

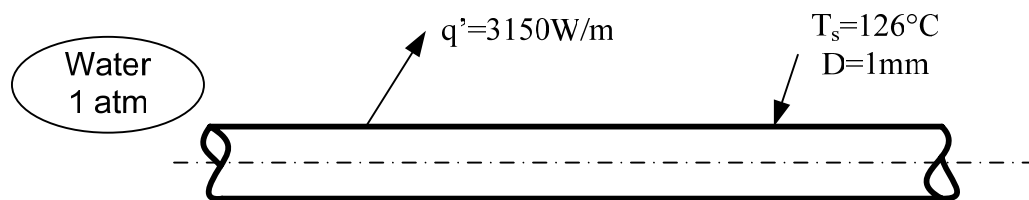
Module 8: Solved Problems

1. A long, 1-mm-diameter wire passes an electrical current dissipating 3150W/m and reaches a surface temperature of 126°C when submerged in water at 1atm. What is the boiling heat transfer coefficient? Estimate the value of the correlation coefficient $C_{s,f}$.

Known: long wire, 1–mm-diameter, reaches a surface temperature of 126°C in water at 1atm while dissipating 3150W/m.

Find: (1) Boiling heat transfer coefficient and (2) correlation coefficient $C_{s,f}$ if nucleate boiling occurs

Schematic:



Assumptions: (1) Steady-state conditions, (2) Nucleate boiling.

Table: Water (saturated, 1atm) $T_s=100C$, $\rho_l=1/v_f=957.9kg/m^3$, $\rho_f=1/v_g=0.5955kg/m^3$, $c_{p,l}=4217 J/kg.K$, $\mu_l=279*10^{-6} N.s/m^2$, $p_{rl}=1.76$, $h_{fg}=2257 KJ/kg$, $\sigma=58.9*10^{-3}N/m$.

Analysis: (a) For the boiling process, the rate equation can be rewritten as

$$\bar{h} = \frac{q_s''}{(T_s - T_{sat})} = \frac{q_s'}{\pi D (T_s - T_{sat})}$$

$$\bar{h} = \frac{3150W/m}{\pi \times 0.001m} / (126 - 100)^\circ C = 1.00 \times 10^6 \frac{W}{m^2} / 26^\circ C = 38,600W/m^2.K$$

Note that heat flux is very close to q''_{\max} , and nucleate boiling exists.

(b) For nucleate boiling, the Rohsenow correlation may be solved for $C_{s,f}$, to give

$$C_{s,f} = \left\{ \frac{\mu_{\ell} h_{f,g}}{q_s'} \right\}^{\frac{1}{3}} \left[\frac{g(\rho_{\ell} - \rho_v)}{\sigma} \right]^{1/6} \left[\frac{c_{p,\ell} \Delta T_e}{h_{f,g} \text{Pr}_{\ell}^n} \right]$$

Assuming the liquid surface combination is such that $n=1$ and substituting numerical values with $\Delta T_e = T_s - T_{\text{sat}}$, find

$$C_{s,f} = \left\{ \frac{279 \times 10^{-6} \text{ N.s/m} \times 22257 \times 10^3 \text{ J/kg}}{1.00 \times 10^6 \text{ W/m}^2} \right\}^{1/3} \left[\frac{9.8 \text{ m/s}^2 (957.9 - 0.5955 \text{ kg/m}^3)}{58.9 \times 10^{-3} \text{ N/m}} \right]^{1/6} \\ \times \left(\frac{4217 \text{ J/kg.K} \times 26 \text{ K}}{2257 \times 10^3 \text{ J/kg} \times 1.76} \right)$$

