MODULE 6

CONVECTION

6.1 Objectives of convection analysis:

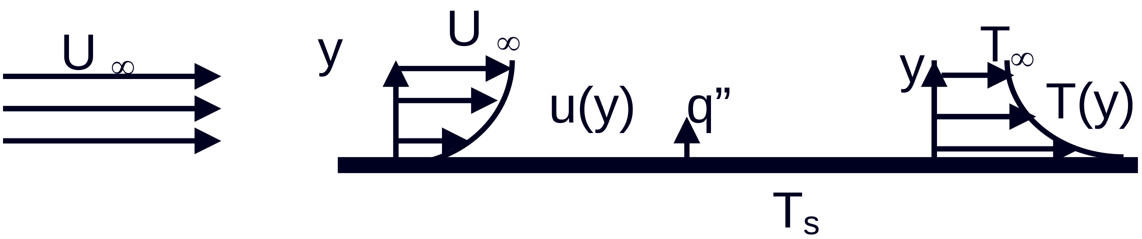
Main purpose of convective heat transfer analysis is to determine:

- flow field
- temperature field in fluid
- heat transfer coefficient, h

How do we determine h ?

Consider the process of convective cooling, as we pass a cool fluid past a heated wall. This process is described by Newton's law of Cooling:

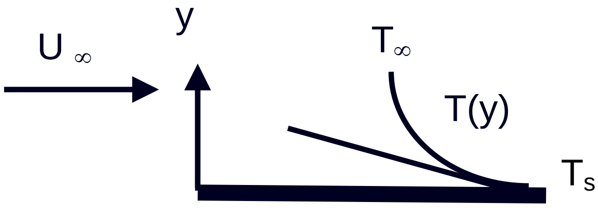
$$q = h \cdot A \cdot (T_S - T_\infty)$$



Near any wall a fluid is subject to the no slip condition; that is, there is a stagnant sub layer. Since there is no fluid motion in this layer, heat transfer is by conduction in this region. Above the sub layer is a region where viscous forces retard fluid motion; in this region some convection may occur, but conduction may well predominate. A careful analysis of this region allows us to use our conductive analysis in analyzing heat transfer. This is the basis of our convective theory.

At the wall, the convective heat transfer rate can be expressed as the heat flux.

$$q''_{conv} = -k_f \left.\frac{\partial T}{\partial y}\right)_{y=0} = h\left(T_s - T_\infty\right)$$



Hence, $h = \dfrac{-k_f \left.\dfrac{\partial T}{\partial y}\right)_{y=0}}{\left(T_s - T_\infty\right)}$

But $\left.\dfrac{\partial T}{\partial y}\right)_{y=0}$ depends on the whole fluid motion, and both fluid flow and heat transfer equations are needed

The expression shows that in order to determine h, we must first determine the temperature distribution in the thin fluid layer that coats the wall.

2.2 Classes of Convective Flows

Convection →
- Free or natural convection (induced by buoyancy forces)
- Forced convection (induced by external means)

} May occur with phase change (boiling, condensation)

- extremely diverse
- several parameters involved (fluid properties, geometry, nature of flow, phases etc)
- systematic approach required
- classify flows into certain types, based on certain parameters
- identify parameters governing the flow, and group them into **meaningful non-dimensional numbers**
- need to understand the physics behind each phenomenon

Common classifications:

A. ***Based on geometry:***

External flow / Internal flow

B. ***Based on driving mechanism***

Natural convection / forced convection / mixed convection

C. ***Based on number of phases***

Single phase / multiple phase

D. ***Based on nature of flow***

Laminar / turbulent

Table 6.1. Typical values of h (W/m^2K)

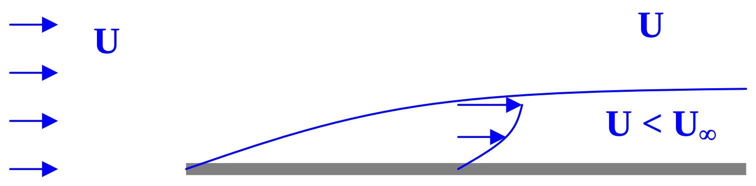
Free convection	gases: 2 - 25 liquid: 50 – 100
Forced convection	gases: 25 - 250 liquid: 50 - 20,000
Boiling/Condensation	2500 -100,000

