

# Module

11

# Design of Joints for Special Loading

# Lesson 2

## Design of Eccentrically Loaded Welded Joints

## Instructional Objectives:

At the end of this lesson, the students should be able to **understand**:

- Ways in which eccentric loads appear in a welded joint.
- General procedure of designing a welded joint for eccentric loading.
- How to avoid eccentric loading in simple cases..

There are many possible ways in which an eccentric loading can be imposed on a welded joint. A few cases are discussed below.

### 1. Eccentrically loaded transverse fillet joint:

Consider a cantilever beam fixed to a wall by two transverse fillet joints as shown in **figure 11.2.1**. The beam is subjected to a transverse load of magnitude **F**.

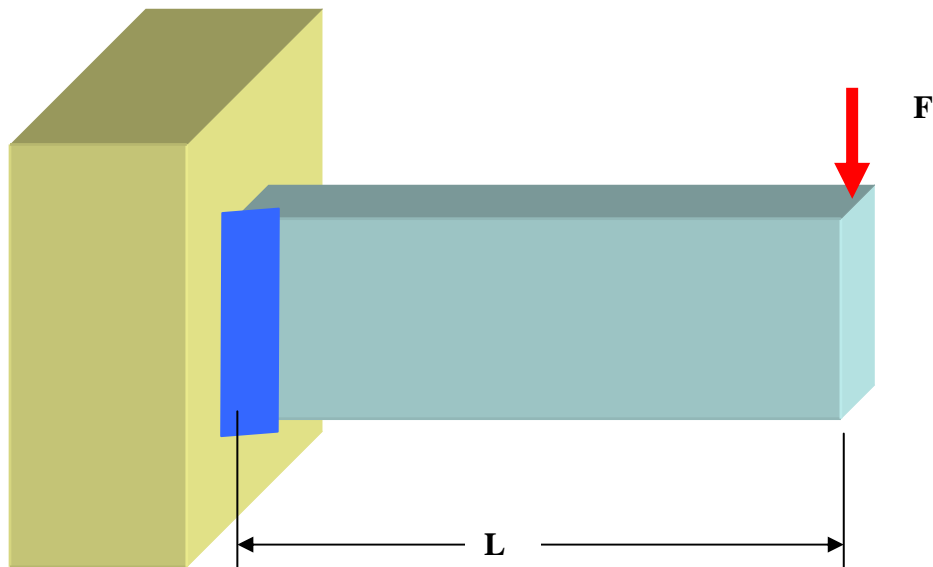


Figure 11.2.1: Eccentrically loaded welded joint

Like any welded joint, the design is based upon the strength of the joint against failure due to shear force along the throat section. In this case any small section of the throat is subjected to

(a) direct shear stress of magnitude  $\frac{F}{2bt}$ ,

where  $b$  = length of the weld,

$t =$  thickness at the throat

and the factor 2 appears in the denominator for double weld.

(b) Indirect shear stress due to bending of the beam, whose magnitude is calculated in the following manner and whose direction is perpendicular to that of the direct shear stress. Consider a small area  $dA$  in throat section lying at a distance  $y$  from the centerline, which is also the centroidal axis of the weld. An important assumption is made regarding the magnitude of the shear stress at a point within the area  $dA$ . It is assumed that the shear stress is proportional to the distance from the centroidal axis, that is  $y$  in this case, and directed along the horizontal. The proportionality constant is calculated using the moment equilibrium equation about centroid of the throat section. This gives,

$$\oint \tau(y)y \, dA = FL \quad \text{where } \tau(y) = cy.$$

Hence,  $c = \frac{FL}{\oint y^2 \, dA}$ . Therefore the magnitude of the shear stress is

$$\tau = \frac{FLy}{I_y}$$

where the second moment of area of the throat section  $I_p = \oint y^2 \, dA = \frac{tb^3}{12}$ . So,

for an eccentrically loaded joint shown in [figure 11.2.2](#) the maximum shear stress occurs at the extreme end and its magnitude is

$$\tau_{\max} = \sqrt{\left(\frac{F}{2bt}\right)^2 + \left(\frac{3FL}{tb^2}\right)^2}.$$

In order to design a safe welded joint

$$\tau_{\max} \leq S_s,$$

where  $S_s$  is the allowable shear stress of the weld material.

