Module 1: Worked out problems

Problem 1:

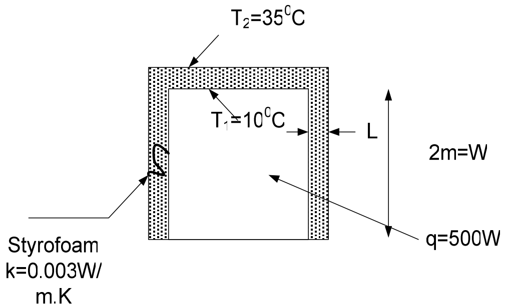
A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of Styrofoam insulation (k=0.030W/m.K) which must be applied to the top and side walls to ensure a heat load less than 500 W, when the inner and outer surfaces are -10 ºC and 35^0C?

Solution:

Known: Dimensions of freezer component, inner and outer surfaces temperatures.

Find: Thickness of Styrofoam insulation needed to maintain heat load below prescribed value.

Schematic:



Assumptions: (1) perfectly insulted bottom, (2) one-dimensional conduction through five walls of areas A=4m^2, (3) steady-state conditions

Analysis: Using Fourier's law, the heat rate is given by

$$q = q''.A = k\frac{\Delta T}{L}A_{total}$$

Solving for L and recognizing that $A_{total} = 5*W^2$

$$L = \frac{5k\Delta TW^2}{q}$$

$$L = \frac{5*0.03W/m.k*45^0C*4m^2}{500W}$$

$$L = 0.054m = 54mm$$

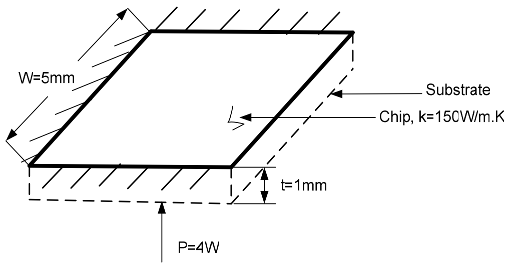
Comments: The corners will cause local departures from one–dimensional conduction and, for a prescribed value of L, a slightly larger heat loss.

Problem 2:
A square silicon chip (k=150W/m.k) is of width W=5mm on a side and of thickness t=1mm. the chip is mounted in a substrate such that its side and back surfaces are insulated, while the front surface is exposed to a coolant. If 4W are being dissipated in circuits mounted to the back surface of the chip, what is the steady-state temperature difference between back and front surfaces?

Known: Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

Find: temperature drop across the chip

Schematic:



Assumptions: (1) steady-state conditions, (2) constant properties, (3) uniform dissipation, (4) negligible heat loss from back and sides, (5) one-dimensional conduction in chip.

Analysis: All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, Fourier's law,

$$P = q = kA\frac{\Delta T}{t}$$

$$\Delta T = \frac{t.P}{kW^2} = \frac{0.001m * 4W}{150W/m.K(0.005m^2)}$$

$$\Delta T = 1.1^0 C$$

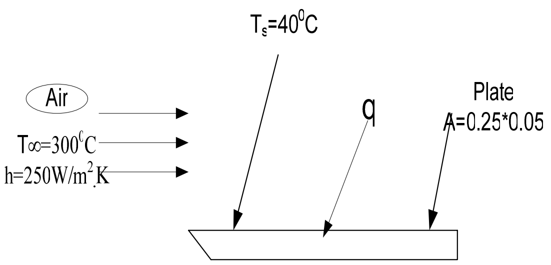
Comments: for fixed P, the temperature drop across the chip decreases with increasing k and W, as well as with decreasing t.

Problem 3:
Air at 300^0C flows over a plate of dimensions 0.50 m, by 0.25 m. if the convection heat transfer coefficient is 250 W/m2.K; determine the heat transfer rate from the air to one side of the plate when the plate is maintained at 40^0C.

Known: air flow over a plate with prescribed air and surface temperature and convection heat transfer coefficient.

