Combination of Loading Modes

Two simple approaches can be presented for this, based on the assumption that all stress components are always in time phase with each other. The procedure is illustrated in the example. The resulting mathematical relations are summarized for convenience.

Method I

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{N}$$

$$\sigma'_a = \left[\left(K_{fb}\sigma_{xa}\right)^2 + \alpha\left(K_{fs}\tau_a\right)^2\right]^{\frac{1}{2}}$$

$$\text{Where} \quad \sigma'_m = \left[\left(\sigma_{xm}\right)^2 + \sigma\left(\tau_m\right)^2\right]^{\frac{1}{2}}$$

Method II

$$\sigma_{eq} = \sigma_m + K_{fb}\sigma_a\left(S_{ut}/S_e\right)$$

$$\tau_{eq} = \tau_m + K_{fs}\tau_a\left(sS_{ut}/sS_e\right)$$

$$\text{and} \quad \sqrt{\sigma_{eq}^2 + \alpha\tau_{eq}^2} = \frac{S_{ut}}{N}$$

Torsional fatigue strength under pulsating stresses

Extensive tests by Smith provide some very interesting results on the pulsating torsional fatigue. Smith's first results based on 72 tests, shows that the existence of a torsional mean stress not more than the torsional yield strength has no effect on the torsional endurance limit, provided the material is ductile, polished, notch free, and cylindrical. However he finds that for materials with stress concentration, notches, or surface imperfections, the torsional fatigue limit decreases steadily with torsional mean stress. Hence modified Goodman's relation is recommended for pulsating torsion also, since great majority of parts will have surface imperfections. Thus the theory could be directly applied with the load factor k_c = 0.577 for torsion.

The above approach is illustrated by solving a problem.

Recall that in the last lesson we have designed an axle taking into account the bending load alone.

In previous solutions torque on the axle is neglected. If the torque is also accounted the problem is going to be of combined loading involving bending torsion.

We need to know the torque on the axle.

The torque on the axle is going to be coefficient of friction times the normal bond. According to T= 2XfXN

Where f is the co-efficient of friction between the wheel and the rail and

N is the normal reaction at each of the wheel

i.e $T = 0.25 * 82*10^3*2 = 164$ N. m

Adopting approach I

$$\sigma_m' = \left[\sigma_{bm}^2 + 3c_m^2\right]^{\frac{1}{2}}$$

$\sigma_{bm} = 0$ Fully reversible bending and hence zero mean stress

$$\sigma_m' = 0 + \sqrt{3}.\frac{167}{\pi d^3}$$

$$\sqrt{3} * \frac{16 * 164 * 10^3}{\pi d^3} = \frac{1.447 * 10^6}{d^3}$$

$$\sigma_m '.[(K_{fs}.\sigma_{ba})^2 + 3(k_{fs}\sigma_a)^2]$$

Neglecting the stress concentration effect and assuming the torque is going to be constant

$$K_{fs}=0$$

As constant torque is assumed $\tau_a = 0$

$$\therefore \sigma_a ' = \sigma_{ba}^2 = \frac{32M}{\pi d^3} = \frac{32 * 12 * 10^3 * 200}{\pi d^3}$$

$$= \frac{167.04 * 10^6}{d^3}$$

Substituting in the main equation

$$\frac{\sigma_m'}{s_{ut}} + \frac{\sigma_a'}{s_e}.\frac{1}{N}$$

$$\frac{1.447*10^6}{d^3.670} + \frac{167.04*10^6}{d^3.206} = \frac{1}{1.5}$$

$$\text{or } d^3 = 1.5\left[\frac{1.447*10^6}{d^3.670} + \frac{167.04*10^6}{d^3.206}\right]$$

$$1.214*10^6$$

$$d = 106\text{mm}$$

Finite Life Design

As noted earlier the finite life region covers a life ranging from 10^3 to 10^6 stress reversals. Design or analysis in this range can be accomplished either by using a stress life and fatigue strength based approaches, or through the strain life relations based. At very low and moderate cycles the plastic strain induced has a greater effect on the fatigue life rather than the stress magnitude.

