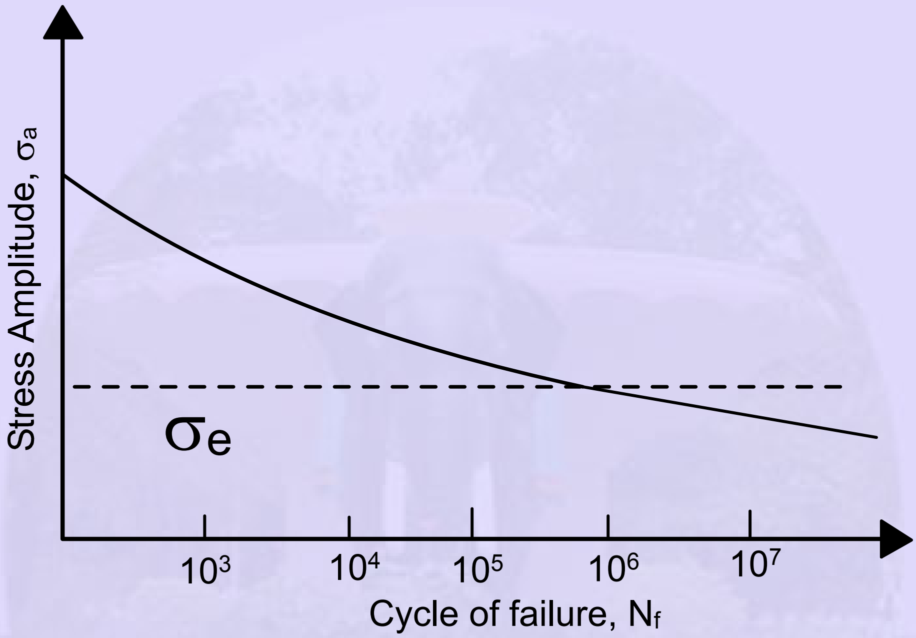
DESIGN APPROACH FOR FATIGUE LOADINGS

Design for Infinite Life

It has been noted that if a plot is made of the applied stress amplitude verses the number of reversals to failure to (S-N curve) the following behaviour is typically observed.



Completely Reversible Loading

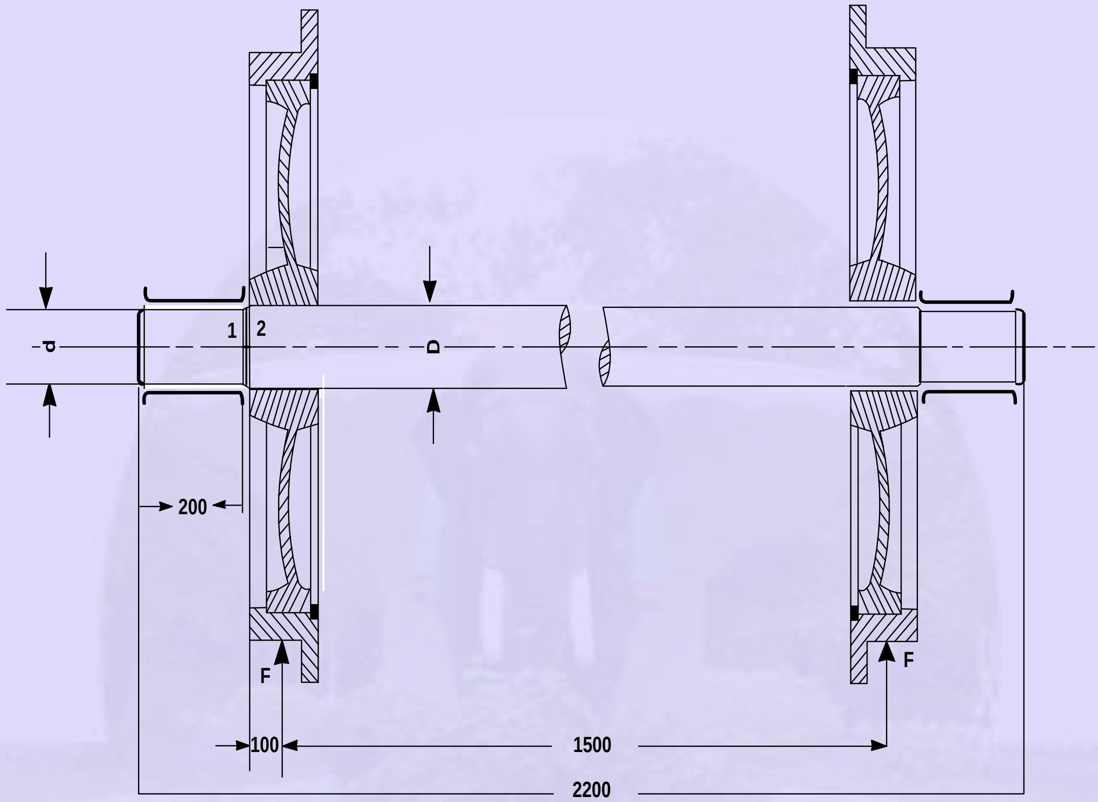
If the stress is below the (the endurance limit or fatigue limit), the component has effectively infinite life. $\sigma_e \approx 0.35\sigma_{TS} - 0.50\sigma_{TS}$ for the most steel and copper alloys. If the material does not have a well defined σ_e, often σ_e, is arbitrarily defined as Stress that gives $N_f = 10^7$ For a known load (Moment) the section area/(modulus) will be designed such that the resulting amplitude stress will be well below the endurance limit.

