely reversible loading $\sigma_m = \sigma_{max} = \sigma_{min}$; R = - 1 and A = 0

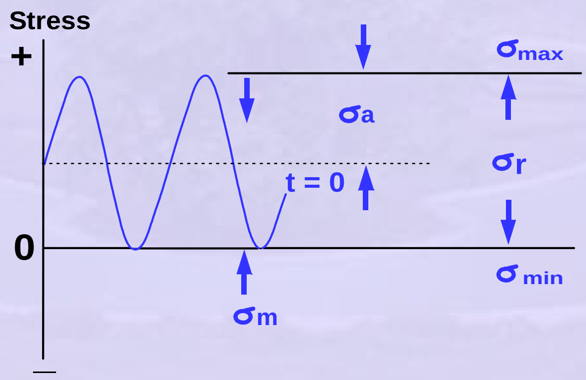


(a) Fully reversed

(b) Repeated stressing:

Stress variation is such that the minimum stress is zero. Mean and amplitude stress have the same value for repeated loading

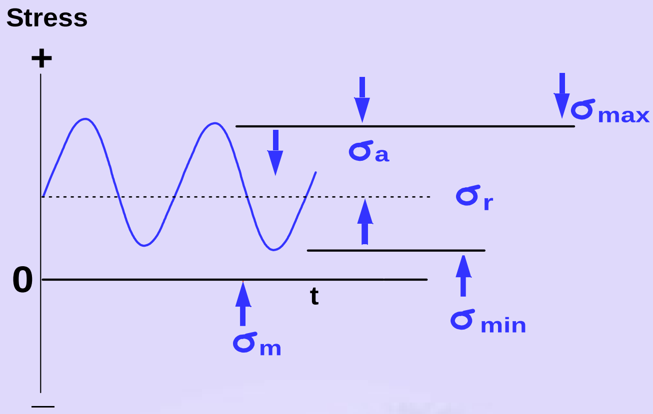
$$\sigma_{min} = 0$$
$$\sigma = \sigma_a = \sigma_{max}/2$$
$$R = 0 \text{ and } A = 1$$



(b) Repeated

(c) Fluctuating stressing:

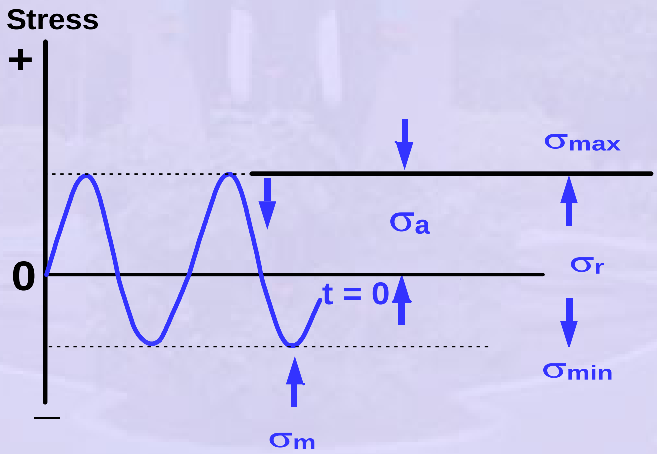
Both minimum and maximum stresses are positive and mean stress also being positive (tensile)



(b) Flutuating

(d) Alternating stressing:

Positive maximum stress and negative minimum stress; mean stress is generally positive but can also be negative.



General Cyclic Loading – Influencing Parameters

What are the important parameters to characterize a given cyclic loading history such as the typical ones highlighted above?