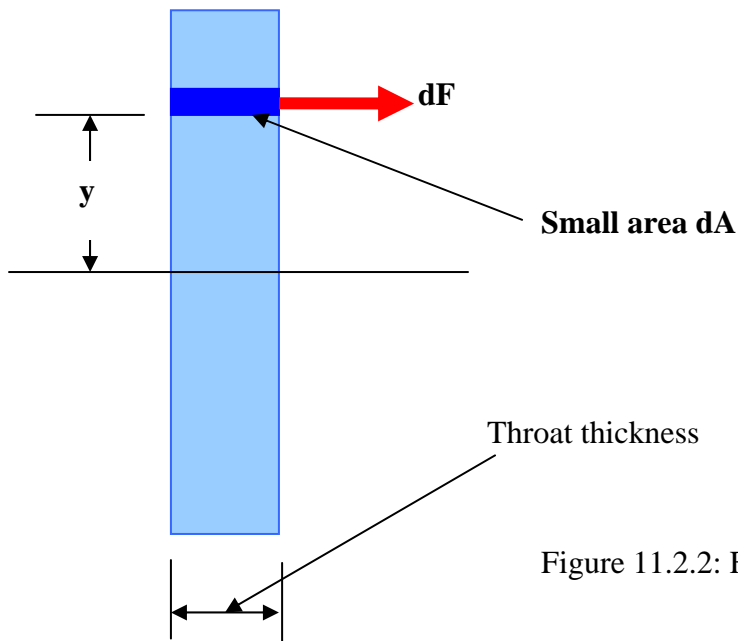


Figure 11.2.2: Forces on weld in bending

## 2. Eccentrically loaded parallel fillet joint:

Consider a cantilever beam connected to a wall by means of two parallel joints as shown in figure 11.2.3. The beam is required to carry a load  $F$  in transverse direction.

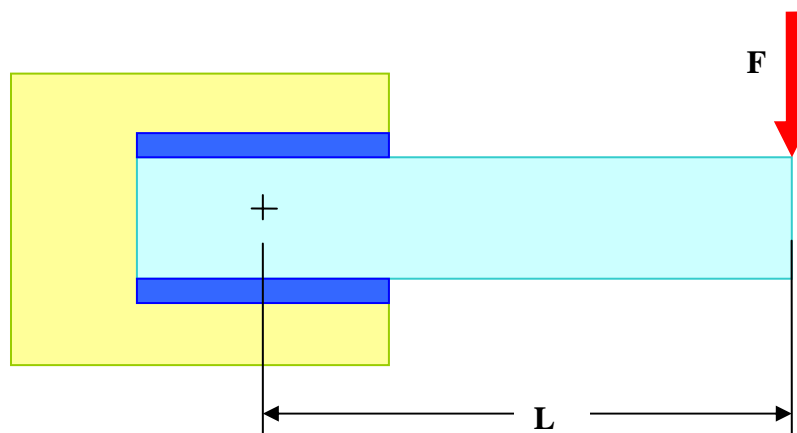


Figure 11.2.3: Eccentrically loaded parallel fillet joint

In order to select the size of the weld it is once again considered that the joint fails in shear along the throat section. For the given loading, the throat area is subjected to two shear stresses.

(a) Direct shear of magnitude  $\frac{F}{2lt}$

where  $l$  = length of the weld

$t$  = thickness of the throat.

(b) Indirect shear stress owing to eccentricity of the loading. The magnitude and direction of the shear stresses are calculated using the similar assumption as in the last section. The magnitude of shear stress at any point is assumed to be proportional to its distance from the centroid of the throat area and the direction is perpendicular to the line joining the point and the centroid. The sense is the same as that of the rotation of the welded joint as a whole (if permitted). With this assumption the shear stress at a point at a distance  $r$  from the centroid is given by

$$\tau(r) = cr$$

where the proportionality constant  $c$  is to be calculated using the moment equilibrium equation. Taking moment about the centroid one finds

$$\oint \tau(r)r \, dA = FL,$$

where  $L$  = distance of the line of action of  $\mathbf{F}$  from centroid.

Thus,

$$c = \frac{FL}{J},$$

where  $J = \oint r^2 \, dA$  is the polar moment of the throat section about its centroid.

The net shear stress at a point is calculated by vector addition of the two kinds of shear stresses discussed above. (Note that the vector addition of stresses is in general not defined. In this case the resultant force at a point within an infinitesimal area is obtained using vector addition of forces calculated from the individual stress values. The resultant stress is the force divided by area. Since everywhere the same value of area is involved in calculation, the net stress is therefore the vector sum of the component stresses.) The weld size is

designed such that the maximum shear stress does not exceed its allowable limiting value.