2.3 How to solve a convection problem ?

- Solve governing equations along with boundary conditions
- Governing equations include
  1. conservation of mass
  2. conservation of momentum
  3. conservation of energy
- In Conduction problems, only (3) is needed to be solved. Hence, only *few parameters* are involved
- In Convection, all the governing equations need to be solved.
  ⇒ large number of parameters can be involved

2.4 FORCED CONVECTION: external flow (over flat plate)

An internal flow is surrounded by solid boundaries that can restrict the development of its boundary layer, for example, a pipe flow. An external flow, on the other hand, are flows over bodies immersed in an unbounded fluid so that the flow boundary layer can grow freely in one direction. Examples include the flows over airfoils, ship hulls, turbine blades, etc



- Fluid particle adjacent to the solid surface is at rest
- These particles act to retard the motion of adjoining layers
- ⇒ boundary layer effect

Inside the boundary layer, we can apply the following conservation principles:
Momentum balance: inertia forces, pressure gradient, viscous forces, body forces
Energy balance: convective flux, diffusive flux, heat generation, energy storage

2.5 Forced Convection Correlations

Since the heat transfer coefficient is a direct function of the temperature gradient next to the wall, the physical variables on which it depends can be expressed as follows:
h=f(fluid properties, velocity field ,geometry,temperature etc.)

As the function is dependent on several parameters, the heat transfer coefficient is usually expressed in terms of **correlations involving pertinent non-dimensional numbers**.

Forced convection: **Non-dimensional groupings**

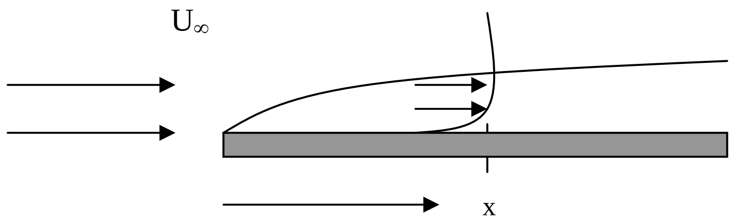
- **Nusselt No**. Nu = hx / k = (convection heat transfer strength)/ (conduction heat transfer strength)
- **Prandtl No.** Pr = ν/α = (momentum diffusivity)/ (thermal diffusivity)
- **Reynolds No**. Re = U x / ν = (inertia force)/(viscous force)

Viscous force provides the dampening effect for disturbances in the fluid. If dampening is strong enough ⇒ **laminar flow**
Otherwise, instability ⇒ **turbulent flow** ⇒ **critical Reynolds number**

For forced convection, the heat transfer correlation can be expressed as

$$Nu = f(Re, Pr)$$

The convective correlation for laminar flow across a flat plate heated to a constant wall temperature is:



$$Nu_x = 0.323 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

where

$$Nu_x \equiv h \cdot x/k$$

$$Re_x \equiv (U_\infty \cdot x \cdot \rho)/\mu$$

$$Pr \equiv c_P \cdot \mu/k$$

Physical Interpretation of Convective Correlation

The Reynolds number is a familiar term to all of us, but we may benefit by considering what the ratio tells us. Recall that the thickness of the dynamic boundary layer, δ, is proportional to the distance along the plate, x.

$$Re_x \equiv (U_\infty \cdot x \cdot \rho)/\mu \;\propto\; (U_\infty \cdot \delta \cdot \rho)/\mu \;=\; (\rho \cdot U_\infty^2)/(\mu \cdot U_\infty/\delta)$$

The numerator is a mass flow per unit area times a velocity; i.e. a momentum flow per unit area. The denominator is a viscous stress, i.e. a viscous force per unit area. The ratio represents the ratio of momentum to viscous forces. If viscous forces dominate, the flow will be laminar; if momentum dominates, the flow will be turbulent.

Physical Meaning of Prandtl Number

The Prandtl number was introduced earlier.