Fatigue Strength Based Finite Life Design

The finite life fatigue domain extends from 10^3 cycles, for steels, to the endurance-limit life s_e, which is about 10^6 cycles or only slightly more. The purpose of this section is to develop methods of approximating the S-N diagram to define the fatigue strength S_f corresponding to any life N between 10^3 and 10^6 cycles. An analytical approach is to approximate the S-N diagram with a line on the log S-log N chart joining 0.9Sut at 10^3 cycles and S_e at 10^6 cycles. Let the equation of the S-N line be $S_f = a N^b$ Then $\log S_f = \log a + b \log N$. This line is to intersect 10^6 cycles at S_e and 10^3 cycles at 0.9 Sut

Substituting these values into the equation and solving for a and b we have,

$$a = \frac{(0.9S_{ut})^2}{S_e} \quad b = -\frac{1}{3}\log\frac{0.9S_{ut}}{S_e}$$

Suppose a completely reversed stress σ_a is given, the number of cycles of life corresponding to this stress can now be found by substituting σ_a for S_f, the result is

$$N = \left(\frac{\sigma_a}{a}\right)^{\frac{1}{b}}$$

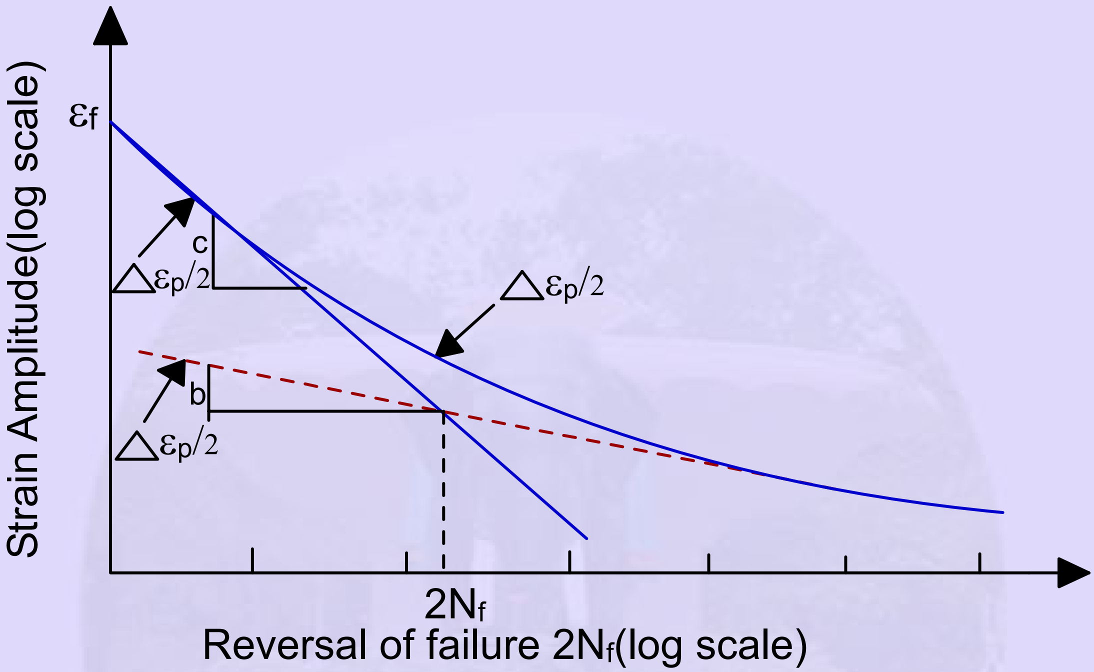
If a component is to be designed for any finite life N ($10^3 < N < 10^6$) then value of S_N determined for this known life could be substituted in place of S_e values in Soderberg or Goodman's equation presented above.

Stress –Life Approach

If a plot is prepared of $\log(\sigma_a)$ Verses $\log(2N_f)$ (where $2N_f$ represents the number of reversals to failure, one cycle is equal to two reversals) a linear relationship is commonly observed. The following relationship between stress amplitude and life time (Basquine, 1910) has been proposed.