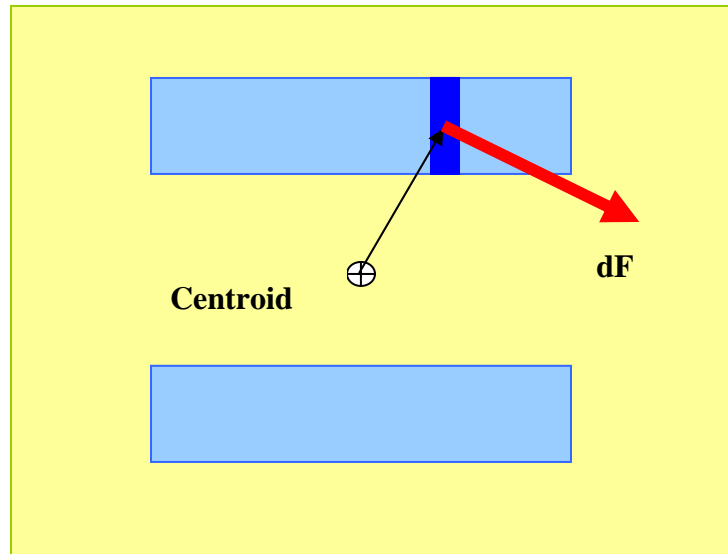


Figure 11..2.4: Forces on throat section due to torsion

### 3. Asymmetric Welded Section:

It is observed from section 1 and 2 that an eccentricity in loading causes extra shear stress in a welded joint. Thus it may be useful to reduce the eccentricity in loading. In some applications this is achieved by making the weld section asymmetric. Consider a plate subjected to an axial load  $F$  as shown in figure 11.2.5. The plate is connected to the wall by means of parallel fillet joint. Assume that the axial load is along the centroidal axis of the beam which is shown by dotted lines. If the welds are made of equal lengths in both sides, then the centroid of the welded section, being along the centerline of the beam will not lie on the centroidal axis of the beam. Thus an eccentricity in loading is introduced. This situation may be avoided by making the two weld lengths unequal in such proportion that the eccentricity is removed. The relationship between  $l_1$  and  $l_2$  will be as following:

$$\frac{l_1}{l_2} = \frac{h_2}{h_1},$$

where  $l_1$  = length of the upper weld

$l_2$  = length of the lower weld,

$h_1$  = distance of the upper weld from centroidal axis,

$h_2$  = distance of the lower weld from centroidal axis.

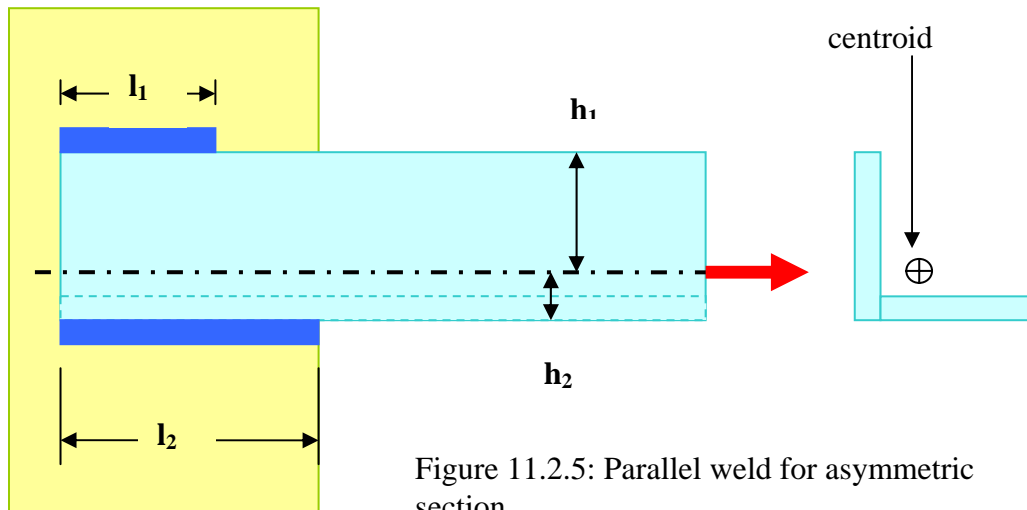


Figure 11.2.5: Parallel weld for asymmetric section

The net length of the weld  $l = l_1 + l_2$  can be calculated from the strength consideration that is