$$C_{s,f} = 0.017$$

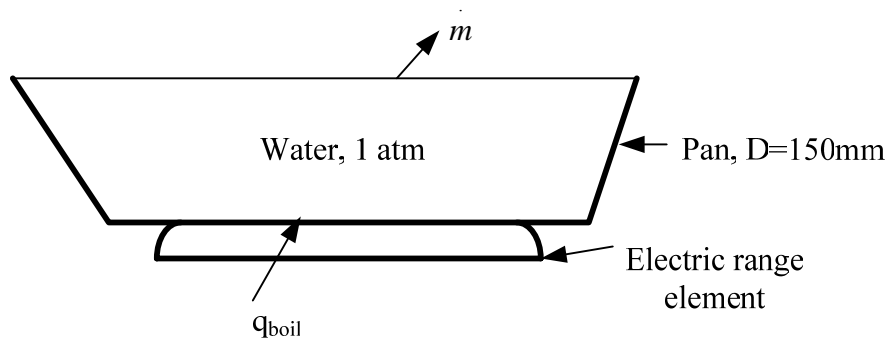
Comments: By comparison with the values $C_{s,f}$ for other water-surface combinations (given in standard tables), the $C_{s,f}$ value for the wire is quite large suggesting that its surface must be highly polished. Note that the value of the boiling heat transfer coefficient is much larger than for other convection processes previously encountered.

2. The bottom of a copper pan, 150 mm in diameter, is maintained at 115°C by the heating element of an electric range. Estimate the power required to boil the water in this pan. Determine the evaporation rate. What is the ratio of the surface heat flux to the critical heat flux? What pan temperature is required to achieve the critical heat flux?

Known: copper pan 150 mm in diameter and filled with water at 1 atm, maintained at 115°C.

Find: the power required to boil the water and the evaporation rate; ratio of the heat flux to the critical heat flux; pan temperature is required to achieve the critical heat flux.

Schematic:



Assumptions: (1) Nucleate pool boiling, (2) Copper pan is polished surface.

Properties: Table: Water (1 atm) $T_{\text{sat}}=100^\circ\text{C}$, $\rho_l=957.9\text{kg/m}^3$, $\rho_v=0.5955\text{kg/m}^3$, $c_{p,l}=4217\text{ J/kg}\cdot\text{K}$, $\mu_l=279\cdot 10^{-6}\text{N}\cdot\text{s/m}^2$, $p_{rl}=1.76$, $h_{fg}=2257\text{ KJ/kg}$, $\sigma=58.9\cdot 10^{-3}\text{N/m}$.

Analysis: the power requirement for boiling and the evaporation rate can be expressed as follows,

$$q_{\text{boil}} = q_s \cdot A_s$$

$$\dot{m} = q_{\text{boil}} / h_{f,g}$$

The heat flux for nucleate pool boiling can be estimated using the Ronsenow Correlation.

$$q''_s = \mu_\ell h_{f,g} \left[\frac{g(\rho_\ell - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{p,\ell} \Delta T_e}{C_{s,f} h_{f,g} \text{Pr}_\ell^n} \right]^3$$

Selecting $C_{s,f}=0.013$ and $n=1$ from standard table for the polished copper finish, find

$$C_{s,f} = \left\{ \frac{279 \times 10^{-6} \text{ N.s/m} \times 2257 \times 10^3 \text{ J/kg}}{\left[\frac{9.8 \text{ m/s}^2 (957.9 - 0.5955 \text{ kg/m}^3)}{58.9 \times 10^{-3} \text{ N/m}} \right]^{1/6}} \right\} \times \left(\frac{4217 \text{ J/kg.K} \times 26 \text{ K}}{2257 \times 10^3 \text{ J/kg} \times 1.76} \right)$$

$$C_{s,f} = 4.619 \times 10^5 \text{ W/m}^2$$

The power and evaporation rate are

$$q_{boil} = 4.619 \times 10^5 \text{ W/m}^2 \times \frac{\pi}{4} (0.150 \text{ m})^2 = 8.16 \text{ kW}$$

$$\dot{m}_{boil} = 8.16 \text{ kW} / 2257 \times 10^3 \text{ J/kg} = 3.62 \times 10^{-3} \text{ kg/s} = 13 \text{ kg/h}$$