Design approach can be better learnt by solving a problem.

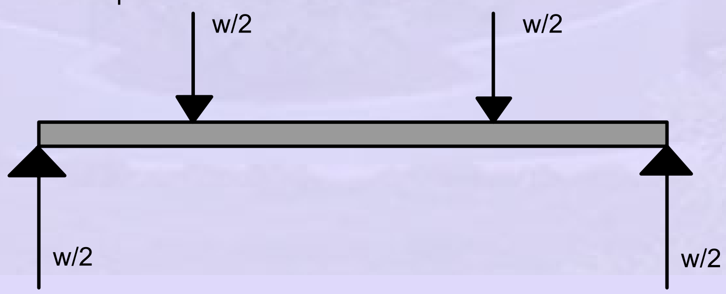
Determine a suitable diameter for the axle of a rail carriage of tentative dimensions and loading shown in the reference to figure below for fatigue endurance.

This design criterion in the case is that to induced stress should be less than the endurance limit of the material used for the axle. So the giving equation is

$$\sigma \leq s_e$$



Loading on the Test Specimen



Bending Moment

A suitable material suggested for the application can be medium carbon material like 45 C8, If is evident that the shaft is subject to binding bonds. By drawing to bending moment diagram the maximum bending moment can be determined. In this case

$$\begin{aligned} M_{max} &= F.l \\ &= 82*10^3*200 \\ &= 16.4*10^6 \text{ Nmm} \end{aligned}$$

The induced stress

$$\tau = \frac{M}{Z}$$

For circular cross section

$$= \frac{32M}{\pi d^3} = \frac{0.16705*10^6}{d^3} \text{MPa}$$

The number of stressing is going to be fully reversed because of rotating shaft with constant load application point. Now we have to estimate the endurance limit for the material of the shaft. The ultimate strength of this steel =670 Mpa. Based on the relation between the EL and UTS the basic endurance limit is =0.5Sut = 335 Ma. The design endurance limit S_e is to be estimated now as noted earlier

$$S_e = S_e * k_a k_b k_c$$

K_a – Surface factor. Assuming shaft surface is machined in nature

$$\begin{aligned} k_a &= aS_{ut}^b = 4.45(670)^{-0.265} \\ &= 0.793 \end{aligned}$$

k_s - size factor . The diameter is unknown. Instead of taking this factor to be one, assuming the diameter can be in the range 60-140 mm, for an average value of 100mm the factor is going to be

$$k_s = 0.859 - 0.0008378 * 100$$
$$= 0.775$$

K_c – load factor

This being a fully reversible bending

K_c=1.0 as the diameter is uniform stress concentration effect is neglected.

