MODULE 2: Worked-out Problems

Problem 1:

The steady-state temperature distribution in a one–dimensional wall of thermal conductivity 50W/m.K and thickness 50 mm is observed to be $T(^0C)= a+bx^2$, where $a=200^0C$, $B=-2000^0c/m^2$, and x in meters.

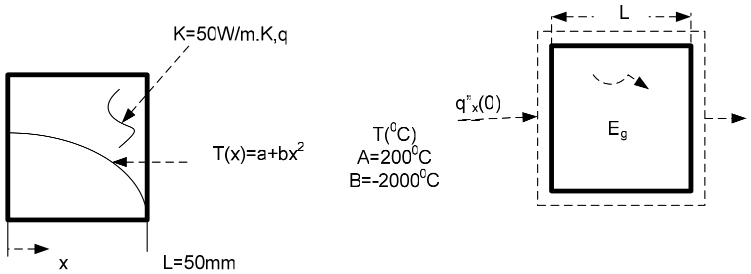
(a) What is the heat generation rate in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

Known: Temperature distribution in a one dimensional wall with prescribed thickness and thermal conductivity.

Find: (a) the heat generation rate, q in the wall, (b) heat fluxes at the wall faces and relation to q.

Schematic:



Assumptions: (1) steady-state conditions, (2) one –dimensional heat flow, (3) constant properties.

Analysis: (a) the appropriate form of heat equation for steady state, one dimensional condition with constant properties is

$$\dot{q} = -K\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

$$\dot{q} = -k\frac{d}{dx}\left[\frac{d}{dx(a+bx^2)}\right] = -k\frac{d}{dx}[2bx] = -2bk$$

$$\dot{q} = -2(-2000^0\,CC/m^2)\times 50W/m.K = 2.0\times 10^5\,W/m^3$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q_x''(x) = -k\frac{dT}{dx}\bigg|_x$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q_x''(x) = -k\frac{d}{dx}[a+bx^2] = -2kbx.$$

The flux at the face, is then x=0

$$q''_x(0) = 0$$

$$\text{at} X = L, q_x''(l) = -2kbL = -2\times 50W/m.K(-2000^0 C/m^2)\times 0.050m$$

$$q''_x(L) = 10{,}000 W/m^2$$

Comments: from an overall energy balance on the wall, it follows that

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q_x''(0) - q''_x(L) + \dot{q}L = 0$$

$$\dot{q} = \frac{q_x''(L) - q_x''(0)}{L} = \frac{10{,}00w/m^2 - 0}{0.050m} = 2.0\times 10^5 W/m^3$$

Problem 2:

A salt gradient solar pond is a shallow body of water that consists of three distinct fluid layers and is used to collect solar energy. The upper- and lower most layers are well mixed and serve to maintain the upper and lower surfaces of the central layer at uniform temperature T_1 and T_2, where $T_1>T_2$. Although there is bulk fluid motion in the mixed layers, there is no such motion in the central layer. Consider conditions for which solar radiation absorption in the central layer provides non uniform heat generation of the form $q=Ae^{-ax}$, and the temperature distribution in the central layer is

$$T(x) = -\frac{A}{ka^2}e^{-ax} + bx + c$$

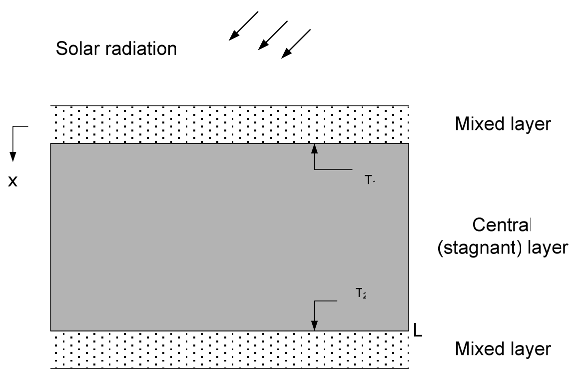
The quantities A (W/m3), a (1/m), B (K/m) and C(K) are known constants having the prescribed units, and k is the thermal conductivity, which is also constant.

(a) Obtain expressions for the rate at which heat is transferred per unit area from the lower mixed layer to the central layer and from central layer to the upper mixed layer.
(b) Determine whether conditions are steady or transient.
(c) Obtain an expression for the rate at which thermal energy is generated in the entire central layer, per unit surface area.

Known: Temperature distribution and distribution of heat generation in central layer of a solar pond.

Find: (a) heat fluxes at lower and upper surfaces of the central layer, (b) whether conditions are steady or transient (c) rate of thermal energy generation for the entire central layer.

Schematic:



Assumptions: (1) central layer is stagnant, (2) one-dimensional conduction, (3)constant properties.