If we multiply and divide the equation by the fluid density, ρ, we obtain:

$$Pr \equiv (\mu/\rho)/(k/\rho \cdot c_P) = \upsilon/\alpha$$

The Prandtl number may be seen to be a ratio reflecting the ratio of the rate that viscous forces penetrate the material to the rate that thermal energy penetrates the material. As a consequence the Prandtl number is proportional to the rate of growth of the two boundary layers:

$$\delta/\delta_t = Pr^{1/3}$$

Physical Meaning of Nusselt Number

The Nusselt number may be physically described as well.

$$Nu_x \equiv h \cdot x/k$$

If we recall that the thickness of the boundary layer at any point along the surface, δ, is also a function of x then

$$Nu_x \propto h \cdot \delta/k \propto (\delta/k \cdot A)/(1/h \cdot A)$$

We see that the Nusselt may be viewed as the ratio of the conduction resistance of a material to the convection resistance of the same material.

Students, recalling the Biot number, may wish to compare the two so that they may distinguish the two.

$$Nu_x \equiv h \cdot x/k_{fluid} \qquad\qquad Bi_x \equiv h \cdot x/k_{solid}$$

The denominator of the Nusselt number involves the thermal conductivity of the **fluid** at the solid-fluid convective interface; The denominator of the Biot number involves the thermal conductivity of the **solid** at the solid-fluid convective interface.

Local Nature of Convective Correlation

Consider again the correlation that we have developed for laminar flow over a flat plate at constant wall temperature

$$Nu_x = 0.323 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

To put this back into dimensional form, we replace the Nusselt number by its equivalent, hx/k and take the x/k to the other side:

$$h = 0.323 \cdot (k/x) \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

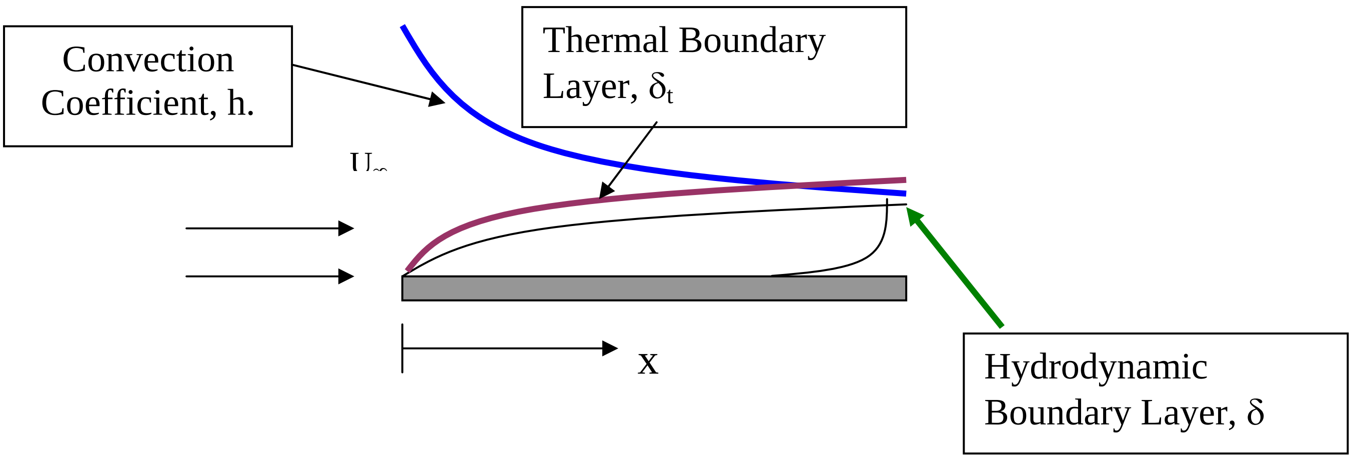
Now expand the Reynolds number

$$h = 0.323 \cdot (k/x) \cdot [(U_\infty \cdot x \cdot \rho)/\mu]^{1/2} \cdot Pr^{1/3}$$

We proceed to combine the x terms:

$$h = 0.323 \cdot k \cdot [(U_\infty \cdot \rho)/(x \cdot \mu)]^{1/2} \cdot Pr^{1/3}$$