Find: heat transfer rate from the air to the plate

Schematic:



Assumptions: (1) temperature is uniform over plate area, (2) heat transfer coefficient is uniform over plate area

Analysis: the heat transfer coefficient rate by convection from the airstreams to the plate can be determined from Newton's law of cooling written in the form,

$$q = q^{"}.A = hA(T_\infty - T_s)$$

where A is the area of the plate. Substituting numerical values,

$$q = 250W/m^2.K*(0.25*0.50)m^2(300-40)^0C$$
$$q = 8125W$$

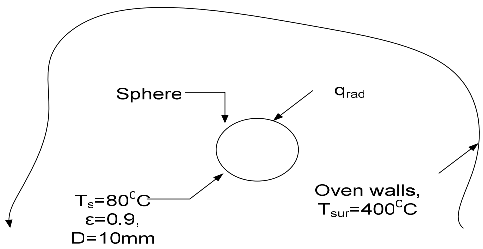
Comments: recognize that Newtown's law of cooling implies a direction for the convection heat transfer rate. Written in the form above, the heat rate is from the air to plate.

Problem 4 :
A water cooled spherical object of diameter 10 mm and emissivity 0.9 is maintained at 4000C. What is the net transfer rate from the oven walls to the object?

Known: spherical object maintained at a prescribed temperature within a oven.

Find: heat transfer rate from the oven walls to the object

Schematic:



Assumptions: (1) oven walls completely surround spherical object, (2) steady-state condition, (3) uniform temperature for areas of sphere and oven walls, (4) oven enclosure is evacuated and large compared to sphere.

Analysis: heat transfer rate will be only due to the radiation mode. The rate equation is

$$q_{rad} = \varepsilon A_s \sigma (T_{sur}{}^4 - T_s{}^4)$$

Where As=πD2, the area of the sphere, substituting numerical values,

$$q_{rad} = 0.9 * \pi (10*10^{-3})^2 m^2 * 5.67*10^{-8} W/m^2.K[(400+273)^4 - (80+273)^4]K^4$$

$$q_{rad} = 3.04W$$

Comments: (1) this rate equation is useful for calculating the net heat exchange between a small object and larger surface completely surrounds the smaller one . this is an essential, restrictive condition.

(2) Recognize that the direction of the net heat exchange depends upon the manner in which T_{sur} and T_s are written.

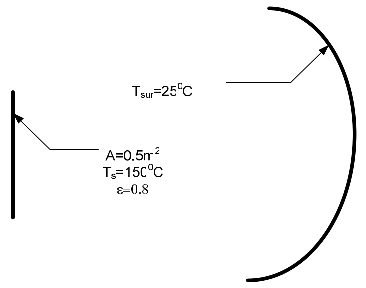
(3) When performing radiant heat transfer calculations, it is always necessary to have temperatures in Kelvin (K) units.

Problem 5:
A surface of area $0.5m^2$, emissivity 0.8 and temperature 150^0C is placed in a large, evacuated chamber whose walls are maintained at $25\ ^0C$. What is the rate at which radiation is emitted by the surface? What is the net rate at which radiation is exchanged between the surface and the chamber walls?

Known: Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

Find: (a) rate of surface radiation emission, (b) net rate of radiation exchange between the surface and chamber walls.

Schematic:



Assumptions: (1) area of the enclosed surface is much less than that of chamber walls.

Analysis (a) the rate at which radiation is emitted by the surface is emitted

$$q_{emit} = q_{emit.}A = \varepsilon A\sigma Ts^4$$
$$qemit = 0.8(0.5m^2)5.67*10^{-8}W/m^2.K^4[(150+273)K]^4$$
$$qemit = 726W$$

(b) The net rate at which radiation is transferred from the surface to the chamber walls is

$$q = \varepsilon A\sigma(Ts4 - Tsurr4)$$
$$q = 0.8(0.5m^2)5.67*10^{-8}W/m^2.K^4[(423K)^4 - (298K)^4$$
$$q = 547W$$

Comments: the foregoing result gives the net heat loss from the surface which occurs at the instant the surface is placed in the chamber. The surface would, of course, cool due to this heat loss and its temperature, as well as the heat loss, would decrease with increasing time. Steady-state conditions would eventually be achieved when the temperature of the surface reached that of the surroundings.