Analysis (1) the desired fluxes correspond to conduction fluxes in the central layer at the lower and upper surfaces. A general form for the conduction flux is

$$q''_{cond} = \left[-k\frac{A}{ka} e^{-ax+} B \right]$$

Hence

$$q''_1 = q''_{cond(x=L0} = \left[-k\frac{A}{ka} e^{-al} + B \right] q''_u = q''_{cond(x=0)} = -k\left[\frac{A}{ka} + B \right]$$

(b) Conditions are steady if ∂T/∂t=0. Applying the heat equation,

$$\frac{\partial^2 T}{\partial^2 t} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -\frac{A}{k}e^{-ax} + \frac{A}{k}e^{-ax} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Hence conditions are steady since

$$\frac{\partial T}{\partial t} = 0 \quad \text{(for all 0<=x<=L)}$$

For the central layer, the energy generation is

$$\dot{E}''_g = \int_0^L q dx = A\int_0^L e - axdx$$

$$\dot{E}_g = -\frac{A}{a} e^{-ax}\Big|_0^L = -\frac{A}{a}(e^{-aL} - 1) = \frac{A}{a}(1 - e^{-aL})$$

Alternatively, from an overall energy balance,

$$q''_2 - q''_1 + \dot{E}_g = 0 \qquad \dot{E}''_g = q''_1 - q''_2 = (-q''_{cond(x=0)}) - (q''_{cond(x=L)})$$

$$\dot{E}_g = k\frac{A}{ka} + B - K\frac{A}{ka}e^{-aL} + B = \frac{A}{a}(1 - e^{-aL})$$

Comments: Conduction is the negative x-direction, necessitating use of minus signs in the above energy balance.

Problem 3:

The steady state temperatures distribution in a one-dimensional wall of thermal conductivity and thickness L is of the form $T=ax^3+bx^2+cx+d$. derive expressions for the heat generation rate per unit volume in the wall and heat fluxes at the two wall faces(x=0, L).

Known: steady-state temperature distribution in one-dimensional wall of thermal conductivity, $T(x)=Ax^3+Bx^2+CX+d$.

Find: expressions for the heat generation rate in the wall and the heat fluxes at the two wall faces(x=0, L).

Assumptions: (1) steady state conditions, (2) one-dimensional heat flow, (3) homogeneous medium.

Analysis: the appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \qquad \text{Or} \qquad \dot{q}=-k\frac{d^2T}{dx^2}$$

Hence, the generation rate is

$$\dot{q}=-\frac{d}{dx}\left[\frac{dT}{dx}\right]=-k\frac{d}{dx}[3Ax^2+2Bx+C+0]$$

$$\dot{q}=-k[6Ax+2B]$$

which is linear with the coordinate x. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x=-k\frac{dT}{dx}=-k[3Ax^2+2Bx+C]$$

Using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

Surface x=0; $\qquad q''_x(0)=-kC$

Surface x=L;

$q''_x(L) = -K\,[3AL^2+2BL+C]$

COMMENTS: (1) from an over all energy balance on the wall, find

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0$$

$$q''_x(0)-q''_x(L)=(-kC)-(-K)[3AL^2+2BL+C]+\dot{E}_g=0$$

$$\dot{E}''_g=-3AkL^2-2BkL$$

From integration of the volumetric heat rate, we can also find

$$\dot{E}''_g=\int_0^L \dot{q}(x)dx=\int_0^L -k[6Ax+2B]dx=-k[3Ax^2+2Bx]_0^L$$

$$\dot{E}''_g=-3AkL^2-2BkL$$

Problem 4:

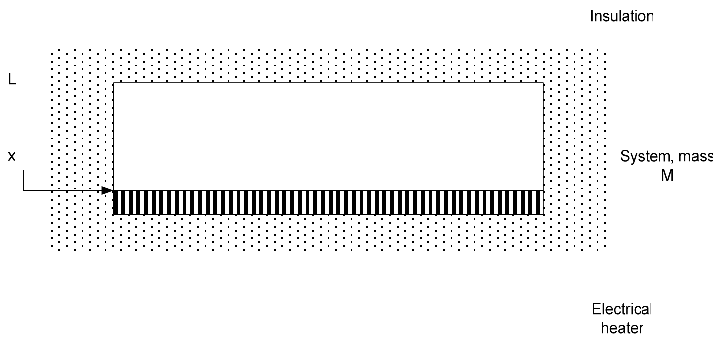
The one dimensional system of mass M with constant properties and no internal heat generation shown in fig are initially at a uniform temperature T_i. The electrical heater is suddenly energized providing a uniform heat flux q''_o at the surface x=0. the boundaries at x=L and else where are perfectly insulated.