$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f \left(2N_f\right)^b$$

The plot of this expression is depicted below:



Stress –Life Approach

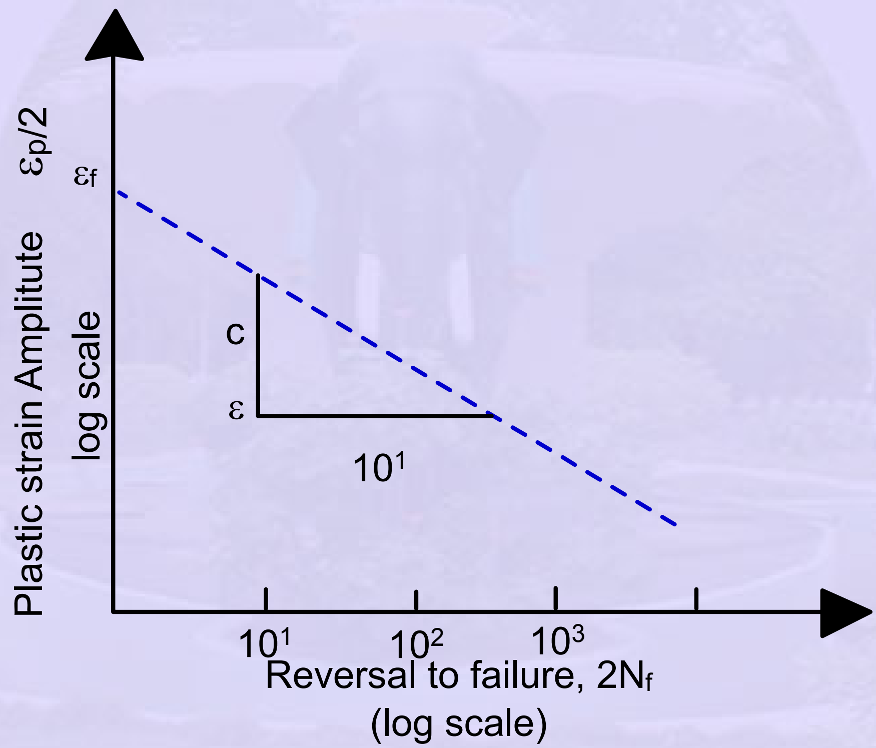
The stress-life approach just described is applicable for the situation involving the primary elastic deformation. Under these conditions the component expected to have a long life time.

Strain-Life Approach

For situation involving high stresses, high temperatures, or high stress concentration such as notches, where significant plasticity can be involved, the stress life approach is not appropriate. How do we handle these situations?

Rather than the stress amplitude σ_a, the loading is characterized by the plastic strain amplitude $\frac{\Delta\varepsilon_p}{2}$

Under these conditions if the plot is made of $\log(2N_f)$ verses $\frac{\Delta\varepsilon_p}{2}$ the following linear behavior is generally observed



To represent this behaviour, the following relationship between the plastic strain amplitude $\frac{\Delta\varepsilon_p}{2}$ and life in terms of stress reversals $2N_f$ has been proposed.

(Coffin – Manson, ca.1955)

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f \left(2N_f\right)^c$$

Where $\approx \varepsilon_f$, is the fatigue ductility coefficient (for the most metals it is equal to the true strain at fracture) and c is the fatigue ductility exponent (-0.5 to -0.7 for many metals).

To represent this behaviour, the following relationship (Coffin – Manson,).