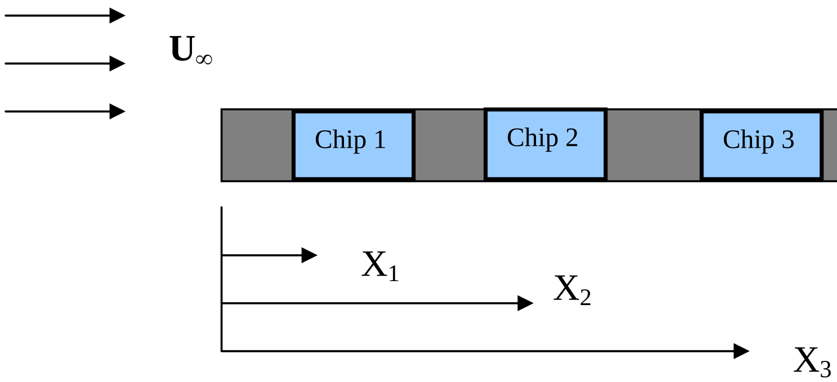
And see that the convective coefficient decreases with $x^{1/2}$.



We see that as the boundary layer thickens, the convection coefficient decreases. Some designers will introduce a series of "trip wires", i.e. devices to disrupt the boundary layer, so that the buildup of the insulating layer must begin anew. This will result in regular "thinning" of the boundary layer so that the convection coefficient will remain high.

Use of the "Local Correlation"

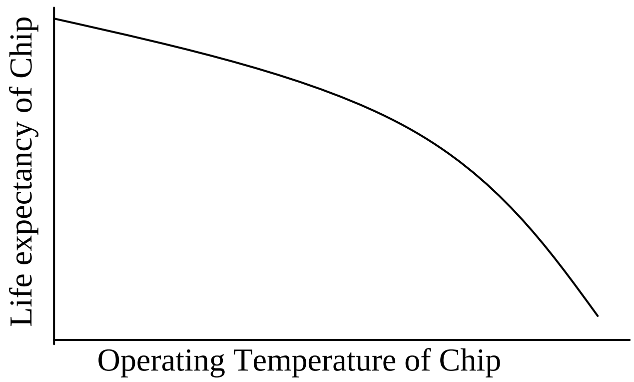
A local correlation may be used whenever it is necessary to find the convection coefficient at a particular location along a surface. For example, consider the effect of chip placement upon a printed circuit board:



Here are the design conditions. We know that as the higher the operating temperature of a chip, the lower the life expectancy.

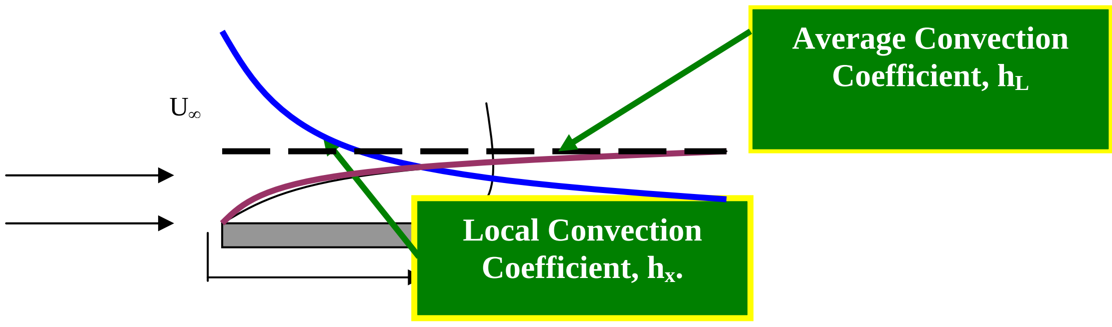
With this in mind, we might choose to operate all chips at the same design temperature.

Where should the chip generating the largest power per unit surface area be placed? The lowest power?