(a) Write the differential equation and identify the boundary and initial conditions that could be used to determine the temperature as a function of position and time in the system.
(b) On T-x coordinates, sketch the temperature distributions for the initial condition (t<=0) and for several times after the heater is energized. Will a steady-state temperature distribution ever be reached?
(c) On q''_x-t coordinates, sketch the heat flux $q''_x(x,t)$ at the planes x=0, x=L/2, and x=L as a function of time.
(d) After a period of time t_e has elapsed, the heater power is switched off. Assuming that the insulation is perfect, the system will eventually reach final uniform temperature T_f. Derive an expression that can be used to determine T_f a function of the parameters q''_o, t_e, T_i, and the system characteristics M, c_p, and A(the heater surface area).

Known: one dimensional system, initially at a uniform temperature Ti, is suddenly exposed to a uniform heat flux at one boundary while the other boundary is insulated.

Find: (a) proper form of heat diffusion equation; identify boundary and initial conditions, (b) sketch temperature distributions for following conditions: initial condition (t<=0), several times after heater is energized ;will a steady-state condition be reached?, (c) sketch heat flux for x=0, L/2, L as a function of time, (d) expression for uniform temperature, Tf, reached after heater has been switched off the following an elapsed time , te, with the heater on.]

Schematic:



Assumptions: (1) one dimensional conduction, (2) no internal heat generation, (3) constant properties.

Analysis: (a) the appropriate form of the heat equation follows. Also the appropriate boundary and initial conditions are:

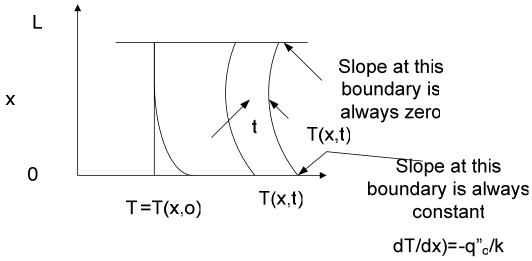
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial condition: $T(x, 0) = T_i$ uniform temperature

Boundary conditions: x=0 $q''_o = -k\partial T/\partial x)_0 \; t > 0$

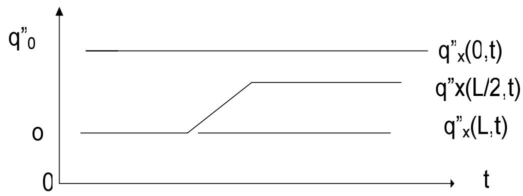
x=L $\partial T/\partial x)_L = 0$ Insulated

(b) The temperature distributions are as follows:



No steady-state condition will be reached since $\dot{E}_{in} - \dot{E}_{out}$ and $\dot{E}_{in}$ is constant.

(c) The heat flux as a function of time for positions x=0, L/2 and L appears as:



(d) If the heater is energized until $t=t_o$ and then switched off, the system will eventually reach a uniform temperature , T_f. Perform an energy balance on the system, for an interval of time $\Delta t=t_e$,

$$\dot{E}_{in} = \dot{E}_{st} \qquad E_{in} = Q_{in} = \int_0^{t_e} q''_0 A_s dt = q''_o A_s t_e \qquad E_{st} = Mc(T_f - T_i)$$

It follows that $q''_o A_s t_e = Mc(T_f - T_i)$ OR $T_f = T_i + \dfrac{q''_o A_s t_e}{Mc}$

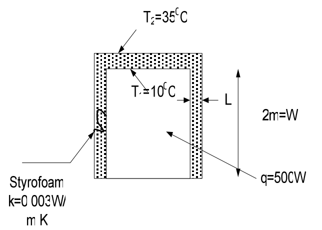
Problem 5:

A 1–m-long steel plate (k=50W/m.K) is well insulated on its sides, while the top surface is at 100^0C and the bottom surface is convectively cooled by a fluid at 20^0C. Under steady state conditions with no generation, a thermocouple at the midpoint of the plate reveals a temperature of 85^0C. What is the value of the convection heat transfer coefficient at the bottom surface?

Known: length, surfacethermal conditions, and thermal conductivity of a Plate. Plate midpoint temperature.

Find: surface convection coefficient

Schematic:



Assumptions: (1) one-dimensional, steady conduction with no generation, (2) Constant properties

Analysis: for prescribed conditions, is constant. Hence,

$$q''_{cond} = \frac{T_1 - T_2}{L/2} = \frac{15^0 C}{0.5m/50W/m.k} = 1500W/m^2$$

$$q'' = \frac{T_1 - T_\infty}{(L/k) + (1/h)} = \frac{30^0 C}{(0.02 + 1/h)m^2.K/W} = 1500W/m^2$$

$$h = 30W/m^2.K$$

Comments: The contributions of conduction and convection to the thermal resistance are

$$R''_{t,cond} = \frac{L}{K} = 0.02m^2.K/W$$

$$R''_{t,cond} = \frac{1}{h} = 0.033m^2.K/W$$

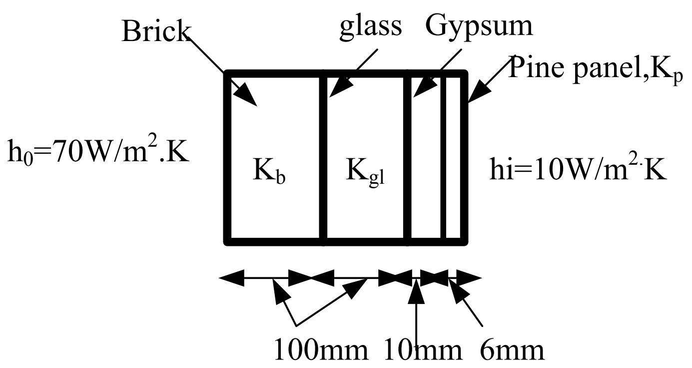
Problem 6:

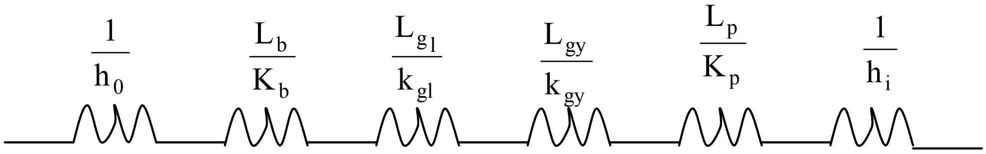
The wall of a building is a composite consisting of a 100-mm layer of common brick, a 100-mm layer of glass fiber(paper faced. $28kg/m^2$), a 10-mm layer of gypsum plaster (vermiculite), and a 6-mm layer of pine panel. If the inside convection coefficient is $10W/m^2.K$ and the outside convection coefficient is $70W/m^2.K$, what are the total resistance and the overall coefficient for heat transfer?

Known: Material thickness in a composite wall consisting of brick, glass fiber, and vermiculite and pine panel. Inner and outer convection coefficients.

Find: Total thermal resistance and overall heat transfer coefficient.

Schematic:





Assumptions: (1) one dimensional conduction, (2) constant properties, (3) negligible contact resistance.

Properties: T= 300K: Brick, k_b=1.3 W/m.K: Glass fiber ($28kg/m^3$), k_{gl}= 0.038W/m.K: gypsum, k_{gy}=0.17W/m.K: pine panel, k_p=0.12W/m.K.

Analysis: considering a unit surface Area, the total thermal resistance

$$R''_{tot} = \frac{1}{h_0} + \frac{L_B}{K_B} + \frac{L_{g1}}{k_{g1}} + \frac{L_{gy}}{k_{gy}} + \frac{L_p}{K_p} + \frac{1}{h_i}$$

$$R''_{tot} = \left[\frac{1}{70} + \frac{0.1}{1.3} + \frac{0.1}{0.038} + \frac{0.01}{0.17} + \frac{0.006}{0.12} + \frac{1}{10}\right]\frac{m^2.K}{W}$$

$$R''_{tot} = (0.0143 + 0.0769 + 2.6316 + 0.0588 + 0.0500 + 0.1)m^2.K/W$$

$$R''_{tot} = 2.93m^2..K/W$$

The overall heat transfer coefficient is

$$U = \frac{1}{R_{tot}A} = \frac{1}{R''_{tot}} = (2.93m^2.K/W)^{-1}$$

$$U = 0.341W/m^2.K.$$

Comments: an anticipated, the dominant contribution to the total resistance is made by the insulation.

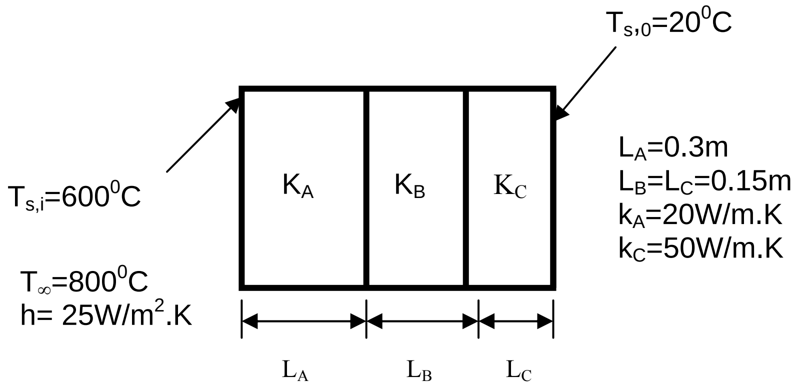
Problem 7:

The composite wall of an oven consists of three materials, two of which are known thermal conductivity, k_A=20W/m.K and k_C=50W/m.K, and known thickness, L_A=0.30m and L_C=0.15m. The third material, B, which is sandwiched between materials A and C, is of known thickness, L_B=0.15m, but unknown thermal conductivity $k_{B.}$ Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,0}$=200C, an inner surface temperature of $T_{s,i}$=600^0C and an oven air temperature of T =800^0C. The inside convection coefficient h is known to be 25W/m^2.K. What is the value of k_B?