Note that the following parameters are common to all such types of variations

Stress Range: $\Delta\sigma = \sigma_{max} - \sigma_{min}$

Stress amplitude: $\sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

Mean stress: $\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min})$

Stress ratio: $R = \frac{\sigma_{min}}{\sigma_{max}}$

Recall the following relations from the earlier discussions

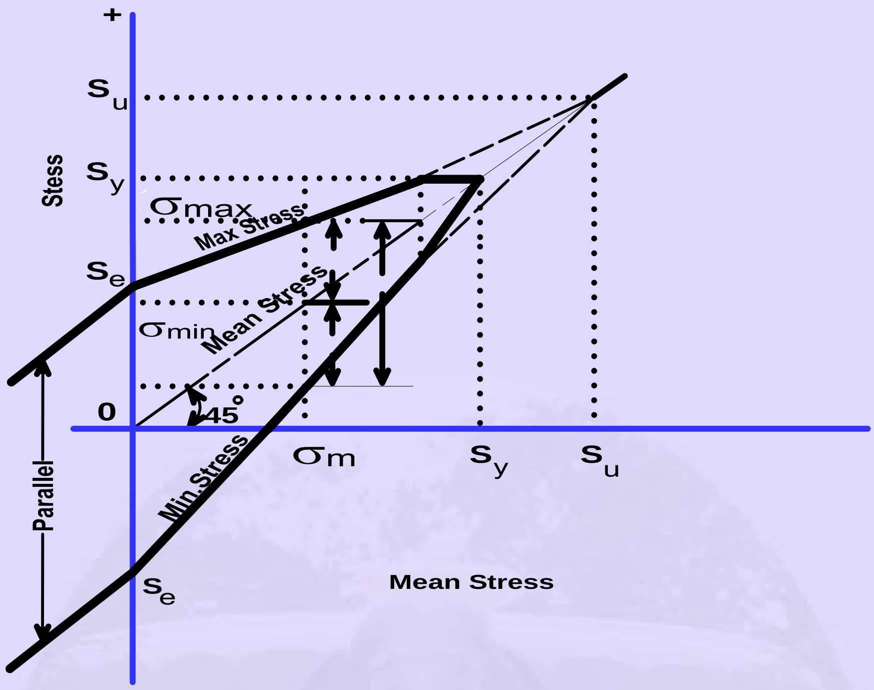
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_m = \frac{\sigma_{max} - \sigma_{min}}{2}$$

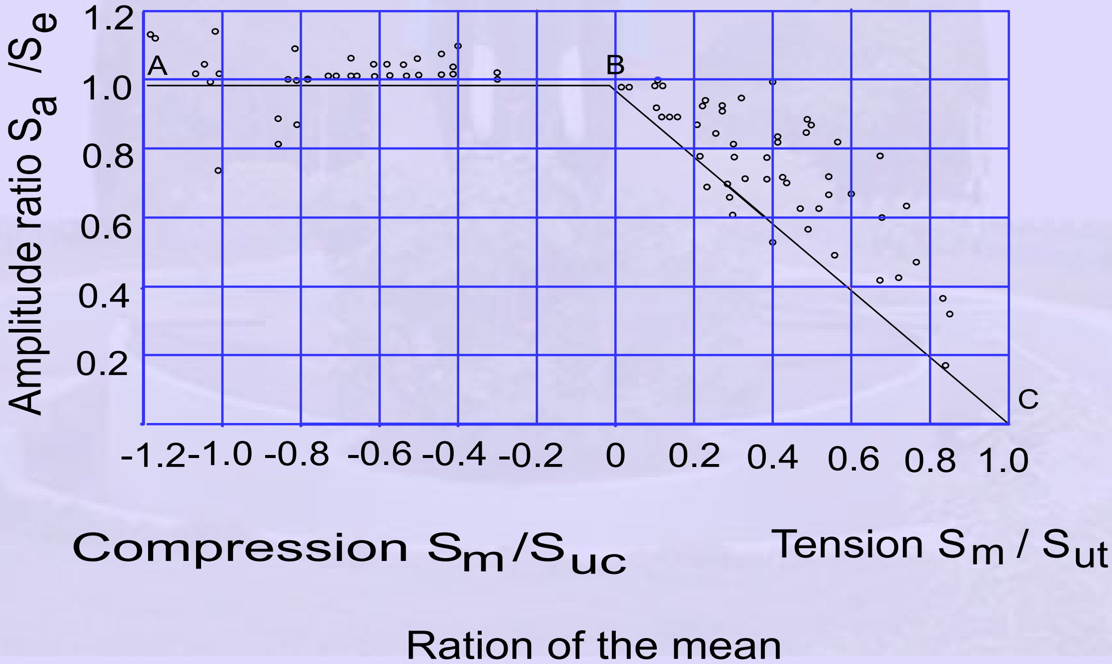
$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad A = \frac{\sigma_a}{\sigma_m}$$