Averaged Correlations

If one were interested in the total heat loss from a surface, rather than the temperature at a point, then they may well want to know something about average convective coefficients. For example, if we were trying to select a heater to go inside an aquarium, we would not be interested in the heat loss at 5 cm, 7 cm and 10 cm from the edge of the aquarium; instead we want some sort of an average heat loss.



The desire is to find a correlation that provides an overall heat transfer rate:

$$Q = h_L \cdot A \cdot [T_{wall} - T_\infty] = \int h_x \cdot [T_{wall} - T_\infty] \cdot dA = \int_0^L h_x \cdot [T_{wall} - T_\infty] \cdot dx$$

where h_x and h_L, refer to local and average convective coefficients, respectively.

Compare the second and fourth equations where the area is assumed to be equal to A = (1·L):

$$h_L \cdot L \cdot [T_{wall} - T_\infty] = \int_0^L h_x \cdot [T_{wall} - T_\infty] \cdot dx$$

Since the temperature difference is constant, it may be taken outside of the integral and cancelled:

$$h_L \cdot L = \int_0^L h_x \cdot dx$$

This is a general definition of an integrated average.

Proceed to substitute the correlation for the local coefficient.

$$h_L \cdot L = \int_0^L 0.323 \cdot \frac{k}{x} \cdot \left[\frac{U_\infty \cdot x \cdot \rho}{\mu}\right]^{0.5} \cdot \Pr^{1/3} \cdot dx$$

Take the constant terms from outside the integral, and divide both sides by k.

$$h_L \cdot L/k = 0.323 \cdot \left[\frac{U_\infty \cdot \rho}{\mu}\right]^{0.5} \cdot \Pr^{1/3} \cdot \int_0^L \left[\frac{1}{x}\right]^{0.5} \cdot dx$$

Integrate the right side.

$$h_L \cdot L/k = 0.323 \cdot \left[\frac{U_\infty \cdot \rho}{\mu}\right]^{0.5} \cdot \Pr^{1/3} \cdot \left.\frac{x^{0.5}}{0.5}\right|_0^L$$

The left side is defined as the average Nusselt number, Nu_L. Algebraically rearrange the right side.

$$Nu_L = \frac{0.323}{0.5} \cdot \left[\frac{U_\infty \cdot \rho}{\mu}\right]^{0.5} \cdot \Pr^{\frac{1}{3}} \cdot L^{0.5} = 0.646 \cdot \left[\frac{U_\infty \cdot L \cdot \rho}{\mu}\right]^{0.5} \cdot \Pr^{\frac{1}{3}}$$

The term in the brackets may be recognized as the Reynolds number, evaluated at the end of the convective section. Finally,

$$Nu_L = 0.646 \cdot Re_L^{0.5} \cdot \Pr^{1/3}$$

This is our average correlation for laminar flow over a flat plate with constant wall temperature.

Reynolds Analogy
In the development of the boundary layer theory, one may notice the strong relationship between the dynamic boundary layer and the thermal boundary layer. Reynold's noted the strong correlation and found that fluid friction and convection coefficient could be related. This refers to the Reynolds Analogy.

Conclusion from Reynold's analogy: Knowing the frictional drag, we know the Nusselt Number. If the drag coefficient is increased, say through increased wall roughness, then the convective coefficient will increase. If the wall friction is decreased, the convective coefficient is decreased.

Turbulent Flow

We could develop a turbulent heat transfer correlation in a manner similar to the von Karman analysis. It is probably easier, having developed the Reynolds analogy, to follow that course. The local fluid friction factor, C_f, associated with turbulent flow over a flat plate is given as:

$$C_f = 0.0592/Re_x^{0.2}$$

Substitute into the Reynolds analogy:

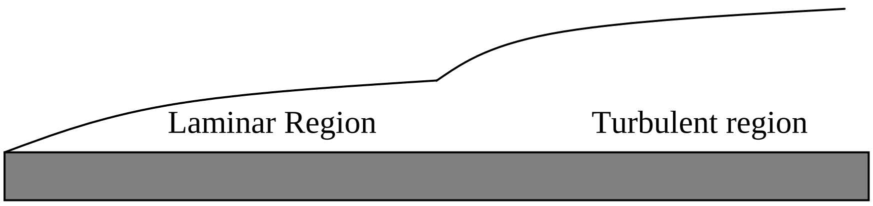
$$(0.0592/Re_x^{0.2})/2 = Nu_x/Re_x Pr^{1/3}$$

Rearrange to find

$$Nu_x = 0.0296 \cdot Re_x^{0.8} \cdot Pr^{1/3}$$

Local Correlation Turbulent Flow Flat Plate.