Known: Thickness of three material which form a composite wall and thermal conductivities of two of the materials. Inner and outer surface temperatures of the composites; also, temperature and convection coefficient associated with adjoining gas.

Find: value of unknown thermal conductivity, $k_{B.}$

Schematic:



Assumptions: (1) steady state conditions, (2) one-dimensional conduction, (3) constant properties, (4) negligible contact resistance, (5) negligible radiation effects.

Analysis: Referring to the thermal circuit, the heat flux may be expressed as

$$q'' = \frac{T_{s,i} - T_{s,0}}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}} = \frac{(600 - 20)^0 C}{\frac{0.3m}{0.018} + \frac{0.15m}{K_B} + \frac{0.15m}{50W/m.K}}$$

$$= \frac{580}{0.018 + 0.15/K_B} W/m^2$$

The heat flux can be obtained from

$$q'' = h(T_\infty - T_{s,i}) = 25W/m^2.K(800 - 600)^0 C$$

$$q'' = 5000W/m^2$$

Substituting for heat flux,

$$\frac{0.15}{K_B} = \frac{580}{q''} - 0.018 = \frac{580}{5000} - 0.018 = 0.098$$

$$K_B = 1.53W/m.K.$$

Comments: radiation effects are likely to have a significant influence on the net heat flux at the inner surface of the oven.

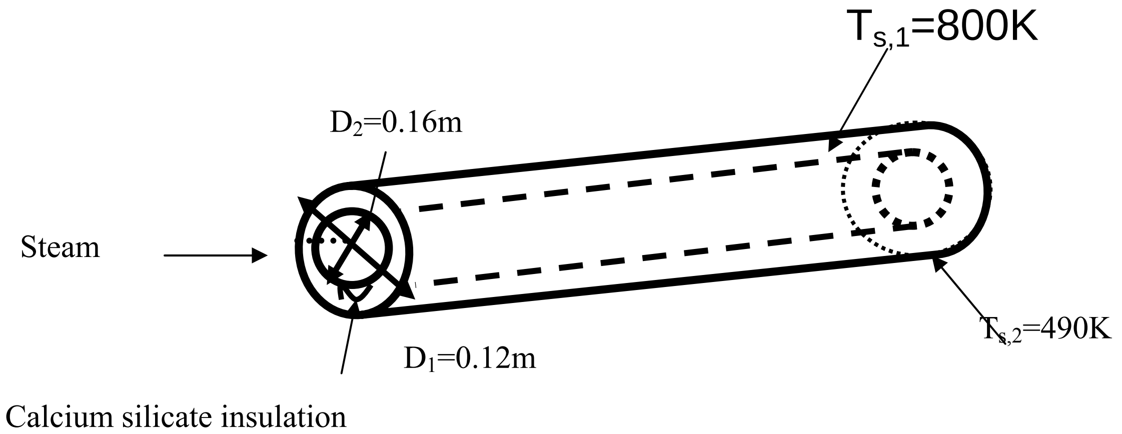
Problem 8:

A steam pipe of 0.12 m outside diameter is insulated with a 20-mm-thick layer of calcium silicate. If the inner and outer surfaces of the insulation are at temperatures of $T_{s,1}$=800K and $T_{s,2}$=490 K, respectively, what is the heat loss per unit length of the pipe?

Known: Thickness and surface temperature of calcium silicate insulation on a steam pipe.

Find: heat loss per unit pipe length.

Schematic:



Assumptions: (steady state conditions, (2) one-dimensional conduction, (3) constant properties.

Properties: calcium silicate (T=645K): k=0.089W/m.K

Analysis: The heat per unit length is

$$q_r' = \frac{q_r}{q_L} = \frac{2\pi K(T_{s,1} - T_{s,2})}{\ln(D_2/D_1)}$$

$$q_r' = \frac{2\pi(0.089\,W/m.K)(800-490)K}{\ln(0.16m/0.12m)}$$

$$q_r' = 603\,W/m$$

Comments: heat transferred to the outer surface is dissipated to the surroundings by convection and radiation.

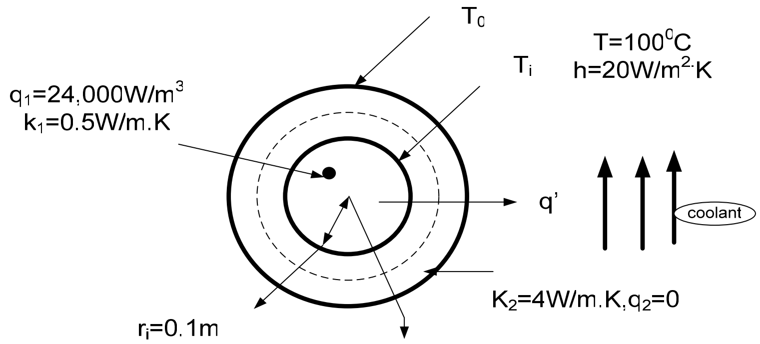
Problem 9:

A long cylindrical rod of 10 cm consists of a nuclear reacting material (k=0.0W/m.K) generating 24,000W/m^3 uniformly throughout its volume. This rod is encapsulated within another cylinder having an outer radius of 20 cm and a thermal conductivity of 4W/m.K. the outer surface is surrounded by a fluid at 100^{0}C, and the convection coefficient between the surface and the fluid is 20W/m^2.K. Find the temperatures at the interface between the two cylinders and at the outer surface.