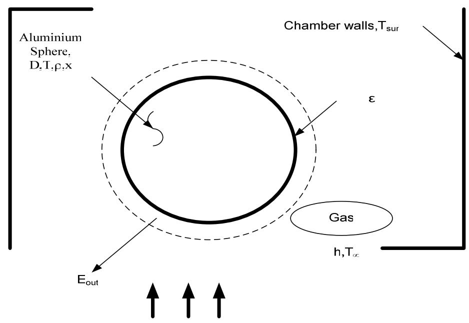
Problem 6:

A solid aluminium sphere of emissivityε is initially at an elevated temperature and is cooled by placing it in chamber. The walls of the chamber are maintained at a lower temperature and a cold gas is circulated through the chamber. Obtain an equation that could be used to predict the variation of the aluminium temperature with time during the cooling process. Do not attempt to solve.

Known: Initial temperature, diameter and surface emissivity of a solid aluminium sphere placed in a chamber whose walls are maintained at lower temperature. Temperature and convection coefficient associated with gas flow over the sphere.

Find: equation which could be used to determine the aluminium temperature as a function of time during the cooling process.

Schematic:



Assumptions: (1) at any time t, the temperature T of the sphere is uniform, (2) constant properties; (3) chamber walls are large relative to sphere.

Analysis: applying an energy balance at an instant of time to a control volume about the sphere, it follows that

$$\dot{E}_{st} = -\dot{E}_{out}$$

Identifying the heat rates out of the CV due to convection and radiation, the energy balance has the form

$$\frac{d}{dt}(\rho VcT) = -(q_{conv} + q_{rad})$$

$$\frac{dT}{dt} = -\frac{A}{\rho Vc}[h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{surr}^4)]$$

$$\frac{dT}{dt} = \frac{6}{\rho cD}[h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{surr}^4)]$$

Where $A=\pi D^2$, $V=\pi D^3/6$ and $A/V=6/D$ for the sphere.

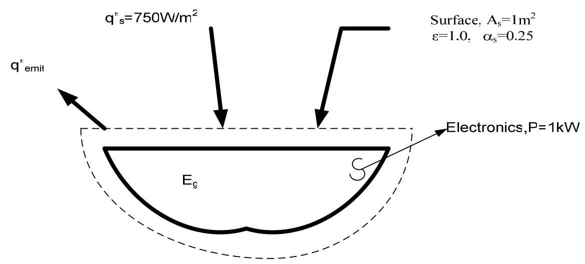
Comments: (1) knowing T=Ti at t =0, the foregoing equation could be solved by numerical integration to obtain T (t). (2) The validity of assuming a uniform sphere temperature depends upon h, D. and the thermal conductivity of the solid (k). The validity of the assumption improves with increasing k and decreasing h and D.

Problem 7: In an orbiting space station, an electronic package is housed in a compartment having surface area $A_s = 1m^2$ which is exposed to space. Under normal operating conditions, the electronics dissipate 1 kW, all of which must be transferred from the exposed surface to space. If the surface emissivity is 1.0 and the surface is not exposed to the sun, what is its steady- state temperature? If the surface is exposed to a solar flux of $750W/m^2$ and its absorptivity to solar radiation is 0.25, what is its steady –state temperature?

Known: surface area of electronic package and power dissipation by the electronics. Surface emissivity and absorptivity to solar radiation. Solar flux.

Find: surface temperature without and with incident solar radiation.

Schematic:



Assumptions: steady state condition

Analysis: applying conservation of energy to a control surface about the compartment, at any instant

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

It follows that, with the solar input,

$$\alpha_S A_s q''_S - A_s q''_{emit} + P = 0$$

$$\alpha_S A_s q''_S - A_s \varepsilon \sigma T_s^4 + P = 0$$

$$T_s = \left(\frac{\alpha_S A_s q''_S + P}{A_s \varepsilon \sigma} \right)^{\frac{1}{4}}$$

In the shade ($q'' = 0$)

$$T_s = \left(\frac{1000W}{1m^2 * 1 * 5.67 * 10^{-8} W/m^2.K^4} \right)^{\frac{1}{4}} = 364K$$

In the sun,

$$T_s = \left(\frac{0.25 * 1m^2 * 750W/m^2 + 1000W}{1m^2 * 1 * 5.67 * 10^{-8} W/m^2.K^4} \right)^{\frac{1}{4}} = 380K$$

Comments: in orbit, the space station would be continuously cycling between shade, and a steady- state condition would not exist.