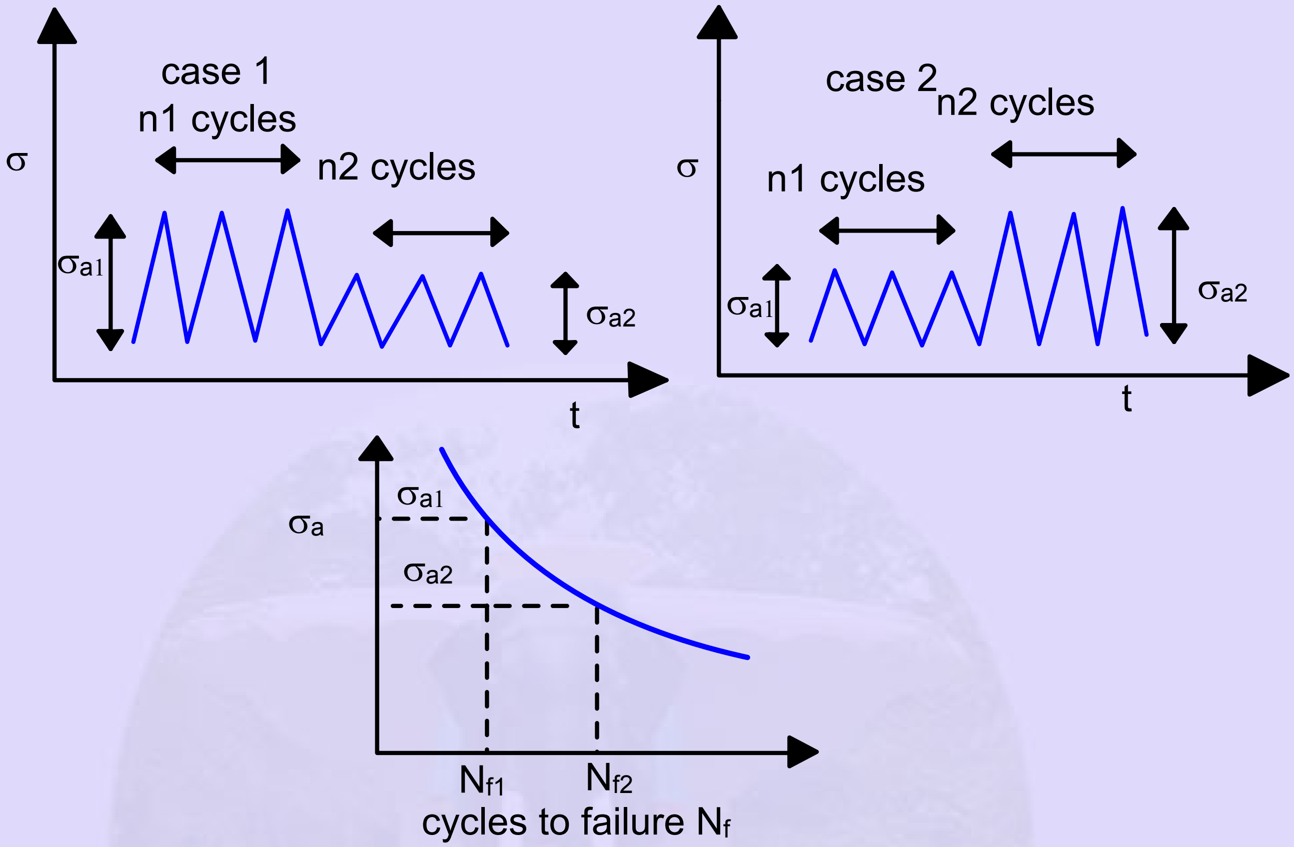
$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f \left(2N_f\right)^c$$

1955) has been proposed. Where $\frac{\Delta\varepsilon_p}{2}$ is the plastic strain amplitude, ε'_f is the fatigue ductility coefficient (for most metals $\approx \varepsilon_f$ is the true fracture ductility) and c is the fatigue ductility exponent (-0.5 to -0.7 for many metals)

The above approach is more useful for analysis rather than for design.

Different amplitudes

How do we handle the situations where we have varying amplitude loads, as depicted below?



Is Fatigue Loading Cumulative?

It is important to realize that fatigue cycles are accumulative. Suppose a part which has been in service is removed and tested for cracks by a certified aircraft inspection station (where it is more likely that the subtleties of Magnaflux inspection are known). Suppose the part passes the inspection, (i.e., no cracks are found) and the owner of the shaft puts it on the "good used parts" shelf.

Later, someone comes along looking for a bargain on such a part, and purchases this "inspected" part. The fact that the part has passed the inspection only proves that there are no detectable cracks RIGHT NOW. It gives no indication at all as to how many cycles remain until a crack forms. A part which has just passed a

Magnaflux inspection could crack in the next 100 cycles of operation and fail in the next 10000 cycles (which at 2000 RPM, isn't very long!).