Known: A cylindrical rod with heat generation is cladded with another cylinder whose outer surface is subjected to a convection process.

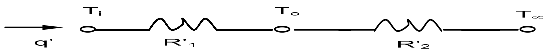
Find: the temperature at the inner surfaces, $T_{1,}$ and at the outer surface, T_c.

Schematic:



Assumptions: (1) steady-state conditions, (2) one-dimensional radial conduction, (3), negligible contact resistance between the cylinders.

Analysis: The thermal circuit for the outer cylinder subjected to the convection process is



$$R_1^{'} = \frac{\ln r_o / r_1}{2\pi k_2}$$

$$R_2^{'} = \frac{1}{h 2\pi r_o}$$

Using the energy conservation requirement, on the inner cylinder,

$$\dot{E}_{out} = \dot{E}_g$$

Find that

$$q^{'} = \dot{q}_1 \times \pi r_1^2$$

The heat rate equation has the form $\dot{q} = \Delta T / R^{'}$, hence

$$T_i - T_\infty = q^{'} \times (R_1^{'} \times R_2^{'}) \text{and} q^{'} = \Delta T / R^{'}$$

Numerical values:

$$R_1^{'} = \ln 0.2/0.1/2\pi \times 4 W/m.K = 0.0276 K.m/W$$
$$R_2^{'} = 1/20 W/m^2.K \times 2\pi \times 0.20 m = 0.0398 K.m/W$$
$$q^{'} = 24{,}000 W/m^3 \times \pi \times (0.1)^2 m^2 = 754.0 W/m$$

Hence

$$T_i = 100^0 C + 754.0 W/m \times (0.0276 + cccc) K.m/W = 100 + 50.8 = 150.8^0 C$$

$$T_C = 100^0 C + 754.0 W/m \times 0.0398 K.m/W = 100 + 30 = 130^0 C$$

Comments: knowledge of inner cylinder thermal conductivity is not needed.

Problem 10:

An electrical current of 700 A flows through a stainless steel cable having a diameter of 5mm and an electrical resistance of $6*10^{-4}$ /m (i.e. perimeter of cable length). The cable is in an environment having temperature of 300C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately $25W/m^2.K$.

(a) If the cable is bar, what is its surface temperature?

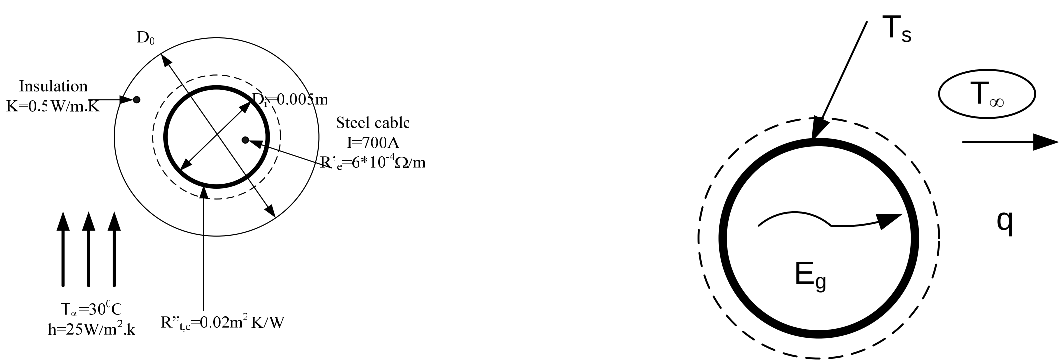
(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of $0.02m^2K/W$, what are the insulation and cable surface temperatures?

(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation (k=0.5W/m.K) will yields the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when the thickness is used?

Known: electric current flow, resistance, diameter and environmental conditions associated with a cable.

Find: (a) surface temperature of bare cable, (b) cable surface and insulation temperatures for a thin coating of insulation, (c) insulation thickness which provides the lowest value of the maximum insulation temperature. Corresponding value of this temperature.

Schematic:



Assumptions: (1) steady-state conditions, (2) one-dimensional conduction in r, (3) constant properties.