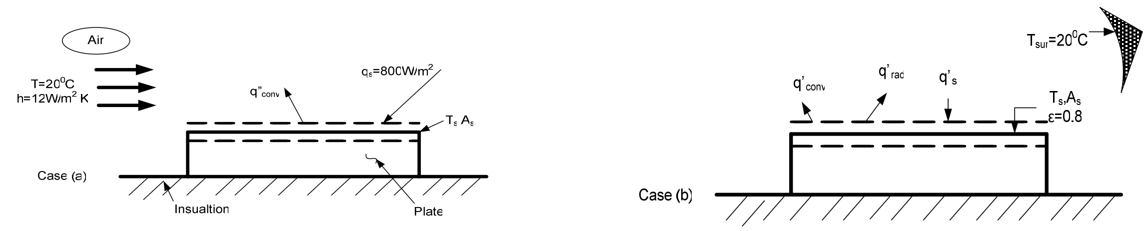
Problem 8: The back side of a metallic plate is perfectly insulated while the front side absorbs a solar radiant flux of 800 W/m^2. The convection coefficient between the plate and the ambient air is 112 w /m^2. K.

(a) Neglecting radiation exchange with the surroundings, calculate the temperature of the plate under steady-state conditions if the ambient air temperature is 20^{0}C.
(b) For the same ambient air temperature, calculate the temperature of the plate if its surface emissivity is 0.8 and the temperature of the surroundings is also20^{0}C.

Known: front surface of insulated plate absorbs solar flux, qs” and experiences for case
(a) Convection process with air at T and for case
(b): the same convection process and radiation exchange with surroundings at T_{sur}

Find: temperature of the plate, T_s, for the two cases.

Schematic:



Assumptions: (1) steady state conditions, (2) no heat loss out backside of plate, (3) surroundings large in comparison plate.

Analysis: (a) apply a surface energy balance, identifying the control surface as shown on the schematic. For an instant of time the conversation requirement is $\dot{E}_{in} - \dot{E}_{out}$ =0. The relevant processes are convection between the plate and the air, q_{conv}, and absorbed solar flux, qs”. Considering the plate to have an area A_s solve for T_s and substitute numerical values to find

$$q''_s.A_s - hA_s(T_s - T_\infty) = 0$$

$$T_s = T_\infty + q''_s / h$$

$$T_s = 20^0 C + \frac{800W/m^2}{12W/m^2.K} = 20^0 C + 66.7^0 C = 87^0 C$$

(b) Considering now the radiation exchange between the surface and its surroundings, the surface energy balance has the form $\dot{E}_{in} - \dot{E}_{out}$ =0.

$$q''_s . A_s - qconv - qrad = 0$$

$$qs'' A_s - hA_s (T_s - T_\infty) - \varepsilon A_s (T_s{}^4 - T_\infty^4) = 0$$

$$800 \frac{W}{m^2} - 12 \frac{W}{m^2.K}(T_s - [20 + 273]K) - 0.8 * 5.67 * 10 - 8 \frac{W}{m^2.K^4}(T_s^4 - [20 + 273]4K^4) = 0$$

$$12T_s + 4.536 * 10 - 8T_s{}^4 = 4650.3$$

By trial and error method, find that T_s=338K=65^{0}C.

Comments: note that by considering radiation exchange, Ts decreases as expected. Note the manner in which q_{conv} is formulated using qconv is formulated using Newton's law of cooling: since qconv is shown leaving the control surface, the rate equation must be $h(T_s - T_\infty)$ and not h (T_∞-T_s).