In order to develop an average correlation, one would evaluate an integral along the plate similar to that used in a laminar flow:



$$h_L \cdot L = \int_0^L h_x dx = \int_0^{L_{crit}} h_{x,laminar} \cdot dx + \int_{L_{crit}}^L h_{x,turbulent} \cdot dx$$

Note: The critical Reynolds number for flow over a flat plate is $5 \cdot 10^5$; the critical Reynolds number for flow through a round tube is 2000.

The result of the above integration is:

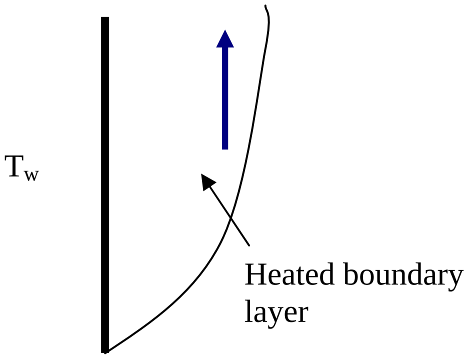
$$Nu_x = 0.037 \cdot (Re_x^{0.8} - 871) \cdot Pr^{1/3}$$

Note: Fluid properties should be evaluated at the average temperature in the boundary layer, i.e. at an average between the wall and free stream temperature.

$$T_{prop} = 0.5 \cdot (T_{wall} + T_\infty)$$

2.6 Free convection

Free convection is sometimes defined as a convective process in which fluid motion is caused by buoyancy effects.

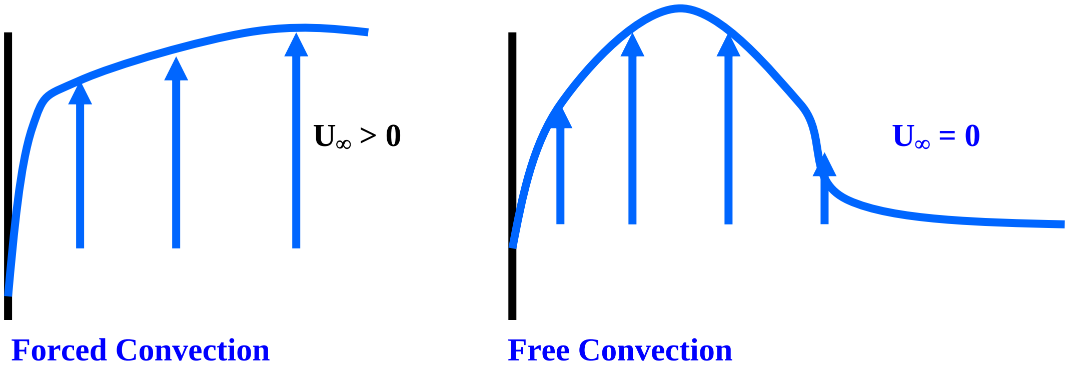


$T_\infty < T_{boundry.\ layer} < T_w$

$\rho_\infty < \rho_{boundry.\ layer}$

Velocity Profiles

Compare the velocity profiles for forced and natural convection shown below:



Coefficient of Volumetric Expansion

The thermodynamic property which describes the change in density leading to buoyancy in the Coefficient of Volumetric Expansion, β.

$$\beta \equiv -\frac{1}{\rho}\cdot\left.\frac{\partial\rho}{\partial T}\right|_{P=Const.}$$