Analysis: (a) the rate at which heat is transferred to the surroundings is fixed by the rate of heat generation in the cable. Performing an energy balance for a control surface about the cable, it follows that $\dot{E}_g = q$ or, for the bare cable, $I^2R_e'L = h(\pi D_i L)(T_s - T_\infty)$.with $q' = I^2R_e' = (700A)^2(6\times10^{-4}\Omega/m) = 294W/m.$

It follows that

$$T_s = T_\infty + \frac{q'}{h\pi D_i} = 30^0C + \frac{294W/m}{(25W/m^2.K)\pi(0.005m)}$$

$$T_s = 778.7^0C$$

(b) With thin coating of insulation, there exists contact and convection resistances to heat transfer from the cable. The heat transfer rate is determined by heating within the cable, however, and therefore remains the same,

$$q = \frac{T_s - T_\infty}{R_{t,c} + \frac{1}{h\pi D_i L}} = \frac{T_s - T\infty}{\frac{R_{t,c}}{\pi D_i L} + \frac{1}{h\pi D_i L}}$$

$$q' = \frac{\pi D_i(T_s - T_\infty)}{R_{t,c} + \frac{1}{h}}$$

And solving for the surface temperature, find

$$T_s = \frac{q'}{\pi D_i}\left(R_{t,c} + \frac{1}{h}\right) + T_\infty = \frac{294W/m}{\pi(0.005m)}\left(0.02\frac{m^2.K}{W} + 0.04\frac{m^2.K}{W}\right) + 30^0C$$

$$T_s = 1153^0C$$

The insulation temperature is then obtained from

$$q = \frac{T_s - T_\infty}{R_{t,e}}$$

Or

$$T_i = T_s - qR_{t,c} = 1153^0\mathrm{C} - q\frac{R''_{t,c}}{\pi D_i L} = 1153^0\mathrm{C} - \frac{294\frac{\mathrm{W}}{\mathrm{m}}\times 0.02\frac{\mathrm{m^2.K}}{\mathrm{W}}}{\pi(0.005\mathrm{m})}$$

$$T_i = 778.7^0\mathrm{C}$$

(c) The maximum insulation temperature could be reduced by reducing the resistance to heat transfer from the outer surface of the insulation. Such a reduction is possible $D_i<D_{cr.}$

$$r_{cr} = \frac{k}{h} = \frac{0.5\mathrm{W/m.K}}{25\mathrm{W/m^2.K}} = 0.02\mathrm{m}$$

Hence, $D_{cr} = 0.04\mathrm{m} > D_i = 0.005\mathrm{m}$. To minimize the maximum temperature, which exists at the inner surface of the insulation, add insulation in the amount.

$$t = \frac{D_0 - D_i}{2} = \frac{D_{cr} - D_i}{2} = \frac{(0.04 - 0.005)m}{2}$$

$$t = 0.0175m$$

The cable surface temperature may then be obtained from

$$q'' = \frac{T_s - T_\infty}{\frac{R''_{t,c}}{\pi D_i} + \frac{\ln(D_{c,r}/D_i)}{2\pi\pi} + \frac{1}{h\pi\pi_{c,r}}} = \frac{T_s - 30^0\mathrm{C}}{\frac{0.02\mathrm{m^2.K/W}}{\pi(0.005\mathrm{m})} + \frac{\ln(0.04/0.005)}{2\pi\pi(0.5\mathrm{W/.})} + \frac{1}{25\frac{\mathrm{W}}{\mathrm{m^2.K}}\pi(0.04\mathrm{m})}}$$

hence,

$$294\frac{\mathrm{W}}{\mathrm{m}} = \frac{T_s - 30^0\mathrm{C}}{(1.27 + 0.66 + 0.32)\mathrm{m.K/W}} = \frac{T_s - 30^0\mathrm{C}}{2.25\mathrm{m.K/W}}$$

$$T_s = 692.5^0\mathrm{C}$$

recognizing that, $q = (T_s - T_i)/R_{t,c,}$

$$T_i = T_s - qR_{t,c,} = T_s - q\frac{R''_{t,c}}{\pi D_i L} = 692.5^0\mathrm{C} - \frac{294\frac{\mathrm{W}}{\mathrm{m}}\times 0.02\frac{\mathrm{m^2.K}}{\mathrm{W}}}{\pi(0.005\mathrm{m})}$$

$$T_i = 318.2^0\mathrm{C}$$

Comments: use of the critical insulation in lieu of a thin coating has the effect of reducing the maximum insulation temperature from 778.7^{0}C to 318.2^{0}C. Use of the critical insulation thickness also reduces the cable surface temperatures to 692.5^{0}C from 778.7^{0}C with no insulation or fro 1153^{0}C with a thin coating.

Problem 11:

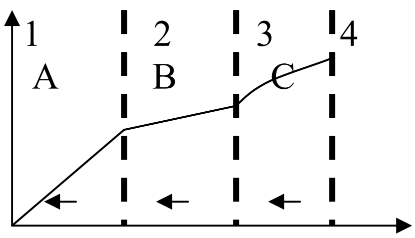
The steady state temperature distribution in a complete plane wall of three different methods, each of constant thermal conductivity, is shown below.