$$\frac{F}{lt} \leq S_s,$$

where  $t$  = thickness of the throat. Thus the individual lengths of the weld are as following:

$$l_1 = \left( \frac{h_2}{b} \right) l$$

and

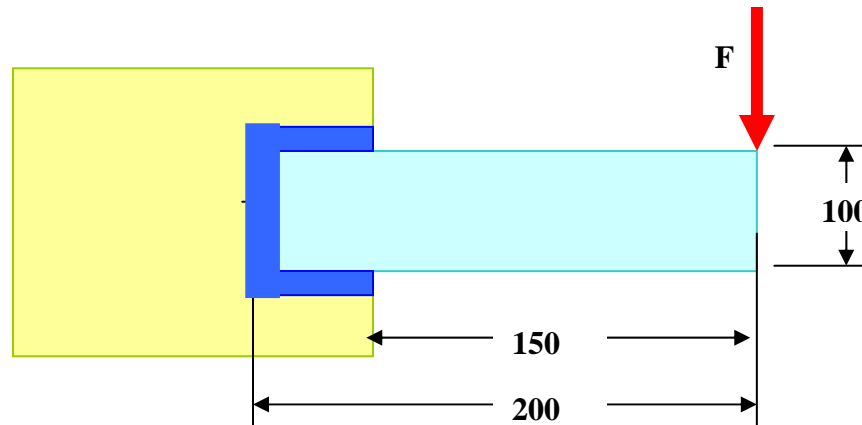
$$l_2 = \left( \frac{h_1}{b} \right) l,$$

where  $b$  = width of the plate.



## Review questions and answers:

**Q.1.** A rectangular steel plate is welded as a cantilever to a vertical column and supports a single concentrated load of 60 kN as shown in figure below. Determine the weld size if the allowable shear stress in the weld material is 140 MPa.



**Ans.** The weld is subjected to two shear stresses

- (1) Direct shear of magnitude  $60,000/\text{Area of the weld}$ . The area of the throat section is easily found out to be  $200 t$  where  $t=0.707 h$ . Thus direct shear stress is  $424/h$  MPa.
- (2) The indirect shear stress as a point  $r$  distance away from the centroid of the throat section has magnitude

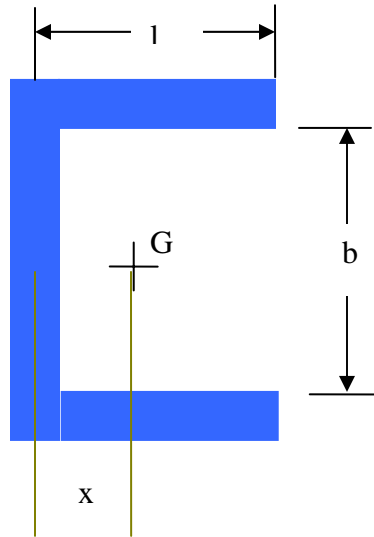
$$\tau = \frac{FLr}{J},$$

where  $J$  is the polar moment of area of the throat section and  $L$  is the eccentricity of the load. From the geometry of the throat section it may be calculated that the distance of centroid from left end =

$$x = \frac{l^2}{2l+b} = 12.5 \text{ mm (see figure below) and the polar moment about G}$$

is

$$J = \frac{h}{\sqrt{2}} \left[ \frac{(b+2l)^3}{12} - \frac{l^2(b+l)^2}{b+2l} \right] = 272530 h \text{ mm}^4.$$



Thus the indirect shear stress has magnitude  $\frac{41.28}{h}r$  MPa. The maximum resultant shear stress depends on both the magnitude and direction of the indirect shear stress. It should be clear that the maximum shear stress appears at the extreme corner of the weld section which is at a distance  $\sqrt{\left(\frac{b}{2}\right)^2 + (l-x)^2} = 62.5$  mm away from the centroid. Noticing that the included angle between the two shear forces as  $\cos^{-1}\left(\frac{l-x}{r_{\max}}\right) \approx 53.13^\circ$ , the maximum value of the resultant shear stress is found out to be  $f_{\max} = \frac{2854.62}{h}$  MPa. Since this value should not exceed 140 MPa the minimum weld size must be  $h = 20.39$  mm.