Evaluation of β

- Liquids and Solids: β is a thermodynamic property and should be found from Property Tables. Values of β are found for a number of engineering fluids in Tables given in Handbooks and Text Books.
- Ideal Gases: We may develop a general expression for β for an ideal gas from the ideal gas law:

$$P = \rho \cdot R \cdot T$$

Then,

$$\rho = P / R \cdot T$$

Differentiating while holding P constant:

$$\left.\frac{d\rho}{dT}\right|_{P=Const.} = -\frac{P}{R \cdot T^2} = -\frac{\rho \cdot R \cdot T}{R \cdot T^2} = -\frac{\rho}{T}$$

Substitute into the definition of β

$$\beta = \frac{1}{T_{abs}}$$

Ideal Gas

Grashof Number

Because U_∞ is always zero, the Reynolds number, $[\rho \cdot U_\infty \cdot D]/\mu$, is also zero and is no longer suitable to describe the flow in the system. Instead, we introduce a new parameter for natural convection, the Grashof Number. Here we will be most concerned with flow across a vertical surface, so that we use the vertical distance, z or L, as the characteristic length.

$$Gr \equiv \frac{g \cdot \beta \cdot \Delta T \cdot L^3}{\nu^2}$$

Just as we have looked at the Reynolds number for a physical meaning, we may consider the Grashof number:

$$Gr \equiv \frac{\rho^2 \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{\mu^2} = \frac{\left(\frac{\rho \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{L^2}\right) \cdot (\rho \cdot U_{\max}^2)}{\mu^2 \cdot \frac{U_{\max}^2}{L^2}} = \frac{\left(\frac{Buoyant\,Force}{Area}\right) \cdot \left(\frac{Momentum}{Area}\right)}{\left(\frac{Viscous\,Force}{Area}\right)^2}$$

Free Convection Heat Transfer Correlations

The standard form for free, or natural, convection correlations will appear much like those for forced convection except that (1) the Reynolds number is replaced with a Grashof number and (2) the exponent on Prandtl number is not generally 1/3 (The von Karman boundary layer analysis from which we developed the 1/3 exponent was for forced convection flows):

$Nu_x = C \cdot Gr_x^m \cdot Pr^n$ Local Correlation

$Nu_L = C \cdot Gr_L^m \cdot Pr^n$ Average Correlation

Quite often experimentalists find that the exponent on the Grashof and Prandtl numbers are equal so that the general correlations may be written in the form:

$Nu_x = C \cdot [Gr_x \cdot Pr]^m$ Local Correlation

$Nu_L = C \cdot [Gr_L \cdot Pr]^m$ Average Correlation

This leads to the introduction of the new, dimensionless parameter, the Rayleigh number, Ra:

$Ra_x = Gr_x \cdot Pr$

$Ra_L = Gr_L \cdot Pr$

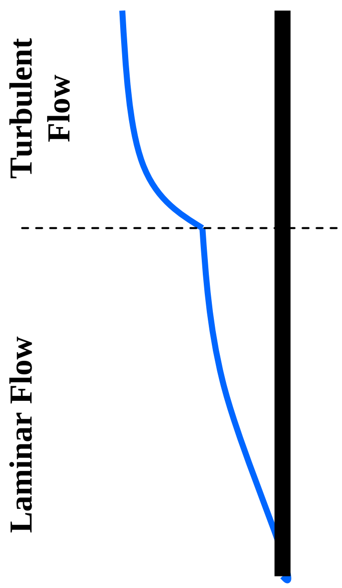
So that the general correlation for free convection becomes:

$Nu_x = C \cdot Ra_x^m$ Local Correlation

$Nu_L = C \cdot Ra_L^m$ Average Correlation

Laminar to Turbulent Transition

Just as for forced convection, a boundary layer will form for free convection. The insulating film will be relatively thin toward the leading edge of the surface resulting in a relatively high convection coefficient. At a Rayleigh number of about 10^9 the flow over a flat plate will transition to a turbulent pattern. The increased turbulence inside the boundary layer will enhance heat transfer leading to relative high convection coefficients, much like forced convection.