The maximum or critical heat flux was found as

$$Q''_{\max} = 1.26 \text{ MW/m}^2.$$

Hence, the ratio of the operating to maximum heat flux is

$$\frac{q''_s}{q''_{\max}} = 4.619 \times 10^5 \text{ W/m}^2 / 1.26 \text{ MW/m}^2 = 0.367$$

From the boiling curve, $\Delta T_e \approx 30^\circ\text{C}$ will provide the maximum heat flux

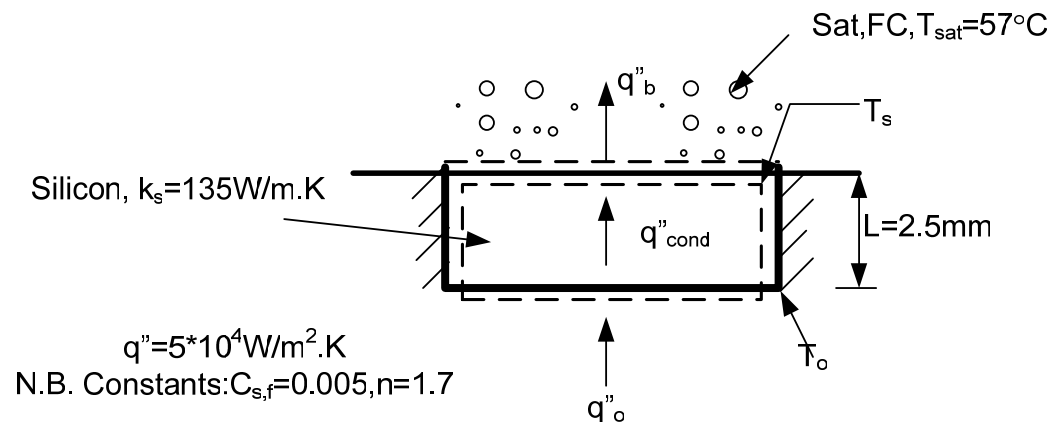
3. A silicon chip of thickness $L=25$ mm and thermal conductivity $k_s=135$ W/m.K is cooled by boiling a saturated fluorocarbon liquid ($T_{\text{sat}}=57^\circ\text{C}$) on its surface. The electronic circuits on the bottom of the chip are perfectly insulated.

Properties of the saturated fluorocarbons are $c_{p,l}=110$ J/kg.K, $h_{fg}=84,400$ J/kg, $\rho_l=1619.2$ kg/m³, $\rho_v=13.4$ kg/m³, $\sigma=8.1 \cdot 10^{-3}$ kg/s², $\mu_l=440 \cdot 10^{-6}$ kg/m.s and $p_{rl}=9.01$. In addition the nucleate boiling constants are $C_{s,f}=0.005$ and $n=1.7$. What is the steady-state temperature T_o at the bottom of the chip? If, during testing of the chip, q''_o is increases to 90% of the critical heat flux, what is the new steady-state value of T_o ?

Known: Thickness and thermal conductivity of a silicon chip. Properties of saturated fluorocarbon liquid on top side.

Find: (a) Temperature at bottom surface of chip for a prescribed heat flux, (b) Temperature of bottom surface at 90% of CHF.

Schematic:



Assumptions: (1) steady-state conditions, (2) uniform heat flux and adiabatic sides, hence one-dimensional conduction in chip, (3) Constant properties, (4) Nucleate boiling in liquid.

Properties: Saturated fluorocarbon (given): $c_{p,\ell} = 1100 \text{ J/kg.K}$,
 $h_{f,g} = 84,400 \text{ J/kg}$, $\rho_\ell = 1619.2 \text{ kg/m}^3$, $\rho_v = 13.4 \text{ kg/m}^3$, $\sigma = 8.1 \times 10^{-3} \text{ kg/s}^2$,
 $\mu_\ell = 440 \times 10^{-6} \text{ kg/m-s}$, $Pr_\ell = 9.01$.