CUMULATIVE FATIGUE DAMAGE

Instead of a single reversed stress σ for n cycles, suppose a part is subjected to σ_1 for n_1 cycles σ_1 for n_2 cycles. etc. Under these conditions our problem is to estimate the fatigue life of a part subjected to these reverse stresses, or to estimate the factor of safety if the part has an infinite life. A search of the literature reveals that this problem has not been solved completely.

Different Amplitudes

A very common approach is the Palmgren-Miner damage summation rule. If we defines $2N_{fi}$ as the number of reversals to failures at σ_{ai} then the partial damage for d for each different loading applied for known number of cycles n_i is σ_{ai}

$$d = \frac{2n_i}{2N_{fi}} = \frac{\text{Reversal at } \sigma_{ai}}{\text{Reversal to failure at } \sigma_{ai}}$$

The component is assumed to fail when the total damage becomes equal to 1, or

$$\sum_i \frac{n_i}{N_{fi}} = 1$$

It is assumed that the sequence in which the loads are applied has no influence on the lifetime of the component. In fact the sequence of load can have a larger influence on the lifetime of the component.

Consider the sequence of the two cyclic loads σ_{a1} and σ_{a2}. Let $\sigma_{a1} > \sigma_{a2}$

Case1: Apply then

In this case $\sum_i \frac{n_i}{N_{fi}}$ can be less than 1. During the first loading (σ_{a1}) numerous microcracks can be initiated, which can be further propagated by second loading (σ_{a2}) Case2: Apply σ_{a2} then σ_{a1}. In this case $\sum_i \frac{n_i}{N_{fi}}$ can be greater than 1. The first loading (σ_{a2}) is not high enough to cause any microcracks, but it is high enough to strain harden the material. Then in the second loading (σ_{a1}), since the material has been hardened it is more difficult to initiate any damage in the material.

Cumulative Fatigue Damage

Thus the theory which is in greatest use at the present time to explain cumulative fatigue damage, i.e the Palmgren-Minor cycle-ratio summation theory also known as Minor's rule can mathematically, stated as

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + ... + \frac{n_i}{N_i} = C$$

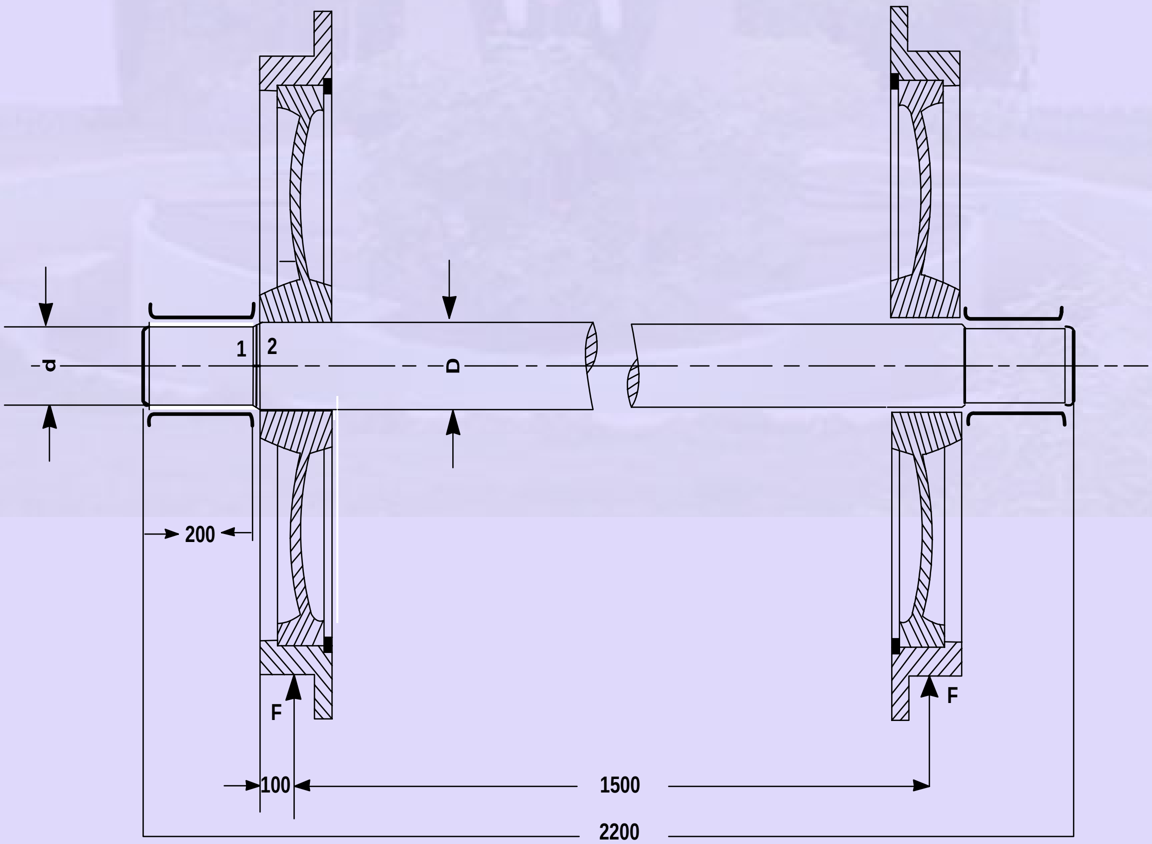
Where n is the number of cycles of stress σ applied to the specimen and N is the fatigue life corresponding to σ. The constant C is determined by experiment and is usually found in the range 0.7 ≤ C ≤2.2. Many authorities recommend C = 1 and the short form of the theory can state as:

$$\sum \frac{n}{N} = 1$$

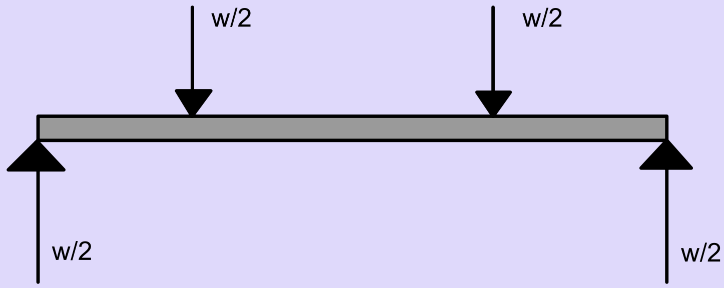
Problem Solutions (Lec 6)

Determine a suitable diameter for the axle of a rail (reference to figure) for fatigue endurance. This design criteria in the case is that to induced stress should be less than the endurance limit of the material used for the axle. So the giving equation is

$$\sigma \leq s_e$$



Loading on the Test Specimen



Bending Moment

A suitable material suggested for the application can be medium carbon material like 45 C8, If is evident that the shaft is subject to binding bonds. By drawing to bending moment diagram the maximum bending moment can be determined. In this case

$$
\begin{aligned}
M_{max} &= F.l \\
&= 82 * 10^3 * 200 \\
&= 16.4 * 10^6 \, \text{Nmm}
\end{aligned}
$$

The induced stress

$$
\begin{aligned}
\tau &= \frac{M}{Z} \\
&= \frac{32M}{\pi d^3} = \frac{0.16705 * 10^6}{d^3} \, \text{MPa}
\end{aligned}
$$

The number of stressing is going to be fully reversed because of rotating shaft with constant load application point. Now we have to estimate the endurance limit for the material of the shaft. The ultimate strength of this steel =670 Mpa.

Based on the relation between the EL and UTS the basic endurance limit is =0.5Sut = 335 Ma. The design endurance limit S_e is to be estimated now as noted earlier

$$S_e = S_e * k_a k_b k_c$$

k_a – Surface factor. Assuming shaft surface is machined in nature

$$k_a = aS_{ut}^b = 4.45(670)^{-0.265}$$
$$= 0.793$$

k_s - size factor . The diameter is unknown. Instead of taking this factors to be one, assuming the diameter can be in the range 60-140 mm, for an average value of 100mm the factor is going to be

$$k_s = 0.859 - 0.0008378 * 100$$
$$= 0.775$$

k_c – load factor K_c – loading

This being a fully reversible bending

k_c=1.0 as the diameter is uniform stress concentration effect is neglected.