(a) On the relative magnitudes of q_2'' and q_3'' and of q_3'' and q_4''.
(b) Comment on the relative magnitudes of k_A and k_B and ok k_B and k_C.
(c) Plot the heat flux as a function of x.

Known: Temperature distribution in a composite wall.

Find: (a) relative magnitudes of interfacial heat fluxes, (b) relative magnitudes of thermal conductivities, and (c) heat fluxes as a function of distance x.

Schematic:



Assumptions: (1) steady-state conditions, (2) one-dimensional conduction, (3) constant properties.

Analysis: (a) for the prescribed conditions (one-dimensional, steady state, constant k), the parabolic temperature distribution in C implies the existence of heat generation. Hence, since dT/dx increases with decreasing x, the heat flux in C increases with decreasing. Hence,

$$q_3'' > q_4''$$

However, the linear temperatures distributions in A and B indicate no generation, in which case

$$q_2'' = q_3''$$

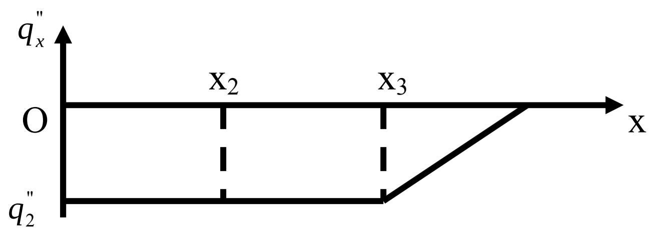
(b) Since conservation of energy requires that $q''_{3,B} = q''_{3,C}$ and dT/dx/$_B$ <dT/dx)$_{C,}$ it follows from Fourier's law that

$K_A > K_C$

Similarly, since $q''_{2,A} = q''_{2,B}$ and $dT/dx)_A > dT/dx)_B$, it follows that

$K_A <$ $K_{B.}$

(d) It follows that the flux distribution appears as shown below.



Comments: Note that, with dT/dx)$_{4,C}$=0, the interface at 4 is adiabatic.

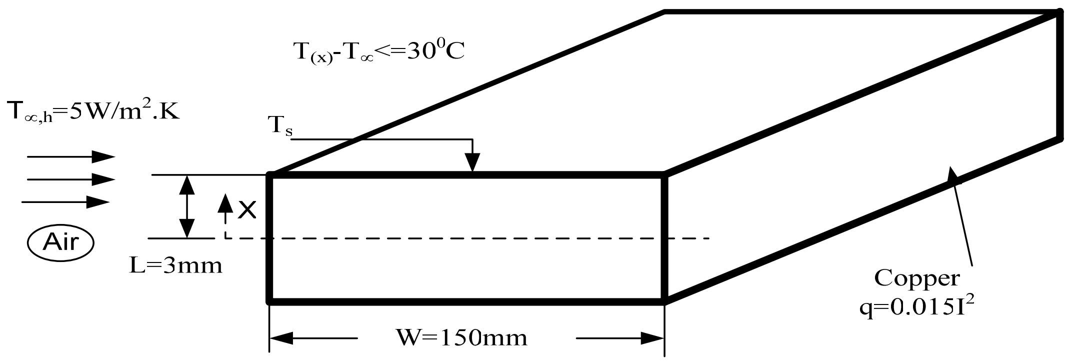
Problem 12:

When passing an electrical current I, a copper bus bar of rectangular cross section (6mm*150mm) experiences uniform heat generation at a rate $\dot{q} = aI^2, \text{where} \alpha = 0.015 W/m^2.A^2$. If the bar is in ambient air with h=5W/m^2. K and its maximum temperature must not exceed that of the air by more than 300C, what is the allowable current capacity for the bar?

Known: Energy generation, q (I), in a rectangular bus bar.

Find: maximum permissible current.

Schematic:



Assumptions: (1) one-dimensional conduction in x (W>>L), (2) steady-state conditions, (3) constant properties, (4) negligible radiation effects.

Properties: copper: k 400W/m. K

Analysis: the maximum mid plane temperature is

$$T_0 = \frac{\dot{q}L^2}{2K} + T_s$$

Or substituting the energy balance results,

$$T_o - T_\infty = \dot{q}L\left(\frac{L}{2k}+\frac{1}{h}\right) = 0.015\, I^2 L\left(\frac{L}{2k}+\frac{1}{h}\right).$$

hence ,

$$T_s = T_\infty + \dot{q}L/h,$$

$$I = \left(\frac{T_o - T_\infty}{0.015\, L(L/2k + 1/h)}\right)^{\frac{1}{2}}$$

$$I_{\max} = \left(\frac{30\,^0C}{0.015\,(W/m^3.A^2)\,0.003\,m\,\frac{0.003\,m}{800\,W/m.K} + \frac{1}{5W/m^2.K}}\right)^{\frac{1}{2}}$$

$$I_{\max} = 1826\ A$$