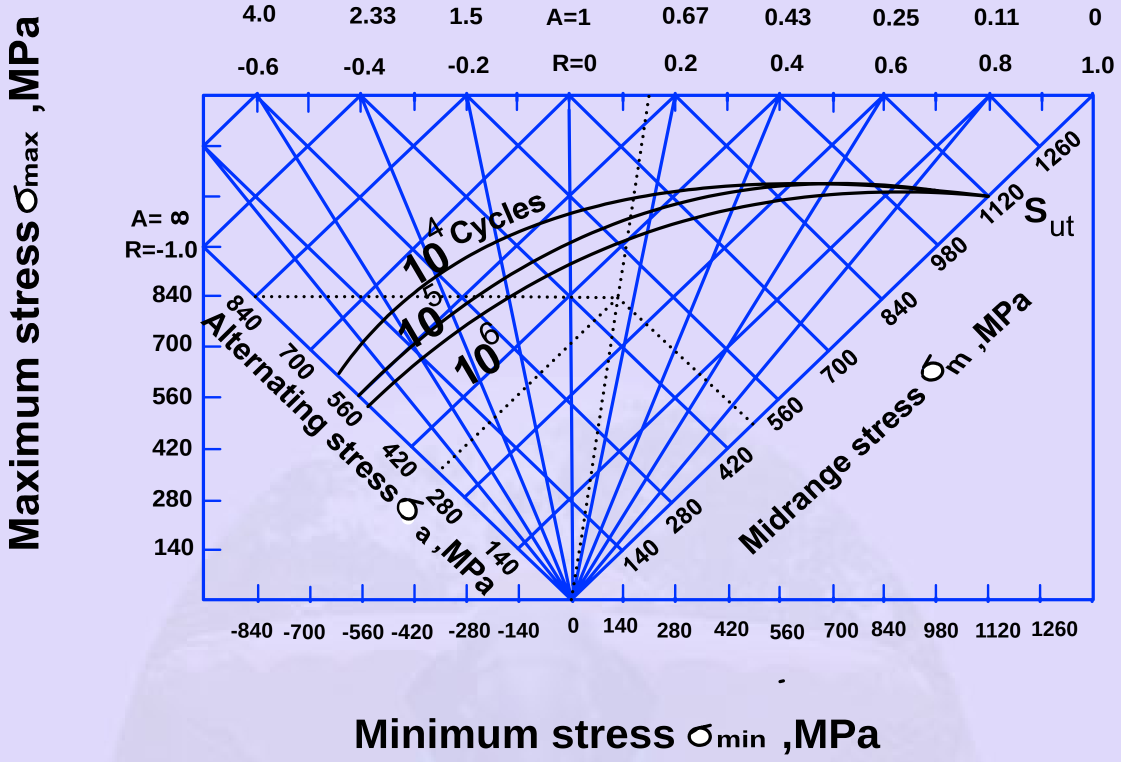
What is the effect of such variations on the Fatigue Strength?

Some typical types of variation in the cyclic stressing of materials have been highlighted All such and several other types of variations can be bounded by two main parameters the variable component of the stress or the stress amplitude and the mean component of stress or the mean stress. The effect of stress amplitude is already noted in the S-N diagram. Now let us note the effect of the mean stress.

By varying both the mean stress and the stress amplitude, or the alternating component, we can learn some thing about the fatigue resistance of parts when subjected to such situations. Three methods of plotting the results of such tests are in general use and are shown in figures below

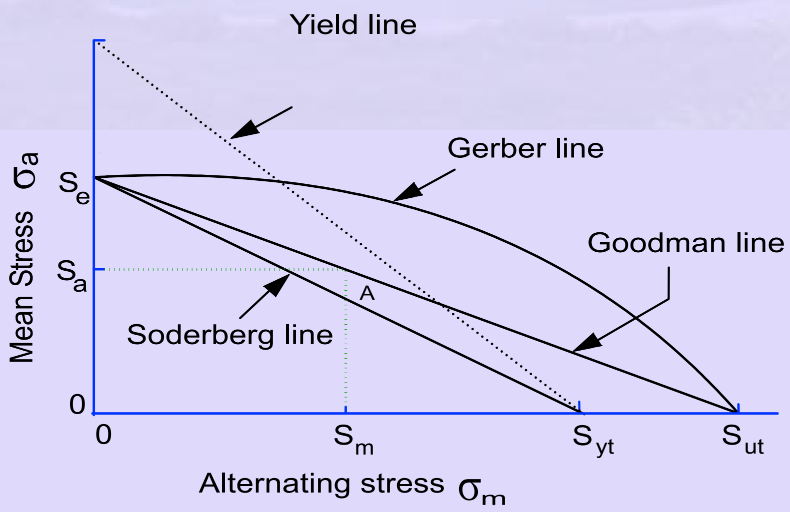

+
Su
Sy
Se
Stess
σmax
Max Stress
σmin
Mean Stress
0
45°
Min.Stress
Parallel
σm
Sy
Su
Se
Mean Stress


Amplitude ratio Sa /Se
1.2
1.0
0.8
0.6
0.4
0.2
A
B
C
-1.2 -1.0 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1.0
Compression Sm/Suc
Tension Sm / Sut
Ration of the mean



It is evident from the above figures that the presence of mean stress reduces the magnitude of variable component or the stress amplitude that can be sustained before failure. The higher the magnitude of mean stress the lower is the magnitude of amplitude stress that can be sustained. However note that if the nature of mean stress is compressive, then it has no effect on the magnitude of the variable component or the stress amplitude value.

Failure Criteria



Four criteria of failure are diagrammed in figure, the Soderberg's, the modified Goodman, the Gerber, and yielding. It is evident that only the Soderberg’s criterion guards against yielding. The linear theories of Figure can be placed in equation form: The equation for the Soderberg’s criteria (line) is

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$