Hence the actual endurance strength is likely to be $S_e = 0.5S_{ut} * k_a k_b k_c$

. = 0.5*670*0.793*0.775

Now the final design equation is

$$\frac{32.m}{\pi d^3} = 206 \text{ Mpa} \text{ or } d = \left[\frac{32.m}{\pi\left[s_e\right]}\right]^{\frac{1}{3}}$$

Assuming a factor safety (N) of 1.5 the design Endurance strength is going to be 137.31

Substituting the values

$$d = \left[\frac{3.2*16.4*10^6}{\pi*137.31}\right]^{\frac{1}{3}}$$

106.75 mm

This values can be rounded off to the nearest Preferred size of = 110mm. In the next step, let us perform a critical analysis of the problem. Because of the step in diameter between the bearing and wheel region (1-2) stress Concentration is going to be there and this section may be critical where failure can Occur. Accounting for the stress concentration effect we can write

$$\sigma = K_f \frac{32M}{\pi d^3}$$

$$K_f = 1 + (k_t - 1)q$$

$$\text{For } \frac{D}{d} = 1.22 \text{ and } \frac{r}{d} = \frac{5}{90} = 0.05 k_t = 1.96$$

For 45 C4 steel with S_{ut} =670 and notch radius r= 5 q= 0.9

$$K_f = 1 + (1.9 - 1)0.9 = 1.81$$

Now that the surface condition is not the same and correction factor for size is to be modified. The surface factor for ground finish condition is

$$k_a = aS_{ut}^b = 1.58(670)^{-0.086}$$
$$= 0.903$$

The size Correlation factor is going to be k_s= 0.703

Hence the actual endurance strength now is

0.5*670*0.903*0.783*1.0 = 231.86

$$\sigma = \frac{1.81 * 32 * 82 * 10^3 * 100}{\pi.90^3} = 207.379$$

The corresponding factor of safety now is

$$N = \frac{S_e}{\sigma} = \frac{231.86}{207.379} = 1.11$$

The factor of safety may not be adequate and the diameter can be modified accordingly.

In previous solutions torque on the axle is neglected. If the torque is also accumulated the problem is going to be of combined binding involving bonding torsion. The torque on the axle is going to be coefficient of friction lesser than the normal bond. According to T= Friction factor* 2(for the?) = 0.25 * 82*103*2= 164 N. m

Adopting approach I

$$\sigma_m' = \left[\sigma_{bm}^2 + 3c_m^2\right]^{\frac{1}{2}}$$

$\sigma_{bm} = 0$ Fully reversible binding with zero mean

$$\sigma_m' = 0 + \sqrt{3}.\frac{167}{\pi d^3}$$

$$\sqrt{3} * \frac{16 * 164 * 10^3}{\pi d^3} = \frac{1.447 * 10^6}{d^3}$$

$$\sigma_m '.[(K_{fs}.\sigma_{ba})^2 + 3(k_{fs}\sigma_a)^2]$$

For the center radius

$$K_{fs}=0$$

As constant torque is assumed $\tau_a = 0$

$$\therefore \sigma_a ' = \sigma_{ba}^2 = \frac{32M}{\pi d^3} = \frac{32 * 12 * 10^3 * 200}{\pi d^3}$$

$$= \frac{167.04 * 10^6}{d^3}$$

Substituting in the four main equation

$$\frac{\sigma_m'}{s_{ut}} + \frac{\sigma_a'}{s_e} . \frac{1}{N}$$

$$\frac{1.447 * 10^6}{d^3 .670} + \frac{167.04 * 10^6}{d^3 .206} = \frac{1}{1.5}$$

$$\text{or } d^3 = 1.5 \left[\frac{1.447 * 10^6}{d^3 .670} + \frac{167.04 * 10^6}{d^3 .206} \right]$$

$$1.214 * 10^6$$

$$d = 106\text{mm}$$