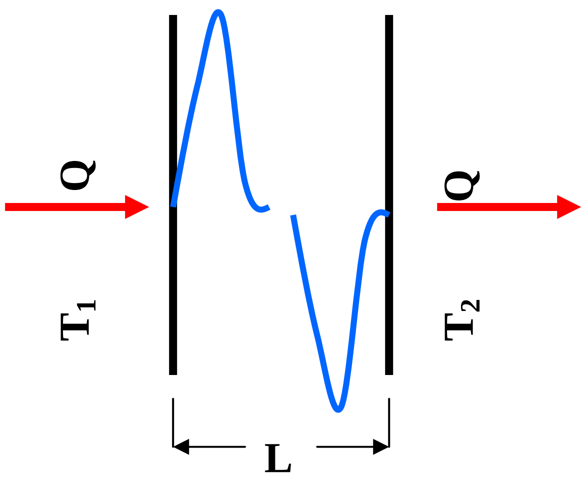
$Ra < 10^9$ Laminar flow. [Vertical Flat Plate]

$Ra > 10^9$ Turbulent flow. [Vertical Flat Plate]

Generally the characteristic length used in the correlation relates to the distance over which the boundary layer is allowed to grow. In the case of a vertical flat plate this will be x or L, in the case of a vertical cylinder this will also be x or L; in the case of a horizontal cylinder, the length will be d.

Critical Rayleigh Number

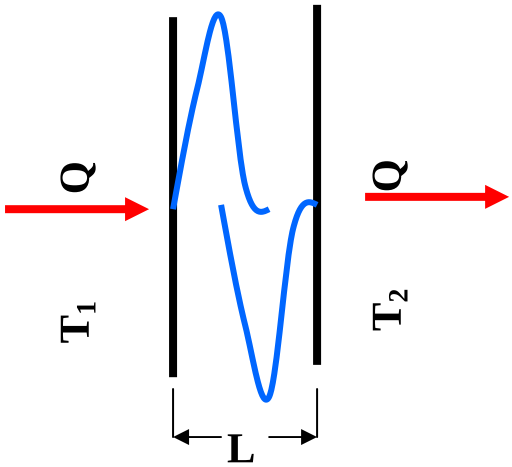
Consider the flow between two surfaces, each at different temperatures. Under developed flow conditions, the interstitial fluid will reach a temperature between the temperatures of the two surfaces and will develop free convection flow patterns. The fluid will be heated by one surface, resulting in an upward buoyant flow, and will be cooled by the other, resulting in a downward flow.

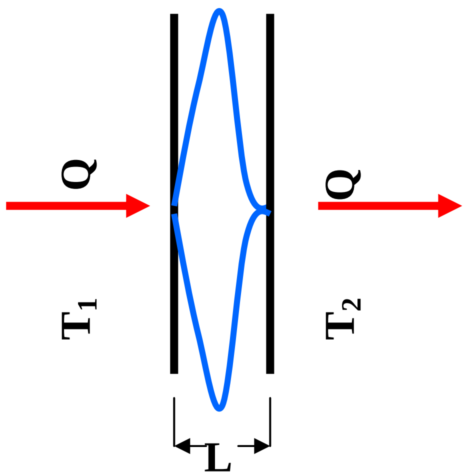


Note that for enclosures it is customary to develop correlations which describe the overall (both heated and cooled surfaces) within a single correlation.

Free Convection Inside an Enclosure

If the surfaces are placed closer together, the flow patterns will begin to interfere:





Free Convection Inside an Enclosure With Partial Flow Interference

Free Convection Inside an Enclosure With Complete Flow Interference

In the case of complete flow interference, the upward and downward forces will cancel, canceling circulation forces. This case would be treated as a pure convection problem since no bulk transport occurs.

The transition in enclosures from convection heat transfer to conduction heat transfer occurs at what is termed the "**Critical Rayleigh Number**". Note that this terminology is in clear contrast to forced convection where the critical Reynolds number refers to the transition from laminar to turbulent flow.

$Ra_{crit} = 1000$ (Enclosures With Horizontal Heat Flow)

$Ra_{crit} = 1728$ (Enclosures With Vertical Heat Flow)

The existence of a Critical Rayleigh number suggests that there are now three flow regimes: (1) No flow, (2) Laminar Flow and (3) Turbulent Flow. In all enclosure problems the Rayleigh number will be calculated to determine the proper flow regime before a correlation is chosen.