Analysis: (a) Energy balances yield $q_o'' = q_{cond}'' = k_s(T_o - T_s)/L = q_b''$.

Obtain T_s from the Rohsenow correlation.

$$T_s - T_{sat} = \frac{C_{s,f} h_{f,g} Pr_\ell^n}{c_{p,\ell}} \left(\frac{q_s'}{\mu_\ell h_{f,g}} \right) \left[\frac{\sigma}{g(\rho_\ell - \rho_v)} \right]^{1/6}$$

1/3

$$T_s - T_{sat} = \frac{0.005(84,400 \text{ J/kg})9.01^{1.7}}{1100 \text{ J/kg.K}} \left(\frac{5 \times 10^4 \text{ W/m}^2}{440 \times 10^{-6} \text{ kg/m.s} \times 84,400 \text{ J/kg}} \right) \times$$

$$\left[\frac{8.1 \times 10^{-3} \text{ kg/s}^2}{9.807 \text{ m/s}^2 (1619.2 - 13.4) \text{ kg/m}^3} \right]^{1/6} = 15.9^\circ \text{C}$$

$$T_s = (15.9 + 57)^\circ \text{C} = 72.9^\circ \text{C}$$

From the rate equation,

$$T_o = T_s + \frac{q_o''}{k_s} = 72.9^\circ \text{C} + \frac{5 \times 10^4 \text{ W/m}^2 \times 0.0025 \text{ m}}{135 \text{ W/m.K}} = 73.8^\circ \text{C}$$

(b) With the heat rate 90% of the critical heat flux (CHF)

$$q''_{\max} = 0.149 h f g \rho_v \left[\frac{\sigma g (\rho_l - \rho_v)}{\rho_v^2} \right]^{1/4} = 0.149 \times 84,400 \text{ J/kg} \times 13.4 \text{ kg/m}^3$$

$$\times \left[\frac{8.1 \times 10^{-3} \times 9.807 \text{ m/s}^2 (1619.2 - 13.4) \text{ kg/m}^3}{13.4 \text{ kg/m}^3} \right]^{1/4}$$

$$q''_{\max} = 15.5 \times 10^4 \text{ W/m}^2$$

$$q''_o = 0.9 q''_{\max} = 13.9 \times 10^4 \text{ W/m}^2$$

$$\Delta T_e = \Delta T_{e,a} (q''_o / q''_{o,a})^{1/3} = 15.9^\circ\text{C} \times 1.41 = 22.4^\circ\text{C}$$

$$T_s = 22.4^\circ\text{C} + 57^\circ\text{C} = 79.4^\circ\text{C}$$

$$T_o = 79.4^\circ\text{C} + \frac{13.9 \times 10^4 \text{ W/m}^2 \times 0.0025}{135 \text{ W/m.K}} = 82.0^\circ\text{C}$$

Comments: Pool boiling is not adequate for many VLSI chip design

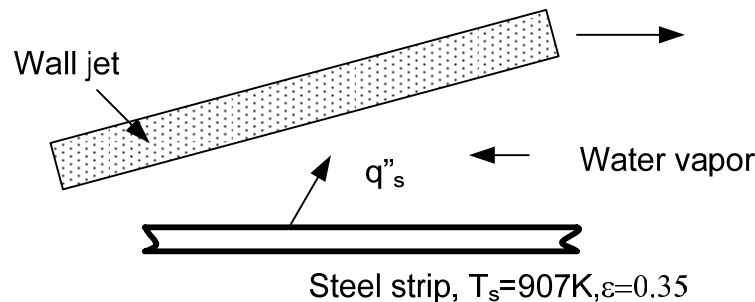
4. As strip steel leaves the last set of rollers in a hot rolling mill, it is quenched by planar water jets before being coiled. Due to the large plate temperatures, film boiling is achieved shortly downstream of the jet impingement region.