Hence the actual endurance strength is likely to be $S_e = 0.5S_{ut} * k_a k_b k_c$

. = 0.5*670*0.793*0.775

Now the final design equation is

$$\frac{32.M}{\pi d^3} = 206 \text{ MPa} \text{ or } d = \left[\frac{32.M}{\pi\left[s_e\right]}\right]^{\frac{1}{3}}$$

Assuming a factor safety (N) of 1.5 the design Endurance strength is going to be 137.31

Substituting the values

$$d = \left[\frac{3.2 * 16.4 * 10^6}{\pi * 137.31}\right]^{\frac{1}{3}}$$

106.75 mm

This values can be rounded off to the nearest Preferred size of = 110mm. In the next step, let us perform a critical analysis of the problem. Because of the step in diameter between the bearing and wheel region (1-2) stress Concentration is going to be there and this section may be critical where failure can Occur. Accounting for the stress concentration effect we can write

$$\sigma = K_f \frac{32M}{\pi d^3}$$

$$K_f = 1 + (k_t - 1)q$$

$$\text{For } \frac{D}{d} = 1.22 \text{ and } \frac{r}{d} = \frac{5}{90} = 0.05 k_t = 1.96$$

For 45 C4 steel with S_{ut} =670 and notch radius r= 5 q= 0.9

$$K_f = 1 + (1.9 - 1)0.9 = 1.81$$

Now that the surface condition is not the same and correction factor for size is to be modified. The surface factor for ground finish condition is

$$k_a = aS_{ut}^b = 1.58(670)^{-0.086}$$
$$= 0.903$$

The size Correlation factor is going to be k_s= 0.703

Hence the actual endurance strength now is

0.5*670*0.903*0.783*1.0 = 231.86

$$\sigma = \frac{1.81 * 32 * 82 * 10^3 * 100}{\pi.90^3} = 207.379$$

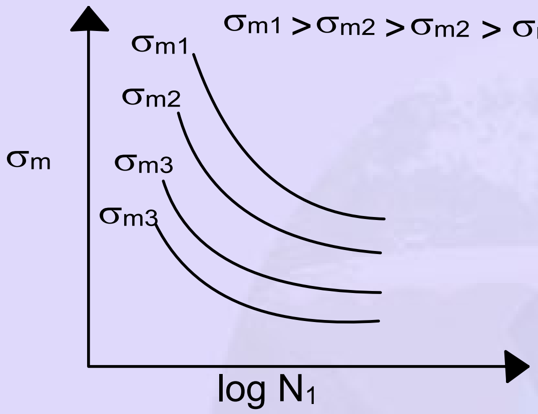
The corresponding factor of safety now is

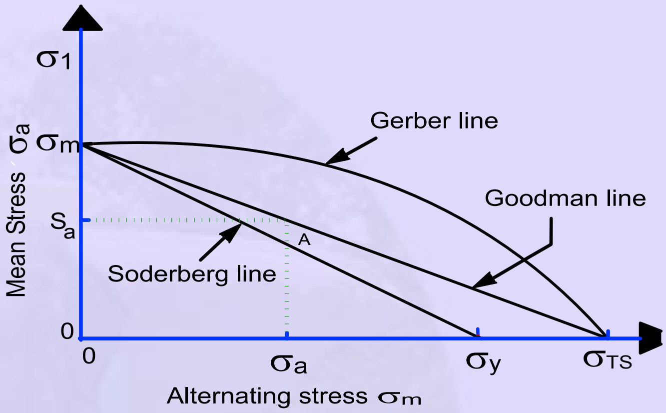
$$N = \frac{S_e}{\sigma} = \frac{231.86}{207.379} = 1.11$$

The factor of safety may not be adequate and the diameter can be modified accordingly.

Design approach for other type of cyclic loadings

The proceeding approaches to design the component assumes fully reversed fatigue load, so that the mean stress σ_m is zero. How do you handle the case where $\sigma_m \neq 0$?





The four different failure criterion and their mathematical equations have been note earlier for such cyclic loadings having a definite mean stress

For design applications the induced stresses σ_a and σ_m can replace S_a and S_m in the above equations and each strength is divided by a factor of safety N. The resulting equation is

Soderberg's criteria (line) is

$$K_f \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{N}$$

Goodman relation or criteria is

$$K_f \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{N}$$

Gerber parabolic relation:

$$K_f \frac{N\sigma_a}{S_e} + \left(\frac{N\sigma_m}{S_{ut}}\right)^2 = 1$$

(Note S_e is corrected endurance limit values and K_f factor accounts for stress concentration effects.) The meaning of these equations is illustrated in Figure, using the modified Goodman theory as an example.

From the above approach we can evolve basic design equations involving the three main type of loadings axial tension or compression, bending and torsion.