Consider conditions for which the strip steel beneath the vapor blanket is at a temperature of 907K and has an emissivity of 0.35. Neglecting the effects of the strip and jet motions and assuming convection within the film to be approximated by that associated with a large horizontal cylinder of 1-m-diameter, estimate the rate of heat transfer per unit surface area from the strip to the wall.

Known: Surface temperature and emissivity of strip steel.

Find: heat flux across vapor blanket.

Schematic:



Assumptions: (1) Steady-state conditions, (2) Vapor/jet interface is at T_{sat} for $P=1\text{atm}$, (3) Negligible effect of jet and strip motion.

Properties: Table: saturated Water (100°C 1atm) $\rho_l=1/v_f=957.9\text{kg/m}^3$,
 $h_{fg}=2257\text{ KJ/kg}$: saturated water vapor ($T_f=640\text{K}$): $\rho_v=175.4\text{kg/m}^3$,
 $c_{p,v}=42\text{ J/kg.K}$, $\mu_v=32*10^{-6}\text{N.s/m}^2$, $k=0.155\text{W/m.K}$, $\nu_v=0.182*10^{-6}\text{m}^2/\text{s}$.

Analysis: The heat flux is

$$q_x'' = \bar{h} \Delta T_e$$

$$\text{Where } \Delta T_e = 907\text{K} - 373\text{K} = 534\text{K}$$

$$\text{and } \bar{h} = \bar{h}_{\text{conv}} + \bar{h}_{\text{rad}}$$

$$\text{with } \bar{h}'_{fg} = h_{fg} + 0.80c_{p,v}(T_s - T_{sat}) = 2.02 \times 10^7 \text{ J/kg}$$

$$\bar{N}u_D = 0.62 \left[\frac{9.8\text{m/s}^2(957.9 - 175.4)\text{kg/m}^3(2.02 \times 10^7 \text{ J/kg})(1\text{m})^3}{0.182 \times 10^{-6} \text{ m}^2/\text{s}(0.155\text{W/m.K})(907 - 373)\text{K}} \right]^{1/4} = 6243$$

hence,

$$\bar{h}_{\text{conv}} = \bar{N}u_D k_v / D = 6243\text{W/m}^2.\text{K}(0.155\text{W/m.K}/1\text{m}) = 968\text{W/m}^2.\text{K}$$

$$\bar{h}_{\text{rad}} = \frac{\epsilon\sigma(T_s^4 - T_{sat}^4)}{T_s - T_{sat}} = \frac{0.35 \times 5.67 \times 10^{-8} \text{ W/m}^2.\text{k}(907^4 - 373^4)\text{K}^4}{(907 - 373)\text{K}}$$

$$\bar{h}_{\text{rad}} = 24\text{W/m}^2.\text{K}$$

$$\text{hence, } \bar{h} = 968\text{W/m}^2.\text{K} + (3/4)(24\text{W/m}^2.\text{K}) = 986\text{W/m}^2.\text{K}$$

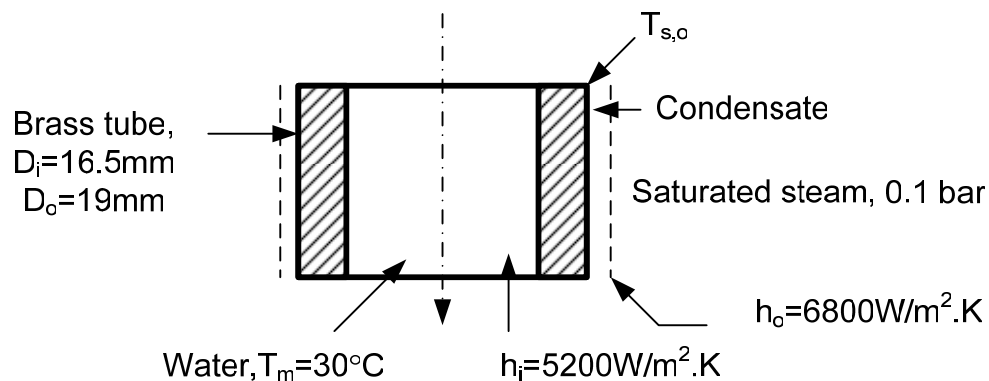
$$\text{and } q_s'' = 986\text{W/m}^2.\text{k}(907 - 373)\text{K} = 5.265 \times 10^5 \text{ W/m}^2$$

5. Saturated steam at 0.1 bars condenses with a convection coefficient of $6800 \text{ W/m}^2\cdot\text{K}$ on the outside of a brass tube having inner and outer diameters of 16.5 and 19mm, respectively. The convection coefficient for water flowing inside the tube is $5200 \text{ W/m}^2\cdot\text{K}$. Estimate the steam condensation rate per unit length of the tube when the mean water temperature is 30°C .

Known: saturated steam condensing on the outside of a brass tube and water flowing on the inside of the tube; convection coefficients are prescribed.

Find: Steam condensation rate per unit length of the tube.

Schematic:



Assumptions: (1) Steady-state conditions.

Properties: Table: Water, vapor (0.1 bar): $T_{\text{sat}} \approx 320 \text{ K}$, $h_{fg} = 2390 \times 10^3 \text{ J/kg}$;

Table: Brass ($\bar{T} = (T_m + T_{\text{sat}}) / 2 \approx 300 \text{ K}$); $k = 110 \text{ W/m}\cdot\text{K}$.

Analysis: The condensation rate per unit length is written as

$$\dot{m}' = \dot{q}' / h_{fg}' \quad (1)$$

Where the heat rate follows from equation using overall heat transfer coefficient

$$q' = U_o \cdot \pi D_o (T_{sat} - T_m) \quad (2)$$

$$U_o = \left[\frac{1}{h_o} + \frac{D_o/2}{k} \ln \frac{D_o}{D_i} + \frac{D_o}{D_i} \frac{1}{h_i} \right]^{-1} \quad (3)$$

$$U_o = \left[\frac{1}{6800W/m^2 \cdot K} + \frac{0.0095m}{110W/m \cdot K} \ln \frac{19}{16.5} + \frac{19}{16.5} \frac{1}{5200W/m^2 \cdot K} \right]^{-1}$$

$$U_o = 147.1 \times 10^{-6} + 12.18 \times 10^{-6} + 192.3 \times 10^{-6} W/m^2 \cdot K = 2627W/m^2 \cdot K$$

Combining equations, (1) and (2) and substituting numerical values, find

$$\dot{m}' = U_o \pi D_o (T_{sat} - T_m) / h'_{fg}$$

$$\dot{m}' = 2627W/m^2 \cdot K \pi (0.019m) (320 - 303)K / 2410 \times 10^3 J/kg = 1.11 \times 10^3 kg/s$$

Comments: (1) Note from evaluation of equation. (3) That the thermal resistance of the brass tube is not negligible.

(2) With $Ja = c_{p,\ell} (T_{sat} - T_s) / h_{f,g}$, $h'_{fg} = h_{fg} [1 + 0.68Ja]$. note from expression for U_o , that the internal resistance is the largest. Hence, estimate $T_{s,o} \approx T_o - (R_o / \Sigma R)(T_o - T_m) \approx 313K$. Hence

$$h'_{fg} \approx 2390 \times 10^3 J/kg [1 + 0.68 \times 4179J/kg \cdot K (320 - 313)K / 2390 \times 10^3 J/kg]$$

$$h'_{fg} = 2410kJ/kg$$

Where $c_{p,\ell}$ for water(liquid) is evaluated at $T_f = (T_{s,o} + T_o) / 2 